

An Introduction to Mathematics for Economics

An Introduction to Mathematics for Economics introduces quantitative methods to students of economics and finance in a succinct and accessible style. The introductory nature of this textbook means a background in economics is not essential, as it aims to help students appreciate that learning mathematics is relevant to their overall understanding of the subject. Economic and financial applications are explained in detail before students learn how mathematics can be used, enabling students to learn how to put mathematics into practice. Starting with a revision of basic mathematical principles the second half of the book introduces calculus, emphasising economic applications throughout. Appendices on matrix algebra and difference/differential equations are included for the benefit of more advanced students. Other features, including worked examples and exercises, help to underpin the readers' knowledge and learning. Akihito Asano has drawn upon his own extensive teaching experience to create an unimimidating yet rigorous textbook.

Akihito Asano is Associate Professor of Economics at the Faculty of Liberal Arts, Sophia University, Tokyo. He has previously held positions at the University of Melbourne and the Australian National University (ANU). In 2008 he received the Award for Teaching Excellence from the College of Business and Economics at the ANU. He currently teaches introductory and intermediate microeconomics, international trade and introduction to game theory to undergraduate students, and mathematical techniques in economics to graduate students.

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AKIHITO ASANO

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