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978-0-521-17523-4 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

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