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Umran S. Inan and Robert A. Marshall
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Numerical Electromagnetics

The FDTD Method

Beginning with the development of Finite Difference Equations, and leading to the complete FDTD algorithm, this is a coherent introduction to the FDTD method (the method of choice for modeling Maxwell's equations). It provides students and professional engineers with everything they need to know to begin writing FDTD simulations from scratch and to develop a thorough understanding of the inner workings of commercial FDTD software. Stability, numerical dispersion, sources, and boundary conditions are all discussed in detail, as are dispersive and anisotropic materials. A comparative introduction of the finite volume and finite element methods is also provided. All concepts are introduced from first principles, so no prior modeling experience is required, and they are made easier to understand through numerous illustrative examples and the inclusion of both intuitive explanations and mathematical derivations.

Umran S. Inan is a Professor in the Department of Electrical Engineering at Stanford University, where he has led pioneering research in plasma physics and electromagnetics for over 30 years. He also currently serves as president of Koç University in Istanbul. As a committed teacher, he has authored two previous books that have become standard textbooks for electromagnetics courses. He has also received numerous awards, including the Tau Beta Pi Excellence in Undergraduate Teaching Award and the Outstanding Service Award from the Electrical Engineering Department for Excellence in Teaching. He is a fellow of the Institute of Electrical and Electronics Engineers (IEEE), the American Geophysical Union (AGU), and the American Physical Society. He is the recipient of the 2008 Appleton Prize of the International Union of Radio Science (URSI) and the Royal Society, and the 2007 Allan V. Cox Medal of Stanford University for faculty excellence in fostering undergraduate research.

Robert A. Marshall is a Research Scientist at Boston University, where he has worked since receiving his Ph.D. in Electrical Engineering from Stanford University in 2009. He has received numerous awards from both Stanford University and the academic community, including the International Radio Science Union (URSI) and the American Geophysical Union (AGU).

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FDTD is presently the method of choice for solving most numerical electromagnetics studies. The Inan and Marshall book is a very thorough, yet readable, account of all the details of the method, very valuable for students and professionals alike, with problems included, ready for a course. Even those who use commercial programs would benefit from a better understanding of dispersion, accuracy, media descriptions, basic limitations and so on, all treated in great detail. I am sure it will be a good reference for a number of years. No special background needed since basic material for a good understanding is given, such as electromagnetics, or numerical solution of differential equations. Highly recommended.

Jørgen Bach Andersen, Professor Emeritus,
Aalborg University

This book is what has been needed to teach in a systematic fashion the theoretical developments and numerical implementations of one of the most powerful numerical techniques in computational electromagnetic with vast applications in a variety of disciplines. The authors have elegantly and masterfully presented the FDTD method from the Maxwell's equations with sufficient rigor to make the book very reachable by students and practicing researchers.

Yahya Rahmat-Samii, Distinguished Professor,
University of California at Los Angeles

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The FDTD Method

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To my parents, my beautiful wife Elif, my dear children Ayse and Ali, and my very special granddaughter Ayla.

USI

To my brother Pete.

RM

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Preface

Our purpose in this text is to provide an introduction to Numerical Electromagnetics, sometimes referred to as Computational Electromagnetics (CEM), a subject much too broad to cover in a single volume. Fifteen years ago, we might have found it difficult to choose between the different techniques to emphasize in our relatively brief coverage. However, due to a number of developments in the late 1980s and the 1990s, partial differential equation (PDE)-based methods, and in particular the so-called Finite Difference Time Domain (FDTD) method, have emerged as the methods with arguably the broadest range of applicability. This is especially true for electromagnetic problems involving complex and dispersive media, photonics applications, and modeling of high-speed circuits and devices. In addition, FDTD modeling of practical problems can now be undertaken with computer resources readily available to individual users. Finally, and quite importantly for our purposes, FDTD methods are relatively straightforward and intuitively follow from a physical understanding of Maxwell's equations, making this topic particularly suitable for both undergraduate and first-year graduate students in the context of a mezzanine-level course. Students with limited or no prior modeling experience will find that the FDTD method is the simplest and most insightful method from which to start their modeling education, and they can write practical and useful simulations in a matter of minutes. With an understanding of the FDTD method under their belts, students can move on to understanding more complicated methods with relative ease.

A number of software packages exist for modeling in the FDTD method, both free, open-source packages and very expensive industrial software suites. Many of these packages are very powerful. In our experience teaching the FDTD method, however, we find that students get far more out of the course by coding their own simple FDTD problems rather than simply pushing buttons in a software package. To this end, this book is targeted toward a comprehensive understanding of the fundamental inner workings and details of the FDTD method. We provide all of the basic background material to understanding how difference equations are formed from differential equations; how instability and numerical dispersion arise; and how to deal with sources, boundary conditions, complex materials, and complicated structures. We provide a number of problems in each chapter, some of which require coding simulations from scratch. These problems are presented in a logical progression, so that codes developed in early chapters can be easily modified to include the new material in later chapters. Furthermore, we

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encourage the students to modify the basic codes outlined in the problems in order to explore problems of interest to them.

Acknowledgments

The material presented in this book was initially put together as class notes for the EE256 Numerical Electromagnetics course, newly introduced and taught by one of us (USI) at Stanford University in the spring quarter of 1999. The course was then taught regularly every other year, for graduate students from Electrical Engineering, Applied Physics, and Physics. Then Ph.D. student Dr. Michael Chevalier served as the Teaching Assistant (TA) for the course in the spring of 1999, 2001, and 2003, while the other one of us (RAM) served as TA in the spring of 2005 and 2007 and also taught the course in the summer of 2008. Ph.D. student Forrest Foust served as the TA in spring 2009, when the course was taught for the last time by one of us (USI). The original set of course notes from 1999 significantly improved over the years, with the help of the TAs cited above, and by the feedback, comments, and suggestions of the many students who took the course. We would like to extend our sincere gratitude especially to Forrest Foust, who put in countless hours of proofreading on short notice and made many valuable suggestions. Sarah Matthews, Julie Lancashire, and Emma Walker at Cambridge were extraordinarily helpful and patient during the development of this text. Finally, both of us want to acknowledge our families for their patience and encouragement during the process.

On the cover: 2D FDTD simulation of a Luneburg lens. A Luneburg lens, proposed by Rudolf Luneburg in 1944, is a dielectric sphere whose permittivity varies with radius as $\epsilon = \epsilon_0[2 - (r/R)^2]$*, where R is the radius of the sphere. When a point source is excited at the surface of the lens, the varying index of refraction creates a plane wave at the output of the lens. The opposite is also true: a plane wave incident on the lens will focus to a point at the opposite side. Recently, an adaptation of the Luneburg lens has been proposed as a compact, high-speed receiver for internet connectivity on trains.*

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