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978-0-521-17561-6 - Random Fields on the Sphere: Representation, Limit Theorems
and Cosmological Applications

Domenico Marinucci and Giovanni Peccati

Table of Contents

[More information](#)

Contents

	<i>Preface</i>	<i>page xi</i>
1	Introduction	1
	1.1 Overview	1
	1.2 Cosmological motivations	2
	1.3 Mathematical framework	5
	1.4 Plan of the book	7
2	Background Results in Representation Theory	12
	2.1 Introduction	12
	2.2 Preliminary remarks	13
	2.3 Groups: basic definitions	15
	2.4 Representations of compact groups	19
	2.5 The Peter-Weyl Theorem	39
3	Representations of $SO(3)$ and Harmonic Analysis on S^2	45
	3.1 Introduction	45
	3.2 Euler angles	46
	3.3 Wigner's D matrices	51
	3.4 Spherical harmonics and Fourier analysis on S^2	63
	3.5 The Clebsch-Gordan coefficients	77
4	Background Results in Probability and Graphical Methods	85
	4.1 Introduction	85
	4.2 Brownian motion and stochastic calculus	86
	4.3 Moments, cumulants and diagram formulae	94
	4.4 The simplified method of moments on Wiener chaos	99
	4.5 The graphical method for Wigner coefficients	104
5	Spectral Representations	114
	5.1 Introduction	114

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and Cosmological Applications

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Table of Contents

[More information](#)

viii	<i>Contents</i>	
	5.2 The Stochastic Peter-Weyl Theorem	114
	5.3 Weakly stationary random fields in \mathbb{R}^m	126
	5.4 Stationarity and weak isotropy in \mathbb{R}^3	130
6	Characterizations of Isotropy	134
	6.1 Introduction	134
	6.2 First example: the cyclic group	135
	6.3 The spherical harmonics coefficients	137
	6.4 Group representations and polyspectra	147
	6.5 Angular polyspectra and the structure of Δ_{l_1, \dots, l_n}	151
	6.6 Reduced polyspectra of arbitrary orders	156
	6.7 Some examples	160
7	Limit Theorems for Gaussian Subordinated Random Fields	169
	7.1 Introduction	169
	7.2 First example: the circle	171
	7.3 Preliminaries on Gaussian-subordinated fields	173
	7.4 High-frequency CLTs	176
	7.5 Convolutions and random walks	181
	7.6 Further remarks	188
	7.7 Application: algebraic/exponential dualities	190
8	Asymptotics for the Sample Power Spectrum	194
	8.1 Introduction	194
	8.2 Angular power spectrum estimation	196
	8.3 Interlude: some practical issues	201
	8.4 Asymptotics in the non-Gaussian case	205
	8.5 The quadratic case	210
	8.6 Discussion	219
9	Asymptotics for Sample Bispectra	223
	9.1 Introduction	223
	9.2 Sample bispectra	224
	9.3 A central limit theorem	230
	9.4 Limit theorems under random normalizations	234
	9.5 Testing for non-Gaussianity	238
10	Spherical Needlets and their Asymptotic Properties	245
	10.1 Introduction	245
	10.2 The construction of spherical needlets	248
	10.3 Properties of spherical needlets	252
	10.4 Stochastic properties of needlet coefficients	257
	10.5 Missing observations	259

Cambridge University Press

978-0-521-17561-6 - Random Fields on the Sphere: Representation, Limit Theorems
and Cosmological Applications

Domenico Marinucci and Giovanni Peccati

Table of Contents

[More information](#)

<i>Contents</i>		ix
	10.6 Mexican needlets	261
11	Needlets Estimation of Power Spectrum and Bispectrum	265
	11.1 Introduction	265
	11.2 A general convergence result	265
	11.3 Estimation of the angular power spectrum	272
	11.4 A functional central limit theorem	273
	11.5 A central limit theorem for the needlets bispectrum	275
12	Spin Random Fields	282
	12.1 Introduction	282
	12.2 Motivations	284
	12.3 Geometric background	286
	12.4 Spin needlets and spin random fields	290
	12.5 Spin needlets spectral estimator	294
	12.6 Detection of asymmetries	302
	12.7 Estimation with noise	305
13	Appendix	312
	13.1 Orthogonal polynomials	312
	13.2 Spherical harmonics and their analytic properties	317
	13.3 The proof of needlets' localization.	320
	<i>References</i>	326
	<i>Index</i>	338