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978-0-521-17523-4 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

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Although topology was recognized by Gauss and Maxwell to play a pivotal role in the formulation of electromagnetic boundary value problems, it is a largely unexploited tool for field computation. The development of algebraic topology since Maxwell provides a framework for linking data structures, algorithms, and computation to topological aspects of three-dimensional electromagnetic boundary value problems. This book attempts to expose the link between Maxwell and a modern approach to algorithms.

The first chapters lay out the relevant facts about homology and cohomology, stressing their interpretations in electromagnetism. These topological structures are subsequently tied to variational formulations in electromagnetics, the finite element method, algorithms, and certain aspects of numerical linear algebra. A recurring theme is the formulation of and algorithms for the problem of making branch cuts for computing magnetic scalar potentials and eddy currents. An appendix bridges the gap between the material presented and standard expositions of differential forms, Hodge decompositions, and tools for realizing representatives of homology classes as embedded manifolds.

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# Electromagnetic Theory and Computation: A Topological Approach

**Paul W. Gross**

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## reface

The authors are long-time fans of MSRI programs and monographs, and are thrilled to be able to contribute to this series. Our relationship with MSRI started when Paul Gross was an MSRI/Hewlett-Packard postdoctoral fellow and had the good fortune of being encouraged by Silvio Levy to coauthor a monograph. Silvio was there when we needed him, and it is in no way an understatement to say that the project would never have been completed without his support.

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Paul Gross and Robert Kotiuga  
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