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Cellular Biophysics and Modeling

A Primer on the Computational Biology of Excitable Cells

Cellular Biophysics and Modeling is what every neuroscientist should know about the mathematical modeling of excitable cells. Combining empirical physiology and nonlinear dynamics, this text provides an introduction to the simulation and modeling of dynamic phenomena in cell biology and neuroscience. It introduces mathematical modeling techniques alongside cellular electrophysiology. Topics include membrane transport and diffusion, the biophysics of excitable membranes, the gating of voltage and ligand-gated ion channels, intracellular calcium signaling, and electrical bursting in neurons and other excitable cell types. It introduces mathematical modeling techniques such as ordinary differential equations, phase plane analysis, and bifurcation analysis of single compartment neuron models. With analytical and computational problem sets, this book is suitable for life sciences majors, in biology to neuroscience, with one year of calculus, as well as graduate students looking for a primer on membrane excitability and calcium signaling.

Greg Conradi Smith is a Professor in the Department of Applied Science and Neuroscience Program Faculty Affiliate at William & Mary, Williamsburg, VA, USA. He was co-organizer of the Cold Spring Harbor Laboratory Summer School on Computational Cell Biology (2008–2014). His research focuses on mathematical aspects of cell physiology and neuroscience.

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GREG CONRADI SMITH
William & Mary, Williamsburg, VA

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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

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www.cambridge.org
Information on this title: www.cambridge.org/9780521183055

DOI: 10.1017/9780511793905

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First published 2019

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data
Names: Smith, Greg Conradi, 1964– author.
Title: Cellular biophysics and modeling : a primer on the computational biology of excitable cells / Greg Conradi Smith.
Description: Cambridge; New York, NY : Cambridge University Press, 2019. | Includes index.
Identifiers: LCCN 2018039283 | ISBN 9781107005365 (hardback : alk. paper) | ISBN 9780521183055 (pbk.)
Subjects: LCSH: Excitation (Physiology) | Mathematical models. | Biophysics. | Cell physiology. | Computational biology.
Classification: LCC QP363 .S595 2019 | DDC 612/.014–dc23
LC record available at https://lccn.loc.gov/2018039283

ISBN 978-1-107-00536-5 Hardback
ISBN 978-0-521-18305-5 Paperback

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Preface

Philosophy is written in this grand book – I mean universe – which stands continuously open to our gaze, but which cannot be understood unless one first learns to comprehend the language in which it is written. It is written in the language of mathematics, and its characters are triangles, circles and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth. — **Galileo Galilei (1564–1642)**

Most students of life science accept Galileo's statement that "triangles, circles and other geometric figures" are necessary to fully understand the cosmos. But many of these students – and perhaps also their professors – have significant doubts about the relevance of mathematics to the science of life on earth.

Admittedly, biology and mathematics sometimes appear immiscible. Like oil and water, the combination does not yield a homogenous mixture of liquids, but an emulsion. When biology and mathematics are viewed as two disparate subjects in an undergraduate education, attempts to forcefully stir one into the other result in something like well-shaken Italian salad dressing, the two dispersed liquid phases having a natural tendency to separate. Because we have no surfactant to stabilize the bio-math emulsion, we shake again. In the process, the students become agitated, too!

Love of science and fear of mathematics have led many to major in biology, psychology and neuroscience. There is no shame in acknowledging this fact. Within the life sciences there are many important research questions that can be asked and answered without mathematics. Many topics covered in biology, psychology and neuroscience courses can be explained and understood without mathematical language. There are numerous scientific and health- and education-related fields that do not require mathematical aptitude, but do need intelligent and resourceful young scientists and science majors.

On the other hand, many life sciences have theoretical foundations that were developed by quantitative scientists using mathematical language (e.g., population genetics). Other life sciences, such as molecular biology and genomics, have become so complex and data rich that most practitioners would appreciate more quantitative aptitude and perspective – if not for themselves, then at least for their trainees. Contemporary life scientists who are at ease with mathematics use quantitative reasoning in the study of life on every scale: molecules, membranes, cells, networks, organisms, behavior, evolution and ecology. Both pure and applied biomedical research is replete with open scientific questions (e.g., protein folding) and technical

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challenges (e.g., rational drug design) whose solutions will be found by biological scientists who are comfortable with mathematics and computation.

In my opinion, mathematics is the language of all natural science, biology and neuroscience no less than astronomy, physics and chemistry. This extension of Galileo's conviction to the realm of neuroscience is, admittedly, a philosophical statement that is open to discussion. I encourage you to think it over, talk to your peers and mentors, and decide for yourself.

Certainly, this book is a combination of biology (cellular biophysics) and mathematics (dynamical systems modeling) written from a Galilean perspective. Is it a homogenous mixture or emulsion? I cannot say. But it is a sweet mix.

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