

# Index

- absolute convergence of series, 225
- acceleration vector, 449
- addition rule for probabilities, 603, 608
- adjoint, *see* Hermitian conjugate
- algebra of
  - complex numbers, 177–8
  - matrices, 378–9
  - power series, 236
  - series, 233
  - unions and intersections, 601
  - vectors, 332–3
    - in a vector space, 369–70
    - in component form, 337–8
- alternating series test, 231
- Ampère's rule (law), 495, 525
- angle between two vectors, 341
- angular momentum
  - of particles, 451
  - of solid body, 512
  - vector representation, 365
- angular velocity, vector representation, 344, 365, 464
- anti-Hermitian matrices, 410
  - eigenvalues, 414
  - eigenvectors, 414
- anticommutativity of vector or cross product, 342
- antisymmetric functions, 126
- antisymmetric matrices, 409
  - general properties, *see* anti-Hermitian matrices
- approximately equal  $\approx$ , definition, 234
- arbitrary parameters for ODE, 555
- arc length of plane curve, 162–4
- arccosech, arccosh, arccoth, arcsech, arcsinh, arctanh, *see* hyperbolic functions, inverses
- Archimedean upthrust, 511, 530
- area element in
  - Cartesian coordinates, 301
  - plane polars, 318
- area of
  - circle, 159
  - ellipse, 159–60, 325
  - parallelogram, 343
  - region, as a line integral, 500
  - region, using multiple integrals, 306–8
  - surfaces, 457
    - as vector, 508–10, 525
- arg, argument of a complex number, 178–9
- Argand diagram, 175
- arithmetic mean, 35
- arithmetic series, 215
- arithmetico-geometric series, 217
- arrays, *see* matrices
- associative law for
  - addition
    - in a vector space of finite dimensionality, 369–70
    - of complex numbers, 177
    - of matrices, 378
    - of vectors, 332
  - convolution, 548
  - linear operators, 375
  - multiplication
    - of a matrix by a scalar, 378
    - of a vector by a scalar, 333–4
    - of complex numbers, 180
    - of matrices, 380
    - multiplication by a scalar
      - in a vector space of finite dimensionality, 370
- associativity, 2
- asymptotes, 126
- atomic orbitals, *s*-states, 624
- auxiliary equation, 572
  - repeated roots, 573
- average value, *see* mean value
- base
  - for logarithms, 9
  - for natural logarithms, 10, 673
- basis vectors, 336–8, 370–1
  - linear independence, 337
  - non-orthogonal, 372
  - orthonormal, 372
  - required properties, 337
- Bayes' theorem, 610–12
- Bernoulli equation, 563
- Bessel inequality, 373

- complex exponential function, 185

- complex logarithms, 194–6
  - principal value of, 194
- complex numbers, 174–212
  - addition and subtraction of, 177–8
  - applications to differentiation and integration, 196–7
  - argument of, 178–9
  - associativity of
    - addition and subtraction, 177
    - multiplication, 180
  - commutativity of
    - addition, 177
    - multiplication, 180
  - complex conjugate of, *see* complex conjugate
  - components of, 175
  - de Moivre's theorem, *see* de Moivre's theorem
  - division of, 184, 187
  - from roots of polynomial equations, 174
  - imaginary part of, 174–5
  - modulus of, 178–9
  - multiplication of, 179–81, 187–8
    - as rotation in the Argand diagram, 180–1
  - notation, 174–5
  - phase, 178n
  - polar representation of, 185–8
  - real part of, 174–5
  - trigonometric representation of, 186
- complex power series, 235
- complex powers, 194–6
- components
  - of a complex number, 175
  - of a vector, 336–8
  - in a non-orthogonal basis, 358–9
  - uniqueness, 371
- compound-angle identities, 28–32
- conditional convergence, 225
- conditional probability, *see* probability, conditional
- cone
  - surface area of, 165–6
  - volume of, 167
- conic sections, 65
  - eccentricity, 67
  - parametric forms, 70
  - standard forms, 66
- conjugate roots of polynomial equations, 193
- connectivity of regions, 497
- conservative fields, 502–4
  - necessary and sufficient conditions, 502–4
  - potential (function), 504
- constant coefficients in ODE, 572–9
  - auxiliary equation, 572
- constants of integration, 145, 554
- constraints, stationary values under, *see* Lagrange undetermined multipliers
- continuity correction for discrete RV, 650
- continuity equation, 521
- contradiction, proof by, 87–8
- convergence of infinite series
  - absolute, 225
  - complex power series, 235
  - conditional, 225
  - necessary condition, 226
  - power series, 234
    - under various manipulations, *see* power series, manipulation
- rearrangement of terms, 225
- tests for convergence, 226–33
  - alternating series test, 231
  - comparison test, 226
  - grouping terms, 230
  - integral test, 229
  - quotient test, 228
  - ratio comparison test, 228
  - ratio test (D'Alembert), 227, 234
  - root test (Cauchy), 230
- convolution
  - Laplace transforms, *see* Laplace transforms, convolution
- convolution theorem for Laplace transforms, 547
- coordinate geometry, 64–73
  - conic sections, 65
  - straight line, 64
- coordinate systems, *see* Cartesian, curvilinear, cylindrical polar, plane polar *and* spherical polar coordinates
- coordinate transformations
  - and integrals, *see* change of variables
  - and matrices, *see* similarity transformations
- coplanar vectors, 347
- correlation of bivariate distributions, 657–60
- correspondence principle in quantum mechanics, 668
- cosh,  $\cosh(x)$ , *see also* hyperbolic functions
  - hyperbolic cosine, 198
  - Maclaurin series for, 244
- cosine,  $\cos(x)$ 
  - geometrical and algebraic definitions, 676
  - geometrical definition, 25
  - in terms of exponential functions, 199
  - Maclaurin series for, 243
  - reciprocal and inverse, 27
- covariance of bivariate distributions, 657–60
- Cramer determinant, 405
- Cramer's rule, 404–6

- cross product, *see* vector product
- crystal lattice, 256
- cube roots of unity, 191
- curl of a vector field, 463
  - as a determinant, 464
  - as integral, 514, 516
  - curl curl, 468
  - in curvilinear coordinates, 480
  - in cylindrical polars, 472
  - in spherical polars, 475
  - Stokes' theorem, 523–6
- curvature, 116–19
  - circle of, 117
  - of a function, 116
  - radius of, 117
- curves, *see* plane curves
- curvilinear coordinates, 476–81
  - basis vectors, 477
  - length and volume elements, 478
  - scale factors, 477
  - surfaces and curves, 477
  - vector operators, 479–81
- cyclic relation for partial derivatives, 266
- cycloid, 485
- cylindrical polar coordinates, 469–73
  - area element, 471
  - basis vectors, 470
  - length element, 471
  - vector operators, 470–3
  - volume element, 471
- $\delta$ -function (Dirac), *see* Dirac  $\delta$ -function
- $\delta_{ij}$ , *see* Kronecker delta,  $\delta_{ij}$
- D'Alembert's ratio test, 227
  - in convergence of power series, 234
- damped harmonic oscillators, 366
- de Moivre's theorem, 189
  - applications, 189–93
    - finding the  $n$ th roots of unity, 191–2
    - solving polynomial equations, 192–3
    - trigonometric identities, 189–91
- de Morgan's laws, 602
- defective matrices, 416, 442
- degenerate eigenvalues, 415, 420–1
- degree
  - of ODE, 554
  - of polynomial equation, 53
- del  $\nabla$ , *see* gradient operator (grad)
- del squared  $\nabla^2$  (Laplacian), 463
  - as integral, 517
  - in curvilinear coordinates, 480
  - in cylindrical polar coordinates, 472
  - in spherical polar coordinates, 475
- del squared  $\nabla^2$  (Laplacian), 269
- delta function (Dirac), *see* Dirac  $\delta$ -function
- dependent random variables, 655–60
- derivative, *see also* differentiation
  - Laplace transform of, 544
  - normal, 461
  - of basis vectors, 450–1
  - of composite vector expressions, 451–2
  - of function of a function, 108–9
  - of hyperbolic functions, 202–5
  - of products, 106–8, 112–13
  - of quotients, 109–10
  - of simple functions, 106
  - of vectors, 448
  - ordinary, first, second and  $n$ th, 103–5
  - partial, *see* partial differentiation
  - total, 263
- determinant form for curl, 464
- determinants, 386–91
  - adding rows or columns, 390
  - and singular matrices, 392
  - as product of eigenvalues, 426
  - evaluation using Laplace expansion, 387
  - identical rows or columns, 390
  - in terms of cofactors, 387
  - interchanging two rows or columns, 390
  - Jacobian representation, 317, 321, 323
  - notation, 386
  - of Hermitian conjugate matrices, 389
  - of order three, in components, 388
  - of transpose matrices, 389
  - product rule, 390
  - properties, 389–91, 444
  - relationship with rank, 396–7
  - removing factors, 390
  - secular, 418
- diagonal matrices, 408
- diagonalisation of matrices, 424–6
  - normal matrices, 425–6
  - properties of eigenvalues, 426
- diamond, unit cell, 361
- dice throwing, 604, 627, 633, 658
- die throwing, *see* probability
- difference method for summation of series, 217–19
- differentiable function of a real variable, 103
- differential
  - definition, 104
  - exact and inexact, 264–5
  - of vector, 452, 455
  - total, 262
- differential equations, *see* ordinary differential equations

- differential equations, particular
  - Bernoulli, 563
- differentiation, *see also* derivative
  - as gradient, 103
  - as rate of change, 102
  - chain rule, 108–9
  - from first principles, 102–6
  - implicit, 110–11
  - logarithmic, 111
  - notation, 105
  - of integrals, 288–90
  - of power series, 237
  - partial, *see* partial differentiation
  - product rule, 106–8, 112–13
  - quotient rule, 109–10
  - theorems, 120–1
  - using complex numbers, 196
- dimensionality of vector space, 370
- dimensions, physical, 15–19
  - dimensional analysis, 16
- dipole matrix elements, 326
- Dirac  $\delta$ -function, 521, 541–4
  - definition, 541
  - impulses, 542
  - point charges, 543
  - properties, 541
  - relation to Heaviside (unit step) function, 543
  - three-dimensional, 543
- direction cosines, 342
- directrix, of a conic section, 66
- disc, moment of inertia, 327
- disjoint events, *see* mutually exclusive events
- distance from a
  - line to a line, 355–6
  - line to a plane, 356–7
  - point to a line, 353–4
  - point to a plane, 354–5
- distributive law for
  - addition of matrix products, 381
  - convolution, 548
  - inner product, 371
  - linear operators, 375
  - multiplication
    - of a matrix by a scalar, 378
    - of a vector by a complex scalar, 342
    - of a vector by a scalar, 333–4
  - multiplication by a scalar
    - in a vector space of finite dimensionality, 370
  - scalar or dot product, 340
  - vector or cross product, 342
- divergence of vector fields, 462
  - as integral, 514, 515
  - in curvilinear coordinates, 479
  - in cylindrical polars, 472
  - in spherical polars, 475
- divergence theorem
  - for vectors, 517–18
  - in two dimensions, 499
  - physical applications, 520–2
  - related theorems, 519
- division of complex numbers, 184
- dot product, *see* scalar product
- double integrals, *see* multiple integrals
- double-angle identities, 30
- dummy variable, 142
- $\epsilon_{ijk}$ , *see* Levi-Civita symbol,  $\epsilon_{ijk}$
- $e^x$ , *see* exponential function
- eccentricity, of conic sections, 67
- eigenvalues, 412–21
  - characteristic equation, 418
  - definition, 412
  - degenerate, 420–1
  - determination, 418–21
  - notation, 413
  - of anti-Hermitian matrices, *see* anti-Hermitian matrices
  - of general square matrices, 415–16
  - of Hermitian matrices, *see* Hermitian matrices
  - of linear operators, 412
  - of unitary matrices, 415
  - under similarity transformation, 426
- eigenvectors, 412–21
  - characteristic equation, 418
  - definition, 412
  - determination, 418–21
  - normalisation condition, 413
  - notation, 413
  - of anti-Hermitian matrices, *see* anti-Hermitian matrices
  - of commuting matrices, 416
  - of general square matrices, 415–16
  - of Hermitian matrices, *see* Hermitian matrices
  - of linear operators, 412
  - of unitary matrices, 415
  - stationary properties for quadratic and Hermitian forms, 429–30
- electromagnetic fields
  - flux, 510
  - Maxwell's equations, 488, 525
- ellipse
  - area of, 159–60, 325, 500
  - as section of quadratic surface, 431
  - equation for, 66
- ellipsoid, volume of, 326
- empty event  $\emptyset$ , 599

- end-points, of a range, 33
- envelopes, 282–4
  - equations of, 282
  - to a family of curves, 282
- equating real and imaginary parts, 176
- equivalence transformations, *see* similarity transformations
- error terms in Taylor series, 242–3
- Euler equation, trigonometric, 186
- even functions, *see* symmetric functions
- events, 597
  - complement of, 599
  - empty  $\emptyset$ , 599
  - intersection of  $\cap$ , 598
  - mutually exclusive, 607
  - statistically independent, 608
  - union of  $\cup$ , 599
- exact differentials, 264–5
- exact equations, 558
  - condition for, 558
- expectation values, *see* probability distributions, mean
- exponent, of a power, 1
- exponential distribution, 653
  - from Poisson, 653
- exponential function
  - and a general power, 4
  - and logarithms, 7
  - and the natural logarithmic base, 9, 673
  - definition, 10
  - equivalence of  $\exp(x)$  and  $e^x$ , 673
  - from the binomial expansion, 23
  - Maclaurin series for, 243
  - of a complex variable, 185
  - properties, 11
  - relation with hyperbolic functions, 198
- Fabry–Pérot interferometer, 254
- factorial function for a positive integer, 10
- factorisation, of a polynomial equation, 60
- Fibonacci series, 594
- fields
  - conservative, 502–4
  - scalar, 458
  - vector, 458
- first law of thermodynamics, 285
- first-order differential equations, *see* ordinary differential equations
- fluids
  - Archimedean upthrust, 511, 530
  - continuity equation, 521
  - flux, 510
  - irrotational flow, 464
  - sources and sinks, 521–2
  - velocity potential, 526
  - vortex flow, 526
- focus, of a conic section, 66
- function of a matrix, 382
- functions of one real variable
  - differentiation of, 102–13
  - integration of, 141–61
  - limits, *see* limits
  - maxima and minima of, 114–15
  - stationary values of, 114–15
  - Taylor series, *see* Taylor series
- functions of several real variables
  - chain rule, 267–8
  - differentiation of, 259–90
  - integration of, *see* multiple integrals, evaluation
  - rates of change, 261–3
  - Taylor series, 270–1
- functions of two real variables
  - maxima and minima, 272–5
  - points of inflection, 272–5
  - saddle points, 272–5
  - stationary values, 272–5
- fundamental theorem of
  - algebra, 174, 175
  - calculus, 143–5
  - complex numbers, *see* de Moivre's theorem
- Gaussian (normal) distribution  $N(\mu, \sigma^2)$ , 645–53
  - and binomial distribution, 649
  - and central limit theorem, 651
  - and Poisson distribution, 651
  - continuity correction, 650
  - cumulative probability function, 646
    - tabulation, 648
  - integration with infinite limits, 318–20
  - mean and variance, 645–9
  - multiple, 652–3
  - sigma limits, 647
  - standard variable, 645
- Gaussian elimination with interchange, 399–401
- geometric distribution, 636
- geometric mean, 35
- geometric series, 215
- Gibbs' free energy, 287
- golden mean, 594
- gradient of a function of
  - one variable, 103
  - several real variables, 261–3
- gradient of scalar, 459–62

- gradient operator (grad), 458
  - as integral, 514
  - in curvilinear coordinates, 479
  - in cylindrical polars, 472
  - in spherical polars, 475
- graph papers, logarithmic, 65
- graphs, 124–32
  - and approximate solutions, 52
  - general considerations, 125
  - horizontal asymptote, 127
  - vertical asymptote, 126
  - worked examples, 127–32
- gravitation, Newton's law, 453
- Green's function for ODE, 298
- Green's theorems
  - in a plane, 498–501, 524
  - in three dimensions, 518
- grouping terms as a test for convergence, 230
- half-angle identities, 31, 150
- harmonic oscillators, damped, 366
- Heaviside function, 543
  - relation to Dirac  $\delta$ -function, 543
- Helmholtz potential, 287
- hemisphere, centre of mass and centroid, 309
- Hermitian conjugate, 384–5
  - and inner product, 385
  - product rule, 384
- Hermitian forms, 427–31
  - positive definite and semi-definite, 428–9
  - stationary properties of eigenvectors, 429–30
- Hermitian matrices, 410
  - eigenvalues, 414–15
  - reality, 414
  - eigenvectors, 414–15
  - orthogonality, 414–15
- higher order differential equations, *see* ordinary differential equations
- homogeneous
  - differential equations, 569
  - dimensionally consistent, 562–3
  - simultaneous linear equations, 397
- hydrogen atom
  - $s$ -states, 624
- hydrogen atom, electron wavefunction, 326
- hyperbola
  - as section of quadratic surface, 431
  - equation for, 66
- hyperbolic functions, 197–205
  - calculus of, 202–5
  - definitions, 198
  - graphs, 198
  - identities, 200
  - in equations, 200–1
  - inverses, 201–2
    - graphs, 202
  - trigonometric analogies, 199–200
- hypergeometric distribution, 637–9
  - mean and variance, 638
- i, j, k** (unit vectors), 338
- $i$ , square root of  $-1$ , 175
- identity matrices, 381, 382
- identity operator, 375
- imaginary part or term of a complex number, 174–5
- improper integrals, 156
- impulses,  $\delta$ -function representation, 542
- independent random variables, 631, 658
- index, of a power, 1
- induction, proof by, 85–6
- inequalities
  - algebraic, 32–9
  - amongst integrals, 160
  - Bessel, 373
  - Schwarz, 373
  - triangle, 373
- inexact differentials, 264–5
- inexact equation, 559
- infinite integrals, 156
- infinite series, *see* series
- inflection
  - general points of, 115
  - stationary points of, 114–15
- inhomogeneous
  - differential equations, 569
  - simultaneous linear equations, 397
- inner product in a vector space, *see also* scalar product
  - of finite dimensionality, 371–2
  - and Hermitian conjugate, 385
  - commutativity, 371
  - distributivity over addition, 371
- integral test for convergence of series, 229
- integrals, *see also* integration
  - definite, 141
  - double, *see* multiple integrals
  - improper, 156
  - indefinite, 145
  - inequalities, 160
  - infinite, 156
  - Laplace transform of, 545
  - limits
    - containing variables, 302
    - fixed, 141
    - variable, 143



- line, *see* line integrals
- multiple, *see* multiple integrals
- of vectors, 453
- properties, 143
- triple, *see* multiple integrals
- undefined, 142
- integrand, 141
- integrating factor (IF)
  - first-order ODE, 559–62
- integration, *see also* integrals
  - applications, 161–7
    - finding the length of a curve, 162–4
    - mean value of a function, 161–2
    - surfaces of revolution, 164–6
    - volumes of revolution, 166–7
  - as area under a curve, 141–2
  - as the inverse of differentiation, 143–5
  - formal definition, 142
  - from first principles, 142–3
  - in plane polar coordinates, 159–60
  - logarithmic, 148
  - multiple, *see* multiple integrals
  - of functions of several real variables, *see* multiple integrals
  - of hyperbolic functions, 202–5
  - of power series, 237
  - of simple functions, 146
  - of singular functions, 156
  - of sinusoidal functions, 146–8
- integration constant, 145
- integration, methods for
  - by inspection, 146
  - by parts, 152–4
  - by substitution, 149–52
    - $t$  substitution, 150–1
  - change of variables, *see* change of variables
  - completing the square, 151–2
  - partial fractions, 148–9
  - reduction formulae, 155–6
  - trigonometric expansions, 146–8
  - using complex numbers, 196–7
- intersection  $\cap$ 
  - algebra of, 601
- intervals, open and closed, 33
- inverse hyperbolic functions, 201–2
- inverse Laplace transforms, 539
  - uniqueness, 539
- inverse matrices, 392–4
  - elements, 392
  - in solution of simultaneous linear equations, 401
  - left and right, 392n
  - product rule, 394
  - properties, 394
- inverse of a linear operator, 375
- irrotational vectors, 464
- isotope decay, 587, 593
- $j$ , square root of  $-1$ , 175
- Jacobians
  - analogy with derivatives, 323
  - and change of variables, 322–3
  - definition in
    - two dimensions, 316
    - three dimensions, 321
  - general properties, 322–3
  - in terms of a determinant, 317, 321, 323
- joint distributions, *see* bivariate distributions *and* multivariate distributions
- Kronecker delta,  $\delta_{ij}$ , 682
  - and orthogonality, 372
- L'Hôpital's rule, 246–8
- Lagrange undetermined multipliers, 276–81
  - and stationary properties of the eigenvectors of quadratic forms, 429
  - for functions of more than two variables, 277–81
  - in deriving the Boltzmann distribution, 280–1
  - with several constraints, 277–81
- Lagrange's identity, 345
- lamina: mass, centre of mass and centroid, 308–9
- Laplace expansion, 387
- Laplace transforms, 536–49
  - convolution
    - associativity, commutativity, distributivity, 548
    - definition, 547
  - convolution theorem, 547
  - definition, 538
  - for ODE with constant coefficients, 576–9
  - inverse, 539
    - uniqueness, 539
  - properties: translation, exponential multiplication, etc., 546
  - table for common functions, 540
- Laplace transforms, examples
  - constant, 538
  - derivatives, 544
  - exponential function, 538
  - integrals, 545
  - polynomial, 538
- Laplacian, *see* del squared  $\nabla^2$  (Laplacian)
- Leibnitz's rule for differentiation of integrals, 289
- Leibnitz's theorem, 112–13, 679



- length of
  - a vector, 338
  - plane curves, 162–4
- Levi-Civita symbol,  $\epsilon_{ijk}$ , 681
- limits, 244–8
  - definition, 245
  - L'Hôpital's rule, 246–8
  - of functions containing exponents, 246
  - of integrals, 141
    - containing variables, 302
  - of products, 245
  - of quotients, 245, 246–8
  - of sums, 245
- line integrals
  - and Stokes' theorem, 523–6
  - of scalars, 491–501
  - of vectors, 491–504
  - physical examples, 495
  - round closed loop, 501
- line, vector equation of, 349
- linear dependence and independence
  - definition in a vector space, 370
  - of basis vectors, 337
  - relationship with rank, 395–6
- linear equations, differential
  - first-order ODE, 561
  - general ODE, 569–79
  - ODE with constant coefficients, 572–9
- linear equations, simultaneous, *see* simultaneous linear equations
- linear independence of functions, 570
  - Wronskian test, 570
- linear operators, 374–5
  - associativity, 375
  - distributivity over addition, 375
  - eigenvalues and eigenvectors, 412
  - in a particular basis, 374–5
  - inverse, 375
  - non-commutativity, 375
  - particular: identity, null or zero, singular and non-singular, 375
  - properties, 375
- linear vector spaces, *see* vector spaces
- $\text{Ln}$  of a complex number, 194–6
- $\ln$  (natural logarithm)
  - choice of base, 673
  - Maclaurin series for, 244
  - of a complex number, 194–6
- logarithmic graph papers, 65
- logarithms, 7–14
  - and data analysis, 12
  - and practical calculations, 8, 12
  - and the value of  $0^0$ , 14
- choice of base, 9
- definition, 8
- nomenclature, 9
- properties, 9
- lottery (UK), and hypergeometric distribution, 638
- lower triangular matrices, 408
- $LU$  decomposition, 401–4
- Maclaurin series, 240
  - standard expressions, 243
- Madelung constant, 256
- magnetic dipole, 340
- magnitude of a vector, 338
  - in terms of scalar or dot product, 342
- mass of non-uniform bodies, 308
- matrices, 369–431
  - as a vector space, 379
  - as arrays of numbers, 376
  - as representation of a linear operator, 376
  - column, 376
  - elements, 376
    - minors and cofactors, 387
  - identity or unit, 381
  - row, 376
  - zero or null, 381
- matrices, algebra of
  - addition, 378–9
  - change of basis, 421–4
  - Cholesky decomposition, 404, 441
  - diagonalisation, *see* diagonalisation of matrices
  - $LU$  decomposition, 401–4
  - multiplication, 379–81
    - and common eigenvalues, 416
    - commutator, 439
    - non-commutativity, 381
  - multiplication by a scalar, 378–9
  - similarity transformations, *see* similarity transformations
  - simultaneous linear equations, *see* simultaneous linear equations
  - subtraction, 378
- matrices, derived
  - adjoint, 384–5
  - complex conjugate, 384–5
  - Hermitian conjugate, 384–5
  - inverse, *see* inverse matrices
  - table of, 433
  - transpose, 376
- matrices, properties of
  - anti- or skew-symmetric, 409
  - anti-Hermitian, *see* anti-Hermitian matrices
  - determinant, *see* determinants

- diagonal, 408
- eigenvalues, *see* eigenvalues
- eigenvectors, *see* eigenvectors
- Hermitian, *see* Hermitian matrices
- normal, *see* normal matrices
- order, 376
- orthogonal, 410
- rank, 395
- square, 376
- symmetric, 409
- trace or spur, 385
- triangular, 408
- unitary, *see* unitary matrices
- maxima and minima (local) of a function of
  - constrained variables, *see* Lagrange
    - undetermined multipliers
  - one real variable, 114–15
    - sufficient conditions, 115
  - two real variables, 272–5
    - sufficient conditions, 274
- Maxwell's
  - electromagnetic equations, 488, 525
  - thermodynamic relations, 285–7
- mean  $\mu$ 
  - of RVD, 624–5
- mean value of a function
  - of one variable, 161–2
  - of several variables, 313
- mean value theorem, 120–1
- median of RVD, 625
- minor of a matrix element, 387
- mode of RVD, 625
- modulus
  - of a complex number, 178–9
  - of a vector, *see* magnitude of a vector
- moments (of distributions)
  - of RVD, 626
- moments (of forces), vector representation of, 344
- moments of inertia
  - definition, 312
  - of disc, 327
  - of rectangular lamina, 313
  - of right circular cylinder, 327
  - of sphere, 321
  - perpendicular axes theorem, 327
- multinomial distribution, 635–6
- multiple angles, trigonometric formulae, 24
- multiple integrals
  - application in finding
    - area and volume, 306–8
    - mass, centre of mass and centroid, 308–10
    - mean value of a function of several variables, 313
  - moments of inertia, 312–13
  - change of variables
    - double integrals, 315–20
    - general properties, 322–3
    - triple integrals, 320–1
  - definitions of
    - double integrals, 301
    - triple integrals, 304
  - evaluation, 302–5
  - notation, 301, 303, 304
  - order of integration, 302–3, 305
- multivariate distributions
  - multinomial, 635–6
- mutually exclusive events, 598, 607
- nabla  $\nabla$ , *see* gradient operator (grad)
- natural logarithm, *see*  $\ln$  and  $\ln$
- natural numbers, in series, 85, 220–1
- necessary and sufficient conditions, 88–90
- negative (semi-)definite function, 36
- negative binomial distribution, 636
- negative powers, 3
- negative vector, 370
- Newton's law of gravitation, *see* gravitation,
  - Newton's law
- non-Cartesian coordinates, *see* curvilinear,
  - cylindrical polar, plane polar *and* spherical
  - polar coordinates
- norm of vector, 372
- normal
  - to coordinate surface, 478
  - to plane, 350
  - to surface, 456, 461, 505
- normal derivative, 461
- normal distribution, *see* Gaussian (normal)
  - distribution
- normal matrices, 411
- normalisation of
  - eigenvectors, 413
  - vectors, 338
- null (zero)
  - matrix, 381, 382
  - operator, 375
  - vector, 333, 370
- $O(x)$ , order of, 234
- observables in quantum mechanics, 414
- odd functions, *see* antisymmetric functions
- ODE, *see* ordinary differential equations (ODE)
- operators linear, *see* linear operators
- order of
  - approximation in Taylor series, 240n
  - ODE, 554

- ordinary differential equations (ODE), *see also*
  - differential equations, particular
  - boundary conditions, 554, 556, 577
  - complementary function, 570
  - degree, 554
  - dimensionally homogeneous, 562
  - exact, 558
  - first-order, 554–68
  - first-order higher degree, 565–8
    - soluble for  $p$ , 565
    - soluble for  $x$ , 566
    - soluble for  $y$ , 567
  - general form of solution, 554–6
  - higher order, 569–79
  - homogeneous, 569
  - inexact, 559
  - linear, 561, 569–79
  - order, 554
  - particular integral (solution), 555, 571, 574–5
  - singular solution, 555, 566, 568
- ordinary differential equations, methods for
  - equations with constant coefficients, 572–9
  - integrating factors, 559–62
  - Laplace transforms, 576–9
  - separable variables, 557
  - undetermined coefficients, 574
- orthogonal lines, condition for, 30
- orthogonal matrices, 410
  - general properties, *see* unitary matrices
- orthogonal systems of coordinates, 477
- orthogonality of
  - eigenvectors of an Hermitian matrix, 414–15
  - vectors, 339, 371
- orthonormal basis vectors, 372
  - under unitary transformation, 423
- outcome, of trial, 597
- Pappus's theorems, 310–12
- parabola, equation for, 66
- parallel axis theorem, 366
- parallel vectors, 343
- parallelepiped, volume of, 346, 347
- parallelogram equality, 373
- parallelogram, area of, 343, 345
- parametric equations
  - of conic sections, 69
  - of cycloid, 485
  - of surfaces, 455
- partial derivative, *see* partial differentiation
- partial differentiation, 259–90
  - as gradient of a function of several real variables, 259
  - chain rule, 267–8
  - change of variables, 268–70
  - definitions, 259–61
  - properties, 266–7
    - cyclic relation, 266
    - reciprocity relation, 266
- partial fractions, 74–83
  - and degree of numerator, 79
  - as a means of integration, 148–9
  - complex roots, 81
  - in inverse Laplace transforms, 540, 577
  - repeated roots, 81
- partial sum, 213
- particular integrals (PI), 555, *see also* ordinary
  - differential equation, methods for
- parts, integration by, 152–4
- path integrals, *see* line integrals
- PDFs, 620
- permutations, 612–17
  - distinguishable, 614
  - symbol " $P_k$ ", 612
- perpendicular axes theorem, 327
- perpendicular vectors, 339, 371
- PF, *see* probability functions
- phase, of a complex number, 178n
- physical constants, values, 684
- physical dimensions, 15–19
  - derived quantities, 15n, 16
  - dimensional analysis, 16
- PI, *see* particular integrals
- plane curves, length of, 162–4
  - in Cartesian coordinates, 162
  - in plane polar coordinates, 164
- plane polar coordinates, 72, 159, 450
  - arc length, 164, 473
  - area element, 318, 473
  - basis vectors, 450
  - velocity and acceleration, 450
- planes
  - and simultaneous linear equations, 406–7
  - vector equation of, 349–51
- point charges,  $\delta$ -function representation, 543
- points of inflection of a function of
  - one real variable, 114–16
  - two real variables, 272–5
- Poisson distribution  $Po(\lambda)$ , 639–42
  - and Gaussian distribution, 651
  - as limit of binomial distribution, 639, 641
  - mean and variance, 641
  - multiple, 642
  - recurrence formula, 640
- polar coordinates, *see* plane polar *and* cylindrical
  - polar *and* spherical polar coordinates
- polar representation of complex numbers, 185–8

- polynomial equations, 53–63
  - conjugate roots, 193
  - factorisation, 60
  - multiplicities of roots, 56
  - number of roots, 174, 175
  - properties of roots, 62
  - real roots, 53
  - solution of, using de Moivre's theorem, 192–3
- positive (semi-)definite function, 36
- positive (semi-)definite quadratic and Hermitian forms, 428
- positive semi-definite norm, 372
- potential energy of
  - ion in a crystal lattice, 256
  - magnetic dipole in a field, 340
- potential function
  - and conservative fields, 504
  - vector, 504
- potential, thermodynamic, 286
- power series
  - interval of convergence, 234
  - Maclaurin, *see* Maclaurin series
  - manipulation: difference, differentiation, integration, product, substitution, sum, 236–8
  - Taylor, *see* Taylor series
- power series in a complex variable, 235
  - circle and radius of convergence, 235
- powers
  - combining, 1
  - complex, 194–6
  - general, 4
  - index or exponent, 1
  - negative, 3
  - rational, 3
  - real, 1
- prime, non-existence of largest, 88
- principal axes of quadratic surfaces, 430
- principal value of
  - complex logarithms, 194
  - improper integral, 158n12
- probability, 602–60
  - axioms, 603
  - conditional, 606–12
    - Bayes' theorem, 610–12
    - combining, 608
  - definition, 603
  - for intersection  $\cap$ , 598
  - for union  $\cup$ , 599, 603–6
- probability distributions, 618, *see also individual distributions*
  - bivariate, *see* bivariate distributions
  - change of variables, 628–31
  - mean  $\mu$ , 624–5
  - mean of functions, 625
  - mode, median and quartiles, 625
  - moments, 626–8
  - multivariate, *see* multivariate distributions
  - standard deviation  $\sigma$ , 626
  - table of
    - continuous distributions, 643
    - discrete distributions, 632
    - variance  $\sigma^2$ , 626
- probability distributions, individual
  - binomial  $\text{Bin}(n, p)$ , 632–5
  - Cauchy (Breit–Wigner), 644
  - chi-squared ( $\chi^2$ ), 653
  - exponential, 653
  - Gaussian (normal)  $N(\mu, \sigma^2)$ , 645–53
  - geometric, 636
  - hypergeometric, 637
  - multinomial, 635–6
  - negative binomial, 636
  - Poisson  $\text{Po}(\lambda)$ , 639–42
  - uniform (rectangular), 644
- probability functions (PFs), 619
  - cumulative, 619, 620
  - density functions (PDFs), 620
- product notation, 62n6
- product rule for differentiation, 106–8, 112–13
- quadratic equations
  - complex roots of, 174
  - properties of roots, 62
  - roots of, 53
- quadratic forms, 427–31
  - positive definite and semi-definite, 428–9
  - quadratic surfaces, 430–1
  - removing cross terms, 427–8
  - stationary properties of eigenvectors, 429–30
- quartiles, of RVD, 625
- quotient rule for differentiation, 109–10
- quotient test for series, 228
- radian, 25
- radius of convergence, 235
- radius of curvature, of plane curves, 117
- random variable distributions, *see* probability distributions
- random variables (RV), 618–23
  - continuous, 620–3
  - dependent, 655–60
  - discrete, 618–20
  - independent, 631, 658
  - uncorrelated, 658

- rank of matrices, 395
  - and determinants, 396–7
  - and linear dependence, 395–6
- rate of change of a function of
  - one real variable, 102
  - several real variables, 261–3
- ratio comparison test, 228
- ratio test (D'Alembert), 227
  - in convergence of power series, 234
- ratio theorem, 334
  - and centroid of a triangle, 335–6
- rational functions, 35
- rational powers, 3
- rationalisation, involving surds, 5
- real part or term of a complex number, 174–5
- real roots, of a polynomial equation, 53
- reciprocal vectors, 357–9
- reciprocity relation for partial derivatives, 266
- rectangular distribution, 644
- recurrence relations (series), 579–84
  - characteristic equation, 582
  - first-order, 580
  - second-order, 582
  - higher order, 584
- reduction formulae for integrals, 155–6
- relative velocities, 338
- remainder term in Taylor series, 240
- repeated roots of auxiliary equation, 573
- rhomboid, volume of, 364
- Riemann theorem for conditional convergence, 225
- Riemann zeta series, 229, 230
- right-hand screw rule, 342
- Rolle's theorem, 57, 120
- root test (Cauchy), 230
- roots
  - of a polynomial equation, 53
  - properties, 62
  - of a real variable, 3
  - of unity, 191–2
- rotation of a vector, *see* curl of a vector field
- row matrix, 376
- RV, *see* random variables
- RVD (random variable distributions), *see* probability distributions
- saddle points, 273
  - sufficient conditions, 274
- sampling
  - space, 597
  - with or without replacement, 607
- scalar fields, 458
  - derivative along a space curve, 459
  - gradient, 459–62
  - line integrals, 491–501
  - rate of change, 459
- scalar product, 339–42
  - and inner product, 371
  - and perpendicular vectors, 339, 371
  - for vectors with complex components, 342
  - in Cartesian coordinates, 341
- scalar triple product, 346–7
  - cyclic permutation of, 347
  - in Cartesian coordinates, 347
  - determinant form, 347
  - interchange of dot and cross, 347
- scalars, 331
- scale factors, 471, 474, 477
- Schwarz inequality, 373
- second-order differential equations, *see* ordinary differential equations
- secular determinant, 418
- semicircle, angle in, 69
- semicircular lamina, centre of mass, 312
- separable variables in ODE, 557
- series, 213–44
  - convergence of, *see* convergence of infinite series
  - differentiation of, 233
  - finite and infinite, 214
  - integration of, 233
  - multiplication by a scalar, 233
  - multiplication of (Cauchy product), 233
  - notation, 214
  - operations, 232
  - summation, *see* summation of series
- series, particular
  - arithmetic, 215
  - arithmetico-geometric, 217
  - geometric, 215
  - Maclaurin, 240, 243
  - power, *see* power series
  - powers of natural numbers, 220–1
  - Riemann zeta, 229, 230
  - Taylor, *see* Taylor series
- similarity transformations, 421–4
  - properties of matrix under, 423
  - unitary transformations, 423–4
- simultaneous linear equations, 397–407
  - and intersection of planes, 406–7
  - homogeneous and inhomogeneous, 397
  - number of solutions, 398–9
  - solution using
    - Cramer's rule, 404–6
    - Gaussian elimination, 399–401

- inverse matrix, 401
- $LU$  decomposition, 401–4
- sine,  $\sin(x)$ 
  - geometrical and algebraic definitions, 676
  - geometrical definition, 25
  - in terms of exponential functions, 199
  - Maclaurin series for, 243
  - reciprocal and inverse, 27
- singular and non-singular
  - linear operators, 375
  - matrices, 392
- singular integrals, *see* improper integrals
- singular solution of ODE, 555, 566, 568
- $\sinh$ ,  $\sinh(x)$ , *see also* hyperbolic functions
  - hyperbolic sine, 198
  - Maclaurin series for, 243
- sinusoidal functions
  - common values, 26
  - identities, 28
- skew-symmetric matrices, 409
- solenoidal vectors, 463, 504
- solid angle
  - as surface integral, 510
  - subtended by rectangle, 530
- solid: mass, centre of mass and centroid, 308–9
- spaces, *see* vector spaces
- span of a set of vectors, 370
- sphere, vector equation of, 351
- spherical polar coordinates, 473–6
  - area element, 474
  - basis vectors, 474
  - length element, 474
  - vector operators, 473–6
  - volume element, 321, 474
- spur of a matrix, *see* trace of a matrix
- square matrices, 376
- standard deviation  $\sigma$ , 626
- stationary values
  - of functions of
    - one real variable, 114–15
    - two real variables, 272–5
  - under constraints, *see* Lagrange undetermined multipliers
- Stokes' theorem, 503, 523–6
  - physical applications, 525
  - related theorems, 524
- submatrices, 396–7
- subscripts
  - dummy, 682
  - free, 682
  - summation convention, 681
- substitution, integration by, 149–52
- summation convention, 432, 681–3
- summation of series, 215–23
  - arithmetic, 215
  - arithmetico-geometric, 217
  - difference method, 217–19
  - geometric, 215
  - powers of natural numbers, 220–1
  - transformation methods, 221–3
    - differentiation, 221
    - integration, 221
    - substitution, 223
- surd, 5
- surface area, as a vector, 508–10
  - as a line integral, 509
- surface integrals
  - and divergence theorem, 517
  - Archimedean upthrust, 511, 530
  - of scalars, vectors, 504–11
  - physical examples, 510
- surfaces, 455–7
  - area of, 457
    - cone, 165–6
    - solid, and Pappus's theorem, 310–12
    - sphere, 457
  - coordinate curves, 456
  - normal to, 456, 461
  - of revolution, 164–6
  - parametric equations, 455
  - quadratic, 430–1
  - tangent plane, 456
- symmetric functions, 126
- symmetric matrices, 409
  - general properties, *see* Hermitian matrices
- $t$  substitution, 150–1
- $\tan^{-1} x$ , Maclaurin series for, 243
- tangent planes to surfaces, 456
- tangent,  $\tan(x)$ 
  - geometrical definition, 25
  - Maclaurin series for, 243
- $\tanh$ ,  $\tanh(x)$ , *see* hyperbolic functions
- Taylor series, 238–44
  - and Taylor's theorem, 239–42
  - approximation errors, 242–3
  - for functions of several real variables, 270–1
  - remainder term, 240
  - required properties, 239
  - standard forms, 240
- tetrahedron
  - mass of, 308
  - volume of, 306
- thermodynamic potential, 286

- thermodynamics
  - first law of, 285
  - Maxwell's relations, 285–7
- torque, vector representation of, 344
- total derivative, 263
- total differential, 262
- trace of a matrix, 385–6
  - as sum of eigenvalues, 418, 426
  - invariance under similarity transformations, 423
- transformation matrix, 422, 427–8
- transformations
  - similarity, *see* similarity transformations
- transforms, Laplace, *see* Laplace transforms
- transpose of a matrix, 376, 383
  - product rule, 383
- trial functions, for PI of ODE, 574
- trials, 597
- triangle inequality, 373
- triangle, centroid of, 335–6
- triangular matrices, 401, 408
- trigonometric identities, 24–32
- triple integrals, *see* multiple integrals
- triple scalar product, *see* scalar triple product
- triple vector product, *see* vector triple product
- turning point, 114
  
- undetermined coefficients, method of, 574
- undetermined multipliers, *see* Lagrange
  - undetermined multipliers
- uniform distribution, 644
- union  $\cup$ 
  - algebra of, 601
- unit step function, *see* Heaviside function
- unit vectors, 338
- unitary matrices, 410
  - eigenvalues and eigenvectors, 415
- unitary transformations, 423–4
- upper triangular matrices, 408
  
- variable, dummy, 142
- variance  $\sigma^2$ , 626
  - of dependent RV, 659
- vector operators, 458–81
  - acting on sums and products, 465–7
  - combinations of, 467–9
  - curl, 463, 480
  - $\text{del } \nabla$ , 458
  - $\text{del squared } \nabla^2$ , 463
  - divergence (div), 462
  - geometrical definitions, 513–17
  - gradient operator (grad), 459–62, 479
  - identities, 467
  - Laplacian, 463, 480
  - non-Cartesian, 469–81
- vector product, 342–5
  - anticommutativity, 342
  - definition, 342
  - determinant form, 345
  - in Cartesian coordinates, 345
  - non-associativity, 342
- vector spaces, 370–3
  - associativity of addition, 369–70
  - basis vectors, 370–1
  - commutativity of addition, 369–70
  - complex, 370
  - defining properties, 370
  - dimensionality, 370
  - inequalities: Bessel, Schwarz, triangle, 373
  - matrices as an example, 379
  - parallelogram equality, 373
  - real, 370
  - span of a set of vectors in, 370
- vector triple product, 348
  - identities, 348
  - non-associativity, 348
- vectors
  - as geometrical objects, 369
  - base, 450
  - column, 376
  - compared with scalars, 331
  - component form, 336–8
  - examples of, 331
  - graphical representation of, 331
  - irrotational, 464
  - magnitude of, 338
  - non-Cartesian, 450, 470, 474
  - notation, 331
  - solenoidal, 463, 504
  - span of, 370
- vectors, algebra of, 331–59
  - addition and subtraction, 332–3
    - in component form, 337–8
  - angle between, 341
  - associativity of addition and subtraction, 332
  - commutativity of addition and subtraction, 332
  - multiplication by a complex scalar, 342
  - multiplication by a scalar, 333–4
  - multiplication of, *see* scalar product *and* vector product
- vectors, applications
  - centroid of a triangle, 335–6
  - equation of a line, 349
  - equation of a plane, 349–51
  - equation of a sphere, 351
  - finding distance from a



- line to a line, 355–6
  - line to a plane, 356–7
  - point to a line, 353–4
  - point to a plane, 354–5
  - intersection of two planes, 350, 351
- vectors, calculus of, 448–81
  - differentiation, 448–52, 454
  - integration, 453–4
  - line integrals, 491–504
  - surface integrals, 504–11
  - volume integrals, 511–13
- vectors, derived quantities
  - curl, 463
  - derivative, 448
  - differential, 452, 455
  - divergence (div), 462
  - reciprocal, 357–9
  - vector fields, 458
    - curl, 523
    - divergence, 462
    - flux, 510
    - rate of change, 461
- vectors, physical
  - acceleration, 449
  - angular momentum, 365
  - angular velocity, 344, 365, 464
  - area, 508–10, 525
  - area of parallelogram, 343, 345
  - force, 331, 332, 340
  - moment or torque of a force, 344
  - velocity, 449
- velocity vectors, 449
- Venn diagrams, 597–602
- volume elements
  - curvilinear coordinates, 478
  - cylindrical polars, 471
  - spherical polars, 321, 474
- volume integrals, 511–13
  - and divergence theorem, 517
- volume of
  - cone, 167
  - ellipsoid, 326
  - parallelepiped, 346
  - rhomboid, 364
  - tetrahedron, 306
- volumes
  - as surface integrals, 512, 517
  - of regions, using multiple integrals, 306–8
- volumes of revolution, 166–7
  - and surface area and centroid, 310–12
- wave equation, from Maxwell's equations, 488
- wavefunction of electron in hydrogen atom, 326
- wedge product, *see* vector product
- work done
  - by force, 495
  - vector representation, 340
- Wronskian test for linear independence, 570
- X-ray scattering, 364
- $z$ , as a complex number, 174
- $z^*$ , as complex conjugate, 181–4
- zero (null)
  - matrix, 381, 382
  - operator, 375
  - vector, 333, 370
- zeros, of a polynomial, 53
- zeta series (Riemann), 229, 230
- $z$ -plane, *see* Argand diagram