INTRODUCTION

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EUCLID

ANYONE writing on Euclid would wish to be in a position to give some particulars of the life and personality of the man who wrote a great classic the reign of which, at least among the civilised nations of the West, can hardly have been paralleled by that of any book except the Bible.

Unfortunately however we have only the most meagre information about him. It was apparently the fate of men of science in Greece as elsewhere to make little noise in the world as such. A still greater man, Archimedes, only appears in history because he figured as a defender of Syracuse during the siege of that place by Marcellus in the Second Punic War; his mechanical appliances (engines of war) were a constant terror to the Romans, and it was these which made him famous, while he himself thought meanly of mechanics and all practical arts and devoted himself, heart and soul, to the theoretical investigation of abstruse problems in pure mathematics.

Euclid is claimed by Proclus as a Platonist, but all that can be assumed is that he received his mathematical training at Athens from the pupils of Plato; for most of the geometers who could have taught him belonged to that school, and it was in Athens that the older writers of Elements had lived and taught.

It is clear that he was in date intermediate between the first pupils of Plato (and, in particular, Eudoxus, who lived approximately from 408 to 355 B.C.) on the one hand and Archimedes on the other (for Archimedes quotes him textually). He is assigned to the reign of the first Ptolemy (306–283 B.C.), and he flourished probably about 300 B.C. or a little earlier. He taught and founded a school at Alexandria, for we are told that Apollonius of Perga, the author of the great treatise on *Conics*, spent a long time with the pupils of Euclid at Alexandria.

There are some stories of him which one would like to believe true. Stobaeus tells us that some one who had begun to read geometry with Euclid, when he had learnt the first proposition, inquired, 'What shall I get by learning these things?' whereon Euclid called his slave and said, 'Give him threepence, since he must needs make gain out of what he learns.'

In another story he appears as the instructor of Ptolemy in geometry. Ptolemy evidently, like some moderns, found the subject long-drawn-out, for he asked whether there was not any shorter way to geometry than that of the *Elements*, to which Euclid replied that there was no royal road to geometry.

The Arabs, who assimilated Greek geometry with avidity, have more circumstantial accounts of Euclid, to the effect, e.g., that he was the son of Naucrates and grandson of Zenarchus, a Greek born at Tyre and domiciled at Damascus. But the Arabian stories seem to be partly due to misunderstandings and partly invented in order to gratify a desire which the Arabs always showed of connecting famous Greeks in some way or other with the East (the same predilection made them describe Pythagoras as a pupil of the wise Salomo, Hipparchus as the exponent of Chaldaean philosophy or as the Chaldaean, Archimedes as an Egyptian, and so on). They even invented an explanation of the name of Euclid, which they variously pronounced as Uclides or Icludes, making it a compound of *Ucli*, a key, and *Dis*, a measure, or, as some say, geometry, so that Uclides is equivalent to the key of geometry!

WORKS OTHER THAN THE ELEMENTS

The *Elements*, though the most famous, was only one of many treatises by Euclid. He seems indeed to have covered the whole field of mathematics as then known.

In enumerating the other known works of Euclid, we will begin with those which, like the *Elements*, were concerned with elementary geometry.

The *Pseudaria*, or *Fallacies*, which seems to be irretrievably lost, was a sort of foil to the *Elements*. In the efforts of geometers to discover new theorems or to solve new problems, many cases had arisen where the authors, although basing themselves on the true principles of the science, had through errors in reasoning or method deduced conclusions which were false. Euclid's book seems to have been a sort of guide to the causes of error, and to have contained illustrations of the false deductions, with practical hints to enable the beginner to detect the flaw in the argument, to refute it, and, by dint of practice in distinguishing the true from the false, to avoid such errors in his own work.

The *Data* (δεδομένα) is fortunately preserved in Greek, and the full text with the commentary by Marinus has been newly edited by Menge (Heiberg and Menge's *Euclidis opera omnia*, vol. VI, Teubner, 1896). A translation of the *Data* was also included in the later editions of Simson's Euclid (though his text left much to be desired). The form of the *Data* is that of propositions proving that, if certain things in a figure are 'given' (in magnitude, in position, or in *species*, i.e. in shape) something else is 'given,' that is to say, can be actually determined. This collection of *Data*, convenient for reference, enabled the solution of small subsidiary problems occurring in a larger investigation to be taken for granted instead of being worked out on each occasion. We may, by way of example, quote one enunciation, namely that of the proposition which gives the equivalent of the solution of a general quadratic equation. 'If two straight lines contain a parallelogram given in magnitude in a given angle, and if the sum of the straight lines be given, then shall each of them be given'; that is, if $xy = b^2$, a given area, and $x + y = a$, a given length, then x and y are both given, i.e. can be determined.

Περὶ διαιρέσεων βιβλίον, a tract *On Divisions* (of figures), is lost in Greek but has been discovered in the Arabic. Woepcke found it in a MS. at Paris and published it with a translation in 1851. It is expressly attributed to Euclid and corresponds to the description of it by Proclus. There is no doubt that the tract edited by Woepcke is not only Euclid's own work but the whole of it. It has been restored and published with full commentary and introductory chapters by R. C. Archibald (Cambridge, 1915). The divisions of figures are divisions into other figures either like or unlike; thus a triangle is divided into triangles or into a triangle and a quadrilateral; a figure bounded by an arc of a circle and two straight lines drawn from a point to the extremities of the arc is divided into two equal parts; from a circle a given fraction of it is cut off between two parallel chords; and so on.

To higher geometry belong the following.

Three books of *Porisms*. These are lost, and all we know of them is contained in the *Collection* of Pappus. There have been several attempts at restoration, but none that can be regarded as disposing of the subject. This much is clear, that the *Porisms* belonged to higher geometry and contained propositions forming part of the modern theory of transversals and of projective geometry. It contained the basis of the theory of anharmonic ratios.

The *Surface-Loci* (τόποι πρὸς ἐπιφανείᾳ). This too is lost, and nothing is known of it except from Pappus, who mentions the book as part of the 'Treasury of Analysis' (τόπος ἀναλυόμενος) and gives two lemmas on it. It seems to have dealt with such loci as are cones, cylinders and spheres. One of Pappus's lemmas states and proves completely the focus-directrix property of the three conic sections, which may therefore be taken to have been assumed in Euclid's work as known.

The *Conics*. This treatise too is lost, having evidently been superseded by the great work of Apollonius about a century later. Pappus says of it, 'The four books of Euclid's Conics were completed by Apollonius, who added four more and gave us eight books of Conics.' It is probable that Euclid's work was already lost in Pappus's time, for he goes on to speak of 'Aristaeus who wrote the *still extant* five books of Solid Loci connected with the Conics.' 'Solid Loci' (στερεοὶ τόποι) in Greek terminology were actually conic sections. Probably Aristaeus's work was on conics regarded as loci, and Euclid's treatise, though general in scope like that of Apollonius, was confined to those properties which were necessary for the analysis of the *Solid Loci* of Aristaeus.

The *Phaenomena*. This is an astronomical work and is still extant. A much interpolated version

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Excerpt
More information

appeared in Gregory's edition of Euclid, but it has now been edited by Menge from an earlier and better source (*Euclidis opera omnia*, vol. VIII, Teubner, 1916). The book consists of 16 (or 18) propositions of *sphaeric* geometry and is based partly on Autolycus's work *On the moving sphere*, and partly on an earlier text-book of *Sphaerica* of exclusively mathematical content.

The *Optics*. This book too survives and is included in Heiberg and Menge's edition (vol. VII, Teubner, 1895). The *Catoptrica* (theory of mirrors) included by Heiberg in the same volume is not by Euclid, and Heiberg suspects that in its present form it may be due to Theon of Alexandria (fourth century A.D.), the editor of the *Elements*.

Euclid is also said to have written on the Elements of Music. Two treatises are attributed to him in our MSS. of the *Musici*. One is the *Sectio Canonis* (κατατομὴ κανόνος), the theory of the musical intervals. Its genuineness is however doubtful. Jan, the editor of the *Musici*, thought it genuine, on two main grounds, (1) that the form and style agree well with what we find in the *Elements* and (2) that in an ancient commentary on Ptolemy's *Harmonica* Euclid is thrice mentioned as the author of the *Sectio*, and almost the whole of the treatise except the preface is quoted *in extenso*. On the other hand Tannery disputed the authenticity, and the latest editor, Menge, who finds in the tract a number of expressions and at all events one proof which are not in Euclid's manner, concludes that in its present form it is not Euclid's but is more likely to have been excerpted by some not too well equipped compiler from the fuller *Elements of Music* mentioned by Proclus and Marinus. The other treatise, an *Introduction to Harmonics* (εἰσαγωγὴ ἁρμονική), is not by Euclid but by Cleonides, a pupil of Aristoxenus. Both treatises are included in Heiberg and Menge's edition, vol. VIII (Teubner, 1916).

EARLIER WRITERS OF ELEMENTS

The first writer of Elements was, we are told, Hippocrates of Chios (fl. in the second half of the fifth century B.C.), a great geometer, who was famous for two other achievements, (1) the reduction of the problem of doubling the cube to that of finding two mean proportionals in continued proportion between two given straight lines, (2) the quadrature of certain *lunes* which can be squared by means of the geometry of the straight line and circle. From the account of these quadratures we can judge to some extent how far the Elements had been developed up to Hippocrates's time; he is quite familiar with the main propositions of Euclid's Book III, and he is himself said to have proved that the areas of circles are to one another as the squares on their diameters (the theorem of Eucl. XII, 2). Next Leon, who came between Plato (429–348 B.C.) and Eudoxus (408–355 B.C.), put together a more careful collection, the propositions in it being at once more numerous and more serviceable. The geometrical text-book of the Academy was written by Theudius of Magnesia, who, with Amyclas of Heraclea, Menaechmus the pupil of Eudoxus, Dinostratus the brother of Menaechmus, and Athenaeus of Cyzicus, consorted together in the Academy and carried on their investigations in common. Hermotimus of Colophon is also said to have 'discovered many of the elements,' but no text-book after that of Theudius is mentioned. Theudius therefore must be taken to be the immediate precursor of Euclid. Euclid himself no doubt made full use of Theudius as well as of all other available material. Euclid is naturally silent as to his relation to his predecessor; but we find in Aristotle (who was fond of geometrical illustrations) indications of proofs of certain propositions, proofs which he must have taken from some recognised text-book (probably that of Theudius) and which differ from Euclid's proofs. Some light is thus thrown on the changes which Euclid made in the methods of his predecessors.

CONTENTS OF EUCLID'S ELEMENTS

A SHORT indication of the contents of the *Elements* will not be out of place. Book I is in three sections. The first section deals mainly with triangles, their construction and their properties in the sense of the relation of their parts, the sides and angles, to one another, and the comparison of different triangles