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# Real and Complex Singularities

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This volume is dedicated to  
*Maria Aparecida Ruas*  
and  
*Terence Gaffney*  
on their 60<sup>th</sup> birthdays



*Maria Aparecida Ruas*



*Terence Gaffney*

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## Preface

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The *Workshops on Real and Complex Singularities* form a series of biennial meetings organized by the Singularities group at Instituto de Ciências Matemáticas e de Computação of São Paulo University (ICMC-USP), Brazil. Their main purpose is to bring together world experts and young researchers in singularity theory, applications and related fields to report recent achievements and exchange ideas, addressing trends of research in a stimulating environment.

These meetings started in 1990 following two pioneer symposia on Singularity Theory (in fact the very first talks on Singularities held in Brazil) organized respectively by G. Loibel and L. Favaro in 1982 and 1988 at ICMC-USP. Since then, with Maria Aparecida Ruas as a driving force, meetings have taken place every two years between singularists from around the world who find in São Carlos a centre to interact and develop new ideas.

The meeting held from the 27th of July to the 2nd of August 2008 was the tenth of these workshops. This was a special occasion, for it was also dedicated to Maria Aparecida Ruas (Cidinha) and Terence Gaffney on their 60th birthdays.

Cidinha and Terry started their scientific connection in 1976 when she was a Ph.D. student at Brown University in the U.S.A. At that time Terry held a position as instructor at that university. Their common interest in singularity theory brought them together and he became her (very young) thesis supervisor. Cidinha returned to Brazil in 1980 and joined the Singularities group created by G. Loibel at the ICMC-USP. Her great capacity and notable enthusiasm has brought the group to a leading position in the Brazilian mathematical community.

The mathematical interaction between Cidinha and Terry has had an important influence on the development of Singularity Theory at the ICMC-USP. From the beginning Terry has attended almost all workshops organized by the

members of Cidinha's group, contributing to their research with stimulating discussions and seminars.

This workshop had a total of about 170 participants from about 15 different countries. The formal proceedings consisted of 27 plenary talks, 27 ordinary sessions and 3 poster sessions, with a total of 19 posters. The topics were divided into six categories: real singularities, classification of singularities, topology of singularities, global theory of singularities, singularities in geometry, and dynamical systems.

The Scientific Committee was composed of Lev Birbrair (Universidade Federal do Ceará, Brazil), Jean-Paul Brasselet (Institut de Mathématiques de Luminy, France), Goo Ishikawa (Hokkaido University, Japan), Shyuichi Izumiya (Hokkaido University, Japan), Steven Kleiman (Massachusetts Institute of Technology, USA), David Massey (Northeastern University, USA), David Mond (University of Warwick, UK), Maria del Carmen Romero Fuster (Universitat de València, Spain), Marcio Gomes Soares (Universidade Federal de Minas Gerais, Brazil), Marco Antonio Teixeira (Universidade Estadual de Campinas, Brazil), David Trotman (Université de Provence, France) and Terry Wall (University of Liverpool, UK).

Thanks are due to many people and institutions crucial in the realization of the workshop. We start by thanking the Organizing Committee: Roberta Wik Atique, Abramo Hefez, Isabel Labouriau, Miriam Manoel, Ana Claudia Nabarro, Regilene Oliveira and Marcelo José Saia. We also thank the members of the Scientific Committee for their support. The workshop was funded by FAPESP, CNPq, CAPES, USP and SBM, whose support we gratefully acknowledge. Finally, it is a pleasure to thank the speakers and the other participants whose presence was the real success of the tenth Workshop.

The editors

## Introduction

This book is a selection of papers submitted for the proceedings of the *10th Workshop on Real and Complex Singularities*. They are grouped into three categories: singularity theory (7 papers), singular varieties (8 papers) and applications to dynamical systems, generic geometry, singular foliations, etc. (10 papers). Among them, four are survey papers: *Local Euler obstruction, old and new, II*, by N. G. Grulha Jr. and J.-P. Brasselet, *Global classifications and graphs*, by J. Martínez-Alfaro, C. Mendes de Jesus and M. C. Romero-Fuster, *Pairs of foliations on surfaces*, by F. Tari, and *Gaffney's work on equisingularity*, by C. T. C. Wall.

We thank the staff members of the London Mathematical Society involved with the preparation of this book. All papers presented here have been refereed.