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978-0-521-18301-7 - Lectures on Profinite Topics in Group Theory

Benjamin Klopsch, Nikolay Nikolov and Christopher Voll

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