Although topology was recognized by Gauss and Maxwell to play a pivotal role in the formulation of electromagnetic boundary value problems, it is a largely unexploited tool for field computation. The development of algebraic topology since Maxwell provides a framework for linking data structures, algorithms, and computation to topological aspects of three-dimensional electromagnetic boundary value problems. This book attempts to expose the link between Maxwell and a modern approach to algorithms.

The first chapters lay out the relevant facts about homology and cohomology, stressing their interpretations in electromagnetism. These topological structures are subsequently tied to variational formulations in electromagnetics, the finite element method, algorithms, and certain aspects of numerical linear algebra. A recurring theme is the formulation of and algorithms for the problem of making branch cuts for computing magnetic scalar potentials and eddy currents. An appendix bridges the gap between the material presented and standard expositions of differential forms, Hodge decompositions, and tools for realizing representatives of homology classes as embedded manifolds.

Mathematical Sciences Research Institute
Publications

48

Electromagnetic Theory and Computation
A Topological Approach

Mathematical Sciences Research Institute Publications

1 Freed/Uhlenbeck: *Instantons and Four-Manifolds*, second edition
2 Chern (ed.): *Seminar on Nonlinear Partial Differential Equations*
3 Lepowsky/Mandelstam/Singer (eds.): *Vertex Operators in Mathematics and Physics*
4 Kac (ed.): *Infinite Dimensional Groups with Applications*
5 Blackadar: *K-Theory for Operator Algebras*, second edition
6 Moore (ed.): *Group Representations, Ergodic Theory, Operator Algebras, and Mathematical Physics*
7 Chorin/Majda (eds.): *Wave Motion: Theory, Modelling, and Computation*
8 Gersten (ed.): *Essays in Group Theory*
9 Moore/Schochet: *Global Analysis on Foliated Spaces*
10–11 Drasin/Earle/Gehring/Kra/Marden (eds.): *Holomorphic Functions and Moduli*
12–13 Ni/Peletier/Serrin (eds.): *Nonlinear Diffusion Equations and Their Equilibrium States*
14 Goodman/de la Harpe/Jones: *Coxeter Graphs and Towers of Algebras*
15 Hochster/Huneke/Sally (eds.): *Commutative Algebra*
16 Ihara/Ribet/Serre (eds.): *Galois Groups over* $\mathbb{Q}$
17 Concus/Finn/Hoffman (eds.): *Geometric Analysis and Computer Graphics*
18 Bryant/Chern/Gardner/Goldschmidt/Griffiths: *Exterior Differential Systems*
19 Alperin (ed.): *Arboreal Group Theory*
20 Dazord/Weinstein (eds.): *Symplectic Geometry, Groupoids, and Integrable Systems*
21 Moschovakis (ed.): *Logic from Computer Science*
22 Ratiu (ed.): *The Geometry of Hamiltonian Systems*
23 Baumslag/Miller (eds.): *Algorithms and Classification in Combinatorial Group Theory*
24 Montgomery/Small (eds.): *Noncommutative Rings*
25 Akbulut/King: *Topology of Real Algebraic Sets*
26 Judah/Just/Woodin (eds.): *Set Theory of the Continuum*
27 Carlsson/Cohen/Hsiang/Jones (eds.): *Algebraic Topology and Its Applications*
28 Clemens/Kollár (eds.): *Current Topics in Complex Algebraic Geometry*
29 Nowakowski (ed.): *Games of No Chance*
30 Grove/Petersen (eds.): *Comparison Geometry*
31 Levy (ed.): *Flavors of Geometry*
32 Cecil/Chern (eds.): *Tight and Taut Submanifolds*
33 Axler/McCarthy/Sarason (eds.): *Holomorphic Spaces*
34 Ball/Milman (eds.): *Convex Geometric Analysis*
35 Levy (ed.): *The Eightfold Way*
36 Gavosto/Krantz/McCallum (eds.): *Contemporary Issues in Mathematics Education*
37 Schneider/Siu (eds.): *Several Complex Variables*
38 Billera/Björner/Green/Simion/Stanley (eds.): *New Perspectives in Geometric Combinatorics*
39 Haskell/Pillay/Steinhorn (eds.): *Model Theory, Algebra, and Geometry*
40 Bleher/Its (eds.): *Random Matrix Models and Their Applications*
41 Schneps (ed.): *Galois Groups and Fundamental Groups*
42 Nowakowski (ed.): *More Games of No Chance*
43 Montgomery/Schneider (eds.): *New Directions in Hopf Algebras*
44 Buhler/Stevenhagen (eds.): *Algorithmic Number Theory*
45 Jensen/Ledet/Yui: *Generic Polynomials: Constructive Aspects of the Inverse Galois Problem*
46 Rockmore/Healy (eds.): *Modern Signal Processing*
47 Uhlmann (ed.): *Inside Out: Inverse Problems and Applications*
48 Gross/Kotiuga: *Electromagnetic Theory and Computation: A Topological Approach*

Volumes 1–4 and 6–27 are published by Springer-Verlag

Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross
MSRI and HP/Agilent

P. Robert Kotiuga
Boston University

CAMBRIDGE
UNIVERSITY PRESS

Paul Gross
pwgross@alum.bu.edu

P. Robert Kotiuga
Department of Electrical
and Computer Engineering
Boston University
8 Saint Mary's Street
Boston, MA 02215
United States
prk@bu.edu

The Mathematical Sciences Research Institute wishes to acknowledge support by the National Science Foundation. This material is based upon work supported by NSF Grant 9810361.

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521175234

First published 2004
First paperback edition 2011

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-80160-7 Hardback
ISBN 978-0-521-17523-4 Paperback

Table of Contents

Preface

The authors are long-time fans of MSRI programs and monographs, and are thrilled to be able to contribute to this series. Our relationship with MSRI started when Paul Gross was an MSRI/Hewlett-Packard postdoctoral fellow and had the good fortune of being encouraged by Silvio Levy to coauthor a monograph. Silvio was there when we needed him, and it is in no way an understatement to say that the project would never have been completed without his support.

The material of this monograph is easily traced back to our Ph.D. theses, papers we wrote, and courses taught at Boston University over the years. Our apologies to anyone who feels slighted by a minimally updated bibliography. Reflecting on how the material of this monograph evolved, we would like to thank colleagues who have played a supporting role over the decades. Among them are Alain Bossavit, Peter Caines, Roscoe Giles, Robert Hermann, Lauri Kettunen, Isaak Mayergoyz, Peter Silvester, and Gilbert Strang. The authors are also indebted to numerous people who read through all or part of the manuscript, produced numerous comments, and provided all sorts of support. In particular, Andre Nicolet, Jonathan Polimeni, and Saku Suuriniemi made an unusually thorough effort to review the draft.

Paul Gross would like to acknowledge Nick Tufillaro at Hewlett-Packard and Agilent Technologies for mentoring him throughout his post-doc at MSRI. Tim Dere graciously provided his time and expertise for illustrations. This book could not have happened without help and encouragement from Tanya.

Robert Kotiuga is grateful to the students taking his courses at Boston University, to Nevine, Michele, Madeleine, Peter and Helen for their support, and to Boston University for granting him a leave while the book was in its final stages.

Paul Gross and Robert Kotiuga
August 2003