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978-0-521-17739-9 - Symmetry and Separation of Variables: Encyclopedia of Mathematics and its Applications: Volume 4

Willard Miller

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## *Subject Index*

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## Subject Index

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