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Not all phrases containing more than one keyword are indexed under each keyword. Thus “abstract graph” appears only under “graph”, “homology group” only under “homology”, etc.

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Peter Giblin

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