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John F. Monahan  
Frontmatter  
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*Numerical Methods of Statistics*  
Second Edition

This book explains how computer software is designed to perform the tasks required for sophisticated statistical analysis. For statisticians, it examines the nitty-gritty computational problems behind statistical methods. For mathematicians and computer scientists, it looks at the application of mathematical tools to statistical problems. The first half of the book offers a basic background in numerical analysis that emphasizes issues important to statisticians. The next several chapters cover a broad array of statistical tools, such as maximum likelihood and nonlinear regression. The author also treats the application of numerical tools; numerical integration and random number generation are explained in a unified manner reflecting complementary views of Monte Carlo methods. Each chapter contains exercises that range from simple questions to research problems. Most of the examples are accompanied by demonstration and source code available on the author's Web site. New in this second edition are demonstrations coded in R, as well as new sections on linear programming and the Nelder-Mead search algorithm.

John F. Monahan is a Professor of Statistics at North Carolina State University, where he joined the faculty in 1978 and has been a professor since 1990. His research has appeared in numerous computational as well as statistical journals. He is also the author of *A Primer on Linear Models* (2008).

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## *Contents*

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	<i>Preface to the Second Edition</i>	<i>page</i> xiii
	<i>Preface to the First Edition</i>	xv
<b>1</b>	<b>Algorithms and Computers</b>	<b>1</b>
	1.1 Introduction	1
	1.2 Computers	3
	1.3 Software and Computer Languages	5
	1.4 Data Structures	8
	1.5 Programming Practice	9
	1.6 Some Comments on R	10
	References	12
<b>2</b>	<b>Computer Arithmetic</b>	<b>13</b>
	2.1 Introduction	13
	2.2 Positional Number Systems	14
	2.3 Fixed Point Arithmetic	17
	2.4 Floating Point Representations	20
	2.5 Living with Floating Point Inaccuracies	23
	2.6 The Pale and Beyond	28
	2.7 Conditioned Problems and Stable Algorithms	32
	Programs and Demonstrations	34
	Exercises	35
	References	38
<b>3</b>	<b>Matrices and Linear Equations</b>	<b>40</b>
	3.1 Introduction	40
	3.2 Matrix Operations	41
	3.3 Solving Triangular Systems	43
	3.4 Gaussian Elimination	44
	3.5 Cholesky Decomposition	50
	3.6 Matrix Norms	54
	3.7 Accuracy and Conditioning	55
	3.8 Matrix Computations in R	60
	Programs and Demonstrations	61
	Exercises	63
	References	65

<b>4</b>	<b>More Methods for Solving Linear Equations</b>	<b>67</b>
4.1	Introduction	67
4.2	Full Elimination with Complete Pivoting	67
4.3	Banded Matrices	71
4.4	Applications to ARMA Time-Series Models	73
4.5	Toeplitz Systems	76
4.6	Sparse Matrices	80
4.7	Iterative Methods	82
4.8	Linear Programming	84
	Programs and Demonstrations	87
	Exercises	88
	References	90
<b>5</b>	<b>Regression Computations</b>	<b>91</b>
5.1	Introduction	91
5.2	Condition of the Regression Problem	93
5.3	Solving the Normal Equations	96
5.4	Gram–Schmidt Orthogonalization	97
5.5	Householder Transformations	100
5.6	Householder Transformations for Least Squares	101
5.7	Givens Transformations	104
5.8	Givens Transformations for Least Squares	105
5.9	Regression Diagnostics	107
5.10	Hypothesis Tests	110
5.11	Conjugate Gradient Methods	112
5.12	Doolittle, the Sweep, and All Possible Regressions	115
5.13	Alternatives to Least Squares	118
5.14	Comments	120
	Programs and Demonstrations	122
	Exercises	122
	References	125
<b>6</b>	<b>Eigenproblems</b>	<b>128</b>
6.1	Introduction	128
6.2	Theory	128
6.3	Power Methods	130
6.4	The Symmetric Eigenproblem and Tridiagonalization	133
6.5	The QR Algorithm	135
6.6	Singular Value Decomposition	137
6.7	Applications	140
6.8	Complex Singular Value Decomposition	144
	Programs and Demonstrations	146
	Exercises	147
	References	150

<b>7</b>	<b>Functions: Interpolation, Smoothing, and Approximation</b>	<b>151</b>
	7.1 Introduction	151
	7.2 Interpolation	153
	7.3 Interpolating Splines	156
	7.4 Curve Fitting with Splines: Smoothing and Regression	159
	7.5 Mathematical Approximation	163
	7.6 Practical Approximation Techniques	168
	7.7 Computing Probability Functions	170
	Programs and Demonstrations	177
	Exercises	179
	References	183
<b>8</b>	<b>Introduction to Optimization and Nonlinear Equations</b>	<b>186</b>
	8.1 Introduction	186
	8.2 Safe Univariate Methods: Lattice Search, Golden Section, and Bisection	188
	8.3 Root Finding	191
	8.4 First Digression: Stopping and Condition	197
	8.5 Multivariate Newton's Methods	199
	8.6 Second Digression: Numerical Differentiation	200
	8.7 Minimization and Nonlinear Equations	203
	8.8 Condition and Scaling	208
	8.9 Implementation	210
	8.10 A Non-Newton Method: Nelder-Mead	211
	Programs and Demonstrations	213
	Exercises	214
	References	217
<b>9</b>	<b>Maximum Likelihood and Nonlinear Regression</b>	<b>219</b>
	9.1 Introduction	219
	9.2 Notation and Asymptotic Theory of Maximum Likelihood	220
	9.3 Information, Scoring, and Variance Estimates	226
	9.4 An Extended Example	228
	9.5 Concentration, Iteration, and the EM Algorithm	230
	9.6 Multiple Regression in the Context of Maximum Likelihood	236
	9.7 Generalized Linear Models	237
	9.8 Nonlinear Regression	242
	9.9 Parameterizations and Constraints	246
	Programs and Demonstrations	251
	Exercises	252
	References	255
<b>10</b>	<b>Numerical Integration and Monte Carlo Methods</b>	<b>257</b>
	10.1 Introduction	257
	10.2 Motivating Problems	258
	10.3 One-Dimensional Quadrature	264

10.4	Numerical Integration in Two or More Variables	271
10.5	Uniform Pseudorandom Variables	278
10.6	Quasi–Monte Carlo Integration	286
10.7	Strategy and Tactics	291
	Programs and Demonstrations	295
	Exercises	297
	References	299
<b>11</b>	<b>Generating Random Variables from Other Distributions</b>	<b>303</b>
11.1	Introduction	303
11.2	General Methods for Continuous Distributions	304
11.3	Algorithms for Continuous Distributions	308
11.4	General Methods for Discrete Distributions	321
11.5	Algorithms for Discrete Distributions	325
11.6	Other Randomizations	330
11.7	Accuracy in Random Number Generation	334
	Programs and Demonstrations	337
	Exercises	338
	References	341
<b>12</b>	<b>Statistical Methods for Integration and Monte Carlo</b>	<b>343</b>
12.1	Introduction	343
12.2	Distribution and Density Estimation	343
12.3	Distributional Tests	350
12.4	Importance Sampling and Weighted Observations	353
12.5	Testing Importance Sampling Weights	359
12.6	Laplace Approximations	361
12.7	Randomized Quadrature	363
12.8	Spherical–Radial Methods	365
	Programs and Demonstrations	370
	Exercises	372
	References	373
<b>13</b>	<b>Markov Chain Monte Carlo Methods</b>	<b>375</b>
13.1	Introduction	375
13.2	Markov Chains	377
13.3	Gibbs Sampling	378
13.4	Metropolis–Hastings Algorithm	383
13.5	Time-Series Analysis	386
13.6	Adaptive Acceptance/Rejection	390
13.7	Diagnostics	394
	Programs and Demonstrations	398
	Exercises	398
	References	400



*Contents*

xi

<b>14</b>	<b>Sorting and Fast Algorithms</b>	<b>403</b>
14.1	Introduction	403
14.2	Divide and Conquer	403
14.3	Sorting Algorithms	405
14.4	Fast Order Statistics and Related Problems	408
14.5	Fast Fourier Transform	409
14.6	Convolutions and the Chirp- $z$ Transform	413
14.7	Statistical Applications of the FFT	415
14.8	Combinatorial Problems	425
	Programs and Demonstrations	429
	Exercises	433
	References	436
	<i>Author Index</i>	439
	<i>Subject Index</i>	444

## *Preface to the Second Edition*

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In the ten years since the first edition of this book went to press, the field of statistical computing has exploded with innovations in many directions. At one time my goal was to write a comprehensive book on the subject. At this moment, however, my goals for a second edition must be more modest. Because the field has grown so much, the scope of this book has now become the core for a subset of this field. To fill in some gaps in this new core, a few sections have been added (e.g., linear programming) and others have been expanded. Many corrections have been made; I can only hope that just a few errors remain.

A second change in this timespan is the rapid widespread adoption of R in the field of statistics. As language and culture shape each other, my own views on computing have changed from teaching this material using R. Small changes scattered throughout reflect this change in viewpoint. Additionally, most of the demonstrations and examples – all that seemed appropriate – have been translated to R and are available on my Web site for this book (<http://www4.stat.ncsu.edu/~monahan/nmos2/toc.html>).

Thanks are due to Lauren Cowles of Cambridge University Press for encouraging this second edition. Karen Chiswell deserves recognition for finding numerous typos and providing other corrections. I would like to also thank Jerry Davis and Wendy Meiring for pointing out others. Bruce McCullough provided invaluable feedback, comments, questions, and suggestions. Thanks are also due to the many students who, perhaps unknowingly, provided feedback with their questions. And this second edition would not be possible without the love, support, and patience of my wife Carol.

## *Preface to the First Edition*

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This book grew out of notes for my Statistical Computing course that I have been teaching for the past 20 years at North Carolina State University. The goal of this course is to prepare doctoral students with the computing tools needed for statistical research, and I have augmented this core with related topics that through the years I have found useful for colleagues and graduate students. As a result, this book covers a wide range of computational issues, from arithmetic, numerical linear algebra, and approximation, which are typical numerical analysis topics, to optimization and non-linear regression, to random number generation, and finally to fast algorithms. I have emphasized numerical techniques but restricted the scope to those regularly employed in the field of statistics and dropped some traditional numerical analysis topics such as differential equations. Many of the exercises in this book arose from questions posed to me by colleagues and students.

Most of the students that I have taught come with a graduate level understanding of statistics, no experience in numerical analysis, and little skill in a programming language. Consequently, I cover only about half of this material in a one-semester course. For those with a background in numerical analysis, a basic understanding of two statistical topics, regression and maximum likelihood, would be necessary.

I would advise any instructor of statistical computing not to shortchange the fundamental topic of arithmetic. I have found that most students resist the idea that computers have limited precision and employ many defense mechanisms to support that denial. Until students are comfortable with finite precision arithmetic, this psychological obstacle will cripple their understanding of scientific computation. As a result, I urge the use of single precision arithmetic in the early part of the course and introduce numerical linear algebra using a low-level language, even though students may eventually use software or languages that completely hide the calculations behind operators and double precision. These operators will continue to be mysterious black boxes until the fundamental concept of finite precision arithmetic is understood and accepted.

Early in this effort, I faced the dilemma of how to describe algorithms. The big picture is easier to present or to understand with pseudocode descriptions of algorithms. But I always felt that skipping over the details was misleading the reader, especially when the details are critical to the success of an implementation. Furthermore, there is no better challenge to one's understanding of a topic than to take a big-picture description and program it to the smallest detail. On the other hand, writing one's own implementation of an algorithm often seems like a futile reinvention of the wheel.

And so my response to this dilemma is to have it both ways: to present algorithms in pseudocode in the text, but also to supplement the pseudocode with Fortran programs and demonstrations on the accompanying disk.

These programs provide the basic tools for extending the realm of statistical techniques beyond the bounds of current statistical software. But my primary goal in providing this code is instructional. Some exercises consist of implementing a particular algorithm, and occasionally I have intentionally included my implementation for the reader to compare with, or, perhaps, improve upon. I encourage the reader to examine the details of the code and to see how the algorithms respond to changes. A secondary goal is to include as many realistic problems as practicable, having endured the frustration of failing to get code to work on anything but toy problems.

I would like to express my appreciation to the many sources of support behind this effort. First of all, three heads of the Department of Statistics have supported my work in statistical computing: Tom Gerig, Dan Solomon, and the late Dave Mason. Some of the work included here is the result of collaborations with many colleagues over the years; especially notable are Al Kinderman on random number generation and Alan Genz on numerical integration. In particular, I would like to thank Sujit Ghosh and Dave Dickey for contributing invaluable advice on Chapter 13. Dennis Boos deserves special acknowledgment as a friend, colleague, and collaborator, and most importantly, for supplying me with many interesting problems over the years. I would like to thank all of the colleagues and students who brought interesting problems to me that have become material in this book. Finally, I appreciate the feedback that students have given me each semester on earlier versions of this manuscript, including their blank stares and yawns, as well as insightful questions.