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978-0-521-15563-2 - More Games of No Chance
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This book is a state-of-the-art look at combinatorial games, that is, games not involving chance or hidden information. It contains articles by some of the foremost researchers and pioneers of combinatorial game theory, such as Elwyn Berlekamp and John Conway, by other researchers in mathematics and computer science, and by top game players.

The articles run the gamut from new theoretical approaches (infinite games, generalizations of game values, two-player cellular automata, alpha-beta pruning under partial orders) to the very latest in some of the hottest games (Amazons, Chomp, Dot-and-Boxes, Go, Chess, Hex). Many of these advances reflect the interplay of the computer science and the mathematics. The book ends with an updated bibliography by A. Fraenkel and an updated version of the famous annotated list of combinatorial game theory problems by R. K. Guy, now in collaboration with R. J. Nowakowski.

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Dalhousie University



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Series Editor
 Silvio Levy
 Mathematical Sciences
 Research Institute
 1000 Centennial Drive
 Berkeley, CA 94720
 United States

Richard J. Nowakowski
 Department of Mathematics
 Dalhousie University
 Halifax, NS
 B3H 3J5
 Canada

MSRI Editorial Committee
 Michael Singer (chair)
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Preface

This volume arose from the second Combinatorial Games Theory Workshop and Conference, held at MSRI from July 24 to 28, 2000. The first such conference at MSRI, which took place in 1994, gave a boost to the relatively new field of Combinatorial Game Theory (CGT); its excitement is captured in *Games of No Chance* (Cambridge University Press, 1996), which includes an introduction to CGT and a brief history of the subject. In this volume we pick up where *Games of No Chance* left off.

Although Game Theory overlaps many disciplines, the majority of the researchers are in mathematics and computer science. This was the first time that the practitioners from both camps were brought together deliberately, and the results are impressive. This bringing together seems to have formed a critical mass. There has already been a follow-up workshop at Dagstuhl (February 2002) and more are planned.

This conference greatly expanded upon the accomplishments of and questions posed at the first conference. What is missing from this volume are the reports of games that were played and analyzed at the conference; of Grossman's Dots-and-Boxes program beating everyone in sight, except for the top four humans who had it beat by the fifth move.

This volume is divided into five parts. The first deals with new theoretical developments. Calistrate, Paulhus, and Wolfe correct a mistake about the ordering of the set of game values, a mistake that has been around for three decades or more. Not only do they show that the ordering is much richer than previously thought, they open up a whole new avenue of investigations. Conway echoes this theme of fantastic and weird structures in CGT (2 being the cube root of ω), and he introduces the smallest infinite games.

The classical games are well represented. Elkies continues his investigations in Chess. There are many new results and tantalizing hints about the deep structure of Go. Moore and Eppstein turn one-dimensional solitaire into a two-player game, and conjecture that the \mathcal{G} -values are unbounded. (In attempting to solve this, Albert, Grossman and Nowakowski defined Clobber, a big hit at the Dagstuhl conference.)

Newer games (some of which were not represented in the first volume) appear here and form the basis of the third section. Grossman and Nowakowski consider

the problem of one-dimensional phutball. Demaine et al. saw the game being played for the first time one night during the conference and by the following morning had a proof that just discovering whether the next player has a game-ending move was difficult. Amazons received much attention in the interval between the two conferences; Müller and Snatzke present some approaches.

The Puzzle section presents the results about puzzles which exhibit the same spirit as games. The game of Life has two papers devoted to it; Demaine et al. also present an entertaining look at solving coin-moving puzzles.

The proceedings is completed with updated versions of the “Unsolved Problems in Combinatorial Game Theory” by Guy and Nowakowski and Fraenkel’s bibliography.

Many thanks must go to the MSRI staff, who helped make the Workshop a success. The facilities (and weather) were great. Thanks must also go to the Workshop Chairs, Elwyn Berlekamp and David Wolfe, who did much of the hard work. Thanks also go to the other organizers (David Blackwell, John Conway, Aviezri Fraenkel, Richard Guy, Jurg Nievergelt, Jonathan Schaeffer, Ken Thompson) who, together with Berlekamp and Wolfe, put together a wonderful program.

As with the last volume, credit must also go to Silvio Levy for his suggestions and expertise in making the volume look good.

Richard J. Nowakowski