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Professor of Mathematics, Hokkaido University, Japan

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*Finite groups and  
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TRANSLATED BY  
A. SEVENSTER AND T. OKUYAMA

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Dedicated to the memory of

TADASI NAKAYAMA

1912–1964

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## *Preface*

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Concepts such as geometric symmetry, symmetric transformations and so on, that are introduced in elementary school mathematics, form the basis for the interrelation between groups and geometries. Symmetric transformation is a special type of motion, i.e. a transformation of Euclidean space that preserves distances, and the group of all motions – the group of automorphisms of the Euclidean space – characterizes Euclidean geometry. This is the basic idea of the so-called Erlangen Program of Klein – the group theoretical observation – according to which the greater part of geometry, especially classical geometry, can be considered in these terms. Recently, this approach has pervaded many parts of mathematics so that for the study of sets endowed with some structure, the group of its automorphisms often plays an important role.

The study of automorphism groups is also a powerful tool in the study of finite geometries. Given a finite geometry, the group of automorphisms is a finite group. Interesting geometric structures often give rise to interesting finite groups. Conversely, from the group theoretical point of view, there are many interesting problems in the areas related to finite geometries, such as the construction of new finite geometries and the determination of the finite geometry corresponding to a given finite group.

The aim of this book is to give a self-contained introduction to some fundamental results about relations between finite groups and finite geometries from the group theoretical point of view. In order to guide the reader to an understanding of the recent problem of the characterization of projective transformation groups as permutation groups, I explain the fundamental results in related areas (elementary to intermediate level difficulty) necessary to gain this understanding. A few recent results are included in the final part of the book.

Apart from high-school mathematics, the reader is expected to have a modest knowledge of linear algebra – reviewed briefly in chapter 1. The theory of finite groups and finite geometries is developed from basic principles to the point where problems mentioned above may be understood.

Since 1960, research on finite groups has mushroomed and diversified rapidly. Therefore, even though this book is restricted to areas related with finite geometries, it covers only a small part of this topic and offers far from a full picture. It is my hope that this book will arouse the reader's interest in this subject.

I was encouraged to write this book by Professor Nagayoshi Iwahori of Tokyo University, who, together with Professor Hiroshi Nagao of Osaka University was kind enough to give many useful suggestions with regard to content and organization of this book. My colleagues at Hokkaido University, Professor Hiroshi Kimura and Dr Shiro Iwasaki, and also Mr Tomoyuki Wada of Otaru University of Commerce corrected the manuscript and the proof-sheets and pointed out many mistakes and imperfections. Mr Hideo Arai of Iwanami Shoten was very helpful and patient while I was preparing the manuscript for this book. I want to express my heartfelt feelings of gratitude for their kindness.

March 1976

TOSIRO TSUZUKU

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## *Preface to the English edition*

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This book is a complete translation of the Japanese text of my *Yugengun to Yugenkika* published by Iwanami Shoten (Tokyo, 1976).

I owe the translation to Dr Arjen Sevenster and Dr Tetsuro Okuyama. I would like to express my heartfelt thanks to Dr Sevenster who has demonstrated how to combine scrupulous respect for the text with its effective adaptation to the spirit of the English language, and to Dr Okuyama who has checked the text during translation and made many useful suggestions for improvement.  
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September 1980

TOSIRO TSUZUKU