Prologue

Mobile particulate systems are encountered in various natural and industrial processes. In the broadest sense, mobile particulate systems include both suspensions and granular media. Suspensions refer to particles dispersed in a liquid or a gas. Familiar examples include aerosols such as sprays, mists, coal dust, and particulate air pollution; biological fluids such as blood; industrial fluids such as paints, ink, or emulsions in food or cosmetics. Suspension flows are also involved in numerous material processing applications, including manufacture of fiber composites and paper, and in natural processes such as sediment transport in rivers and oceans. In common usage, a suspension refers to solid particles as the dispersed state in a liquid, while an emulsion concerns liquid droplets dispersed in another immiscible fluid, and an aerosol is specific to the case of a suspension of fine solid or liquid particles in a gas. We focus on the case of a suspension in this text.

In the flow of suspensions, the viscous fluid between the particles mediates particle interactions, whereas in dry granular media the fluid between the particles is typically assumed to have a minor role, doing no more than providing a resistive drag, and this allows direct contact interactions. Familiar examples of granular media include dry powders, grains, and pills in the food, pharmaceutical, and agricultural industries; sand piles, dredging, and liquefaction of soil in civil engineering; and geophysical phenomena such as landslides, avalanches, and volcanic eruptions. However, certain situations go beyond this simple division between dry and wet granular material. For instance, the flow of dense or highly concentrated particulate media belongs to an intermediate regime between pure suspensions and granular flows.

Suspended particles can be of a wide range of sizes. In practical situations, for the particles to remain suspended for long periods of time, they will usually be smaller than approximately 100 microns in size, as larger particles settle out of suspension due to gravitational forces. (In laboratory settings the densities of solid and liquid can be closely matched to keep larger particles suspended.) The small size of the particles often means that the surrounding flow is dominated by viscous effects, and therefore that inertial forces can be neglected relative to viscous forces. Stated in dimensionless terms, this means that the particle Reynolds number, based on the particle size and the difference in velocities in the immediate neighborhood of the particle, is small. Particles smaller than one micron remain more or less permanently in suspension under gravity, owing to the influence of Brownian motion. Suspensions formed of these small particles are termed colloidal suspensions, as not only Brownian motion, but also colloidal phenomena such as van der Waals forces, electrical double layers, and capillary forces have significant effects at this scale where the surface to volume ratio is large.

A single particle moving in a fluid, e.g. a single solid particle falling in fluid under gravity forces, can be investigated by methods issuing from classical single-phase models, e.g. the Navier–Stokes equations. However, particulate suspensions refer generally to a large number of dispersed particles moving through a moving fluid and thus to a two-phase flow which presents a more intimate mixture of the two moving phases. Their behaviors therefore cannot be described in any practical sense by the classical models, but require the use of novel concepts and theories: the key problem is that of the interaction between the particle and fluid phases which occurs at a complex interface having fluctuating shape, position, and motion. Even with this complexity, suspensions of rigid particles are more easily described than many other multiphase flow systems, and this relative simplicity makes suspensions a model multiphase material for which theoretical descriptions may be developed and tested with some precision. As a result of these factors and their wide occurrence in nature and in engineering, the dynamics of particulate suspensions is a highly relevant, challenging, and largely unresolved area of fluid mechanics.

The present book aims at providing a physically based introduction to the dynamics of particulate suspensions and focuses on hydrodynamical aspects. While we may address in some cases Brownian and colloidal suspensions, this is not the central issue. The general approach is made specific through the most analytically tractable case of low-Reynolds number suspensions but goes beyond viscous suspensions. The goal is not to present the subject as closed but instead to present a selection of well-understood problems as an entry to the study of the many open questions in the field of particulate flows.

The reader is assumed to have completed a course in fluid mechanics or continuum mechanics at the graduate level. This means the reader will be familiar with both invariant vector and tensor notation, as well as index notation. This would imply familiarity with establishing boundary-value problems for the Navier–Stokes equations. Physical content includes the concepts of viscous and inertial forces/stresses, and the related concept of dynamical scaling (Reynolds number, for example), and the assumptions necessary to be in the Stokes regime.

The book is composed of two primary parts, separated by an interlude to discuss statistical techniques needed to employ the results of the first part in the second, followed by an epilogue.

Part I: Microhydrodynamics This part of the book presents the well-developed theory of particles in viscous fluids. The microscopic treatment considers only single- and pair-body dynamics. The philosophy is to introduce the theoretical concepts with the least mathematical burden and to capture their physical meaning through examples and a few additional exercises at the ends of chapters. A brief overview of the contents of Part I follows:

1. The book begins with a review of the Stokes-flow regime, justified through the smallness of the particles and the dominance of viscous effects. Symmetry, superposition, and reversibility properties of the motion are developed through examples.
2. The presentation then considers the flows associated with a single body in viscous fluid, emphasizing through elementary solutions the structure of the flow solutions for basic situations of translation, rotation, and straining around spheres. The long-range nature of the decay associated with the flow is demonstrated mathematically and given physical meaning.
3. More sophisticated solution techniques which allow generalization of the concepts of the single-body flows are then developed. These techniques give access to presently used approaches underlying numerical techniques, including the resistance/mobility matrix formulation, integral representations, and slender-body theory.
4. The presentation then addresses interactions of pairs of spheres, considering the near- and far-field cases. Lubrication interactions and the method of reflections are developed. The latter considers worked

examples to demonstrate the physical effects of long-range flow on particle interaction. The connection of these techniques to the form of the resistance and mobility tensors is made and used to motivate the simulation approach of Stokesian Dynamics, which is presented in an abbreviated format.

Interlude: From the microscopic to the macroscopic The microscopic hydrodynamics presented in Part I will be used to develop predictive models for large collections of particles in Part II. This is properly a problem of statistical mechanical theory. This interlude provides the basic statistical and stochastic concepts employed in Part II.

Part II: Toward a description of macroscopic phenomena in suspensions This part combines the microscopic theory of Part I and the statistical concepts of the Interlude as the foundation for considering the behavior of large assemblies of hydrodynamically interacting particles. Note that Part I treats basic and by now relatively classic material, whereas Part II treats subjects which are still in development and thus is more tentative. In addition, while the fluid mechanics of a single body or of pairs of particles is mathematically linear and thus can be fully developed, many-body dynamics is manifestly nonlinear and irreversible. Part II of this book presents examples and seeks to illustrate the consequences and (to a lesser degree because it is not a fully understood topic) the basis of this nonlinearity.

There are two basic cases: sedimentation and shear flow. In each case, the coupling between microstructure and the bulk collective phenomena is a central theme. While sedimentation and shear often occur together, here they are treated as distinct in order to highlight their essential features. For sedimentation, these include the dominant effects of slip between the particle and fluid phases and the resulting hydrodynamic force on each particle, giving rise to extremely long-range interactions and surprising spatial correlations of motions. For shear flow, the dominant effect of close pair interactions driven by the flow and the resulting spatial correlations are shown to give rise to non-Newtonian stresses and irreversible migration.

This section of the book thus contains:

1. A chapter that presents established results and open questions in the area of sedimentation. It starts by showing that as soon as three or more particles are involved the system can become chaotic. But

for a larger number of particles, coherent structure and collective motion are observed. The mean sedimentation rate as a function of solid fraction and the behavior of the concentration fronts are then discussed. We also briefly discuss the sedimentation of polydisperse spheres and non-spherical particles such as fibers, both of which can differ qualitatively from the case of monodisperse spheres.

2. A chapter devoted to shear flows of suspensions describes the observed rheology of these materials and develops ideas necessary to describe the rheology based on a microstructural understanding. The impact of rheological properties on the bulk fluid mechanics of suspensions is described. The rheological behavior of orientable particles, with fibers the primary example, is also briefly considered.
3. This part ends with a chapter that goes beyond Stokes flow and considers the role of inertia at small but finite particle-scale Reynolds number. The topic is much less complete than the preceding areas, and in particular the mathematical aspects are presented as a sketch. To connect to the earlier material on sedimentation, we consider the wake interactions of falling particles. To connect to that on shear flow, we consider the tubular-pinch migration phenomenon and inertially influenced interactions.

The book ends with an epilogue where we point out some of the open issues in the current research on particulate flows.

PART I

MICROHYDRODYNAMICS

1

Basic concepts in viscous flow

In general, we shall be interested in the motion of a mixture composed of particles in viscous liquid, as illustrated in Figure 1.1. For many cases of interest, the particles are quite small and/or the fluid is viscous and therefore we are in the realm of *microhydrodynamics*, a term coined by G. K. Batchelor in the 1970s. Under these conditions, it is often legitimate to reduce the Navier–Stokes equations to the Stokes equations; in other words, inertia in the flow is negligible relative to viscous effects. The value of this reduction is that it provides a simplification of the fluid-mechanical description as the Stokes equations are linear. Consequently, the mathematical solutions are analytically derivable for a number of basic but important situations. In this chapter we will show under which conditions this approximation is reached and provide a description of the properties of solutions to the Stokes equations.

1.1 The fluid dynamic equations

Consider Figure 1.1 showing flow past particles. For simplicity, assume the particles to be solid bodies idealized as non-deformable (rigid). We consider the particle dynamics later. Here, we address the continuous fluid, whose motion is governed by the Navier–Stokes equations, i.e. the continuity equation for an incompressible fluid,

$$\nabla \cdot \mathbf{u} = 0, \tag{1.1}$$

and the equation for conservation of momentum,

$$\begin{aligned} \rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right] &= \mathbf{f} + \nabla \cdot \boldsymbol{\sigma}^a \\ &= \mathbf{f} - \nabla p^a + \mu \nabla^2 \mathbf{u}, \end{aligned} \tag{1.2}$$

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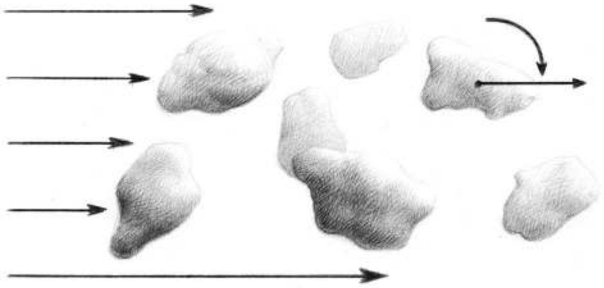

Figure 1.1 Many particles in a flow.

where $\mathbf{f}$ is the external body force per unit volume, the dynamic viscosity is μ, and the constant density is ρ. The superscript a indicates an absolute pressure and a corresponding absolute stress tensor. The term "absolute stress" is used to indicate the actual stress (with the absolute pressure being the true pressure) rather than a modified stress to be defined below, in which the hydrostatic stress field is removed. In the last equality, we assume the constitutive equation for a Newtonian fluid which implies the symmetric stress tensor $\boldsymbol{\sigma}^a$ is given in Einstein notation[1] by

$$\sigma^a_{ij} = \sigma^a_{ji} = -p^a\delta_{ij} + 2\mu e_{ij}, \tag{1.3}$$

[1] In index notation, one writes vectors and tensors using indices, so that a vector $\mathbf{a}$ is expressed as its component a_i, and a second-rank tensor $\mathbf{T}$ by its component T_{ij}, where the indices i and j take on values of 1, 2, or 3 in three dimensions. For a thorough discussion, see the book by Aris (1962). The method is implicitly applied in a Cartesian coordinate frame. Note that results of calculations by the method may be translated to the invariant vector notation (i.e. where a vector is expressed as $\mathbf{a}$).
When using index notation for calculations, the Einstein summation convention is often used. This convention implies summation over repeated indices within a product expression. Thus, the dot product, $\mathbf{a}\cdot\mathbf{a}$, in index notation using the Einstein convention is written simply as $a_ia_i = a_1^2 + a_2^2 + a_3^2$. We could equally as well have written a_ja_j, as the repeated index is a dummy. An index may not be repeated three or more times within a product, as the meaning of such an expression is ambiguous.
Considering quantities arising in fluid mechanics, the divergence of the velocity is a scalar quantity expressed in the Einstein notation as

$$\nabla\cdot\mathbf{u} = \frac{\partial u_i}{\partial x_i} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}.$$

The convective derivative of a vector, $(\mathbf{u}\cdot\nabla)\mathbf{v}$, yields a vector whose ith component is $u_k\partial v_i/\partial x_k$.

where p^a is the absolute pressure, defined as $p^a = (-1/3)\sigma^a_{ii}$, and the rate-of-strain tensor $\mathbf{e}$ is defined as

$$e_{ij} = e_{ji} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{1.4}$$

The quantity δ_{ij} appearing in (1.3) is called the Kronecker delta.[2]

The incompressibility condition (1.1) can also be expressed as

$$e_{kk} = 0. \tag{1.5}$$

As noted, we will consider the Newtonian dynamics of the particles in detail later. Here, the influence of the particles arises from the boundary conditions which they impose upon the fluid motion. It is, of course, necessary to apply conditions at the outer boundary of the domain of interest, whether on a containing vessel or at infinity, but here our interest is in the boundary conditions on the particles. The usual condition is that of no slip, meaning the velocity of the fluid at a point in contact with a particle surface is the same as the particle velocity at this point. This condition may be written at the surface of a particle, with center of mass at $\mathbf{x}_p$, as

$$\mathbf{u}(\mathbf{x}) = \mathbf{U}^{\mathrm{p}} + \boldsymbol{\omega}^{\mathrm{p}} \times (\mathbf{x} - \mathbf{x}_p), \tag{1.6}$$

where $\mathbf{U}^{\mathrm{p}}$ is the translational velocity and $\boldsymbol{\omega}^{\mathrm{p}}$ is the rotational velocity of the particle. For many particles as shown in Figure 1.1, this condition must be written for each particle, and because the particles are mobile, we face a complicated time-dependent boundary-value problem. For the moment we restrict ourselves to the single-body problem, as illustrated in Figure 1.2 for a sphere. Furthermore, we will suppose that viscous effects dominate the fluid dynamics, which will reduce the Navier–Stokes equations to the Stokes equations. This is justified below by a scaling argument.

[2] The Kronecker delta δ_{ij} is defined by

$$\delta_{ij} = \begin{cases} 0 & \text{if} \quad i \neq j, \\ 1 & \text{if} \quad i = j. \end{cases}$$

The Kronecker delta serves as an identity in index notation, in the sense that $\delta_{ij}x_j = x_i$ or $\delta_{ij}\delta_{jk} = \delta_{ik}$. The expression $\delta_{ij}x_j = x_i$ is equivalent to $\mathbf{I} \cdot \mathbf{x} = \mathbf{x}$ in invariant notation using the unit second-rank, or identity, tensor $\mathbf{I}$. Hence, the Kronecker delta is the identity matrix in matrix–vector calculations. In solving problems, it is useful to note that the trace of the Kronecker delta is $\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 3$ when used in a three-dimensional problem, and more generally $\delta_{ii} = d$ where d is the dimensionality of the problem.

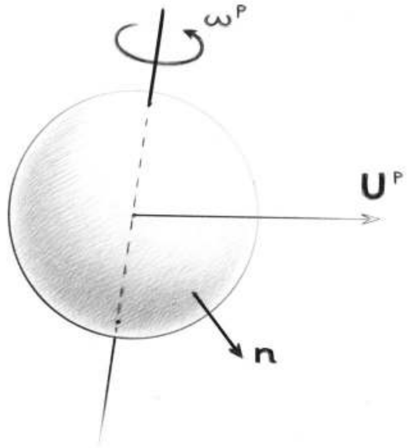

Figure 1.2 One sphere translating and rotating.

1.2 Scaling arguments and the Stokes approximation

The importance of inertial effects compared to viscous effects in equation (1.2) is measured by the Reynolds number. Suppose the sphere, of radius a, translates with a velocity of magnitude U. Then the Reynolds number at the particle scale is

$$Re = \frac{Ua}{\nu} \sim \frac{|(\mathbf{u}\cdot\nabla)\mathbf{u}|}{|\nu\nabla^2\mathbf{u}|}, \qquad (1.7)$$

where $\nu = \mu/\rho$ is the kinematic viscosity.

For suspensions, recall that we are usually interested in small length-scales, typically between 10^{-2} and 10^2 μm. As a result of the smallness of particles, the velocity scale is often small, for example in sedimentation, where the isolated particle settling velocity scales with the square of its size, as we will see later. For a grain of sand of size $a = 1\,\mu\mathrm{m}$, the settling velocity in water is of the order of $U = 1\,\mu\mathrm{m\,s^{-1}}$ and the Reynolds number of the motion is thus of $O(10^{-6})$. Therefore, in many practical flows of suspensions, the Reynolds number is small and we may neglect the convective acceleration in the left-hand side of equation (1.2). Some care must be taken in neglecting the convective acceleration term, as this scaling argument fails at distances far from the particle, i.e. when