Foundation Mathematics for the Physical Sciences

This tutorial-style textbook develops the basic mathematical tools needed by first- and second-year undergraduates to solve problems in the physical sciences. Students gain hands-on experience through hundreds of worked examples, end-of-section exercises, self-test questions and homework problems.

Each chapter includes a summary of the main results, definitions and formulae. Over 270 worked examples show how to put the tools into practice. Around 170 self-test questions in the footnotes and 300 end-of-section exercises give students an instant check of their understanding. More than 450 end-of-chapter problems allow students to put what they have just learned into practice.

Hints and outline answers to the odd-numbered problems are given at the end of each chapter. Complete solutions to these problems can be found in the accompanying *Student Solution Manual*. Fully worked solutions to all the problems, password-protected for instructors, are available at www.cambridge.org/foundation.

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521192736

First published 2011
Reprinted 2014

Printed in the United Kingdom by TJ International Ltd, Padstow, Cornwall

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Riley, K. F. (Kenneth Franklin), 1936–
Foundation mathematics for the physical sciences : a tutorial guide / K. F. Riley, M. P. Hobson.
p. cm.
Includes index.
ISBN 978-0-521-19273-6
1. Mathematics. I. Hobson, M. P. (Michael Paul), 1967– II. Title.
QA37.3.R56 2011
510 – dc22 2010041510

ISBN 978-0-521-19273-6 Hardback

Additional resources for this publication: www.cambridge.org/foundation

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Preface

Since *Mathematical Methods for Physics and Engineering* by Riley, Hobson and Bence (Cambridge: Cambridge University Press, 1998), hereafter denoted by *MMPE*, was first published, the range of material it covers has increased with each subsequent edition (2002 and 2006). Most of the additions have been in the form of introductory material covering polynomial equations, partial fractions, binomial expansions, coordinate geometry and a variety of basic methods of proof, though the third edition of *MMPE* also extended the range, but not the general level, of the areas to which the methods developed in the book could be applied. Recent feedback suggests that still further adjustments would be beneficial. In so far as content is concerned, the inclusion of some additional introductory material such as powers, logarithms, the sinusoidal and exponential functions, inequalities and the handling of physical dimensions, would make the starting level of the book better match that of some of its readers.

To incorporate these changes, and others aimed at increasing the user-friendliness of the text, into the current third edition of *MMPE* would inevitably produce a text that would be too ponderous for many students, to say nothing of the problems the physical production and transportation of such a large volume would entail.

For these reasons, we present under the current title, *Foundation Mathematics for the Physical Sciences*, an alternative edition of *MMPE*, one that focuses on the earlier part of a putative extended third edition. It omits those topics that truly are 'methods' and concentrates on the 'mathematical tools' that are used in more advanced texts to build up those methods. The emphasis is very much on developing the basic mathematical concepts that a physical scientist needs, before he or she can narrow their focus onto methods that are particularly appropriate to their chosen field.

One aspect that has remained constant throughout the three editions of *MMPE* is the general style of presentation of a topic – a qualitative introduction, physically based wherever possible, followed by a more formal presentation or proof, and finished with one or two full-worked examples. This format has been well received by reviewers, and there is no reason to depart from its basic structure.

In terms of style, many physical science students appear to be more comfortable with presentations that contain significant amounts of explanation or comment in words, rather than with a series of mathematical equations the last line of which implies 'job done'. We have made changes that move the text in this direction. As is explained below, we also feel that if some of the advantages of small-group face-to-face teaching could be reflected in the written text, many students would find it beneficial.

In keeping with the intention of presenting a more 'gentle' introduction to university-level mathematics for the physical sciences, we have made use of a modest number of appendices. These contain the more formal mathematical developments associated with

the material introduced in the early chapters, and, in particular, with that discussed in the introductory chapter on arithmetic and geometry. They can be studied at the points in the main text where references are made to them, or deferred until a greater mathematical fluency has been acquired.

As indicated above, one of the advantages of an oral approach to teaching, apparent to some extent in the lecture situation, and certainly in what are usually known as tutorials,[1] is the opportunity to follow the exposition of any particular point with an immediate short, but probing, question that helps to establish whether or not the student has grasped that point. This facility is not normally available when instruction is through a written medium, without having available at least the equipment necessary to access the contents of a storage disc.

In this book we have tried to go some way towards remedying this by making a non-standard use of footnotes. Some footnotes are used in traditional ways, to add a comment or a pertinent but not essential piece of additional information, to clarify a point by restating it in slightly different terms, or to make reference to another part of the text or an external source. However, about half of the more than 300 footnotes in *this* book contain a question for the reader to answer or an instruction for them to follow; neither will call for a lengthy response, but in both cases an understanding of the associated material in the text will be required. This parallels the sort of follow-up a student might have to supply orally in a small-group tutorial, after a particular aspect of their written work has been discussed.

Naturally, students should attempt to respond to footnote questions using the skills and knowledge they have acquired, re-reading the relevant text if necessary, but if they are unsure of their answer, or wish to feel the satisfaction of having their correct response confirmed, they can consult the specimen answers given in Appendix F. Equally, footnotes in the form of observations will have served their purpose when students are consistently able to say to themselves 'I didn't need that comment – I had already spotted and checked that particular point'.

There are two further features of the present volume that did not appear in *MMPE*. The first of these is that a small set of exercises has been included at the end of each section. The questions posed are straightforward and designed to test whether the student has understood the concepts and procedures described in that section. The questions are not intended as 'drill exercises', with repeated use of the same procedure on marginally different sets of data; each concept is examined only once or twice within the set. There are, nevertheless, a total of more than 300 such exercises. The more demanding questions, and in particular those requiring the synthesis of several ideas from a chapter, are those that appear under the heading of 'Problems' at the end of that chapter; there are more than 450 of these.

The second new feature is the inclusion at the end of each chapter, just before the problems begin, of a summary of the main results of that chapter. For some areas, this takes the form of a tabulation of the various case types that may arise in the context of the chapter; this should help the student to see the parallels between situations which in the main text are presented as a consecutive series of often quite lengthy pieces of mathematical development. It should be said that in such a summary it is not possible to

1 But in Cambridge are called 'supervisions'!

state every detailed condition attached to each result, and the reader should consider the summaries as reminders and formulae providers, rather than as teaching text; that is the job of the main text and its footnotes. Fortunately, in this volume, occasions on which subtle conditions have to be imposed upon a result are rare.

Finally, we note, for the record, that the format and numbering of the problems associated with the various chapters have not been changed significantly from those in *MMPE*, though naturally only problems related to included topics are retained. This means that abbreviated solutions to all odd-numbered problems can be found in this text. Fully worked solutions to the same problems are available in the companion volume *Student Solution Manual for Foundation Mathematics for the Physical Sciences*; most of them, except for those in the first chapter, can also be found in the *Student Solution Manual for MMPE*.

Fully worked solutions to all problems, both odd- and even-numbered, are available to accredited instructors on the password-protected website www.cambridge.org/foundation. Instructors wishing to have access to the website should contact solutions@cambridge.org for registration details.