Phase Noise and Frequency Stability in Oscillators

Presenting a comprehensive account of oscillator phase noise and frequency stability, this practical text is both mathematically rigorous and accessible. An in-depth treatment of the noise mechanism is given, describing the oscillator as a physical system, and showing that simple general laws govern the stability of a large variety of oscillators differing in technology and frequency range. Inevitably, special attention is given to amplifiers, resonators, delay lines, feedback, and flicker (1/f) noise. The reverse engineering of oscillators based on phase-noise spectra is also covered, and end-of-chapter exercises are given. Uniquely, numerous practical examples are presented, including case studies taken from laboratory prototypes and commercial oscillators, which allow the oscillator internal design to be understood by analyzing its phase-noise spectrum. Based on tutorials given by the author at the Jet Propulsion Laboratory, international IEEE meetings, and in industry, this is a useful reference for academic researchers, industry practitioners, and graduate students in RF engineering and communications engineering.

Additional materials are available via www.cambridge.org/rubiola.

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Phase Noise and Frequency Stability in Oscillators

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Foreword by Lute Maleki

Given the ubiquity of periodic phenomena in nature, it is not surprising that oscillators play such a fundamental role in sciences and technology. In physics, oscillators are the basis for the understanding of a wide range of concepts spanning field theory and linear and nonlinear dynamics. In technology, oscillators are the source of operation in every communications system, in sensors and in radar, to name a few. As man's study of nature's laws and human-made phenomena expands, oscillators have found applications in new realms.

Oscillators and their interaction with each other, usually as phase locking, and with the environment, as manifested by a change in their operational parameters, form the basis of our understanding of a myriad phenomena in biology, chemistry, and even sociology and climatology. It is very difficult to account for every application in which the oscillator plays a role, either as an element that supports understanding or insight or an entity that allows a given application.

In all these fields, what is important is to understand how the physical parameters of the oscillator, i.e. its phase, frequency, and amplitude, are affected, either by the properties of its internal components or by interaction with the environment in which the oscillator resides. The study of oscillator noise is fundamental to understanding all phenomena in which the oscillator model is used in optimization of the performance of systems requiring an oscillator.

Simply stated, noise is the unwanted part of the oscillator signal and is unavoidable in practical systems. Beyond the influence of the environment, and the non-ideality of the physical elements that comprise the oscillator, the fundamental quantum nature of electrons and photons sets the limit to what may be achieved in the spectral purity of the generated signal. This sets the fundamental limit to the best performance that a practical oscillator can produce, and it is remarkable that advanced oscillators can reach it.

The practitioners who strive to advance the field of oscillators in time-and-frequency applications cannot be content with knowledge of physics alone or engineering alone. The reason is that oscillators and clocks, whether of the common variety or the advanced type, are complex "systems" that interact with their environment, sometimes in ways that are not readily obvious or that are highly nonlinear. Thus the physicist is needed to identify the underlying phenomenon and the parameters affecting performance, and the engineer is needed to devise the most effective and practical approach to deal with them. The present monograph by Professor Enrico Rubiola is unique in the extent to which it satisfies both the physicist and the engineer. It also serves the need to understand both the fundamentals and the practice of phase-noise metrology, a required tool in dealing with noise in oscillators.

Rubiola's approach to the treatment of noise in this book is based on the input–output transfer functions. While other approaches lead to some of the same results, this treatment allows the introduction of a mathematical rigor that is easily tractable by anyone with an introductory knowledge of Fourier and Laplace transforms. In particular, Rubiola uses this approach to obtain a derivation, from first principles, of the Leeson formula. This formula has been used in the engineering literature for the noise analysis of the RF oscillator since its introduction by Leeson in 1966. Leeson evidently arrived at it without realizing that it was known earlier in the physics literature in a different form as the Schawlow–Townes linewidth for the laser oscillator. While a number of other approaches based on linear and nonlinear models exist for analyzing noise in an oscillator, the Leeson formula remains particularly useful for modeling the noise in high-performance oscillators. Given its relation to the Schawlow–Townes formula, it is not surprising that the Leeson model is so useful for analyzing the noise in the optoelectronic oscillator, a newcomer to the realm of high-performance microwave and millimeter-wave oscillators, which are also treated in this book.

Starting in the Spring of 2004, Professor Rubiola began a series of limited-time tenures in the Quantum Sciences and Technologies group at the Jet Propulsion Laboratory. Evidently, this can be regarded as the time when the initial seed for this book was conceived. During these visits, Rubiola was to help architect a system for the measurement of the noise of a high-performance microwave oscillator, with the same experimental care that he had previously applied and published for the RF oscillators. Characteristically, Rubiola had to know all the details about the oscillator, its principle of operation, and the sources of noise in its every component. It was only then that he could implement the improvement needed on the existing measurement system, which was based on the use of a long fiber delay in a homodyne setup.

Since Rubiola is an avid admirer of the Leeson model, he was interested in applying it to the optoelectronic oscillator, as well. In doing so, he developed both an approach for analyzing the performance of a delay-line oscillator and a scheme based on Laplace transforms to derive the Leeson formula, advancing the original, heuristic, approach. These two treatments, together with the range of other topics covered, should make this unique book extremely useful and attractive to both the novice and experienced practitioners of the field.

It is delightful to see that in writing the monograph, Enrico Rubiola has so openly bared his professional persona. He pursues the subject with a blatant passion, and he is characteristically not satisfied with "dumbing down," a concept at odds with mathematical rigor. Instead, he provides visuals, charts, and tables to make his treatment accessible. He also shows his commensurate tendencies as an engineer by providing numerical examples and details of the principles behind instruments used for noise metrology. He balances this with the physicist in him that looks behind the obvious for the fundamental causation. All this is enhanced with his mathematical skill, of which he always insists, with characteristic modesty, he wished to have more. Other ingredients, missing in the book, that define Enrico Rubiola are his knowledge of ancient languages and history. But these could not inform further such a comprehensive and extremely useful book on the subject of oscillator noise.

Lute Maleki
NASA/Caltech Jet Propulsion Laboratory
and OEwaves, Inc.,
February 2008

Foreword by David Leeson

Permit me to place Enrico Rubiola's excellent book *Phase Noise and Frequency Stability in Oscillators* in context with the history of the subject over the past five decades, going back to the beginnings of my own professional interest in oscillator frequency stability.

Oscillator instabilities are a fundamental concern for systems tasked with keeping and distributing precision time or frequency. Also, oscillator phase noise limits the demodulated signal-to-noise ratio in communication systems that rely on phase modulation, such as microwave relay systems, including satellite and deep-space links. Comparably important are the dynamic range limits in multisignal systems resulting from the masking of small signals of interest by oscillator phase noise on adjacent large signals. For example, Doppler radar targets are masked by ground clutter noise.

These infrastructure systems have been well served by what might now be termed the classical theory and measurement of oscillator noise, of which this volume is a comprehensive and up-to-date tutorial. Rubiola also exposes a number of significant concepts that have escaped prior widespread notice.

My early interest in oscillator noise came as solid-state signal sources began to be applied to the radars that had been under development since the days of the MIT Radiation Laboratory. I was initiated into the phase-noise requirements of airborne Doppler radar and the underlying arts of crystal oscillators, power amplifiers, and nonlinear-reactance frequency multipliers.

In 1964 an IEEE committee was formed to prepare a standard on frequency stability. Thanks to a supportive mentor, W. K. Saunders, I became a member of that group, which included leaders such as J. A. Barnes and L. S. Cutler. It was noted that the independent use of frequency-domain and time-domain definitions stood in the way of the development of a common standard. To promote focused interchange the group sponsored the November 1964 NASA/IEEE Conference on Short Term Frequency Stability and edited the February 1966 *Special Issue on Frequency Stability* of the *Proceedings of the IEEE*.

The context of that time included the appreciation that self-limiting oscillators and many systems (FM receivers with limiters, for example) are nonlinear in that they limit amplitude variations (AM noise); hence the focus on phase noise. The modest frequency limits of semiconductor devices of that period dictated the common usage of nonlinear-reactance frequency multipliers, which multiply phase noise to the point where it dominates the output noise spectrum. These typical circuit conditions were second nature then to the "short-term stability community" but might not come so readily to mind today.

The first step of the program to craft a standard that would define frequency stability was to understand and meld the frequency- and time-domain descriptions of phase instability to a degree that was predictive and permitted analysis and optimization. By the time the subcommittee edited the *Proc. IEEE* special issue, the wide exchange of viewpoints and concepts made it possible to synthesize concise summaries of the work in both domains, of which my own model was one.

The committee published its "Characterization of frequency stability" in *IEEE Trans. Instrum. Meas.*, May 1971. This led to the IEEE 1139 Standards that have served the community well, with advances and revisions continuing since their initial publication. Rubiola's book, based on his extensive seminar notes, is a capstone tutorial on the theoretical basis and experimental measurements of oscillators for which phase noise and frequency stability are primary issues.

In his first chapter Rubiola introduces the reader to the fundamental statistical descriptions of oscillator instabilities and discusses their role in the standards. Then in the second chapter he provides an exposition of the sources of noise in devices and circuits. In an instructive analysis of cascaded stages, he shows that, for modulative or parametric flicker noise, the effect of cascaded stages is cumulative without regard to stage gain.

This is in contrast with the well-known treatment of additive noise using the Friis formula to calculate an equivalent input noise power representing noise that may originate anywhere in a cascade of real amplifiers. This example highlights the concept that "the model is not the actual thing." He also describes concepts for the reduction of flicker noise in amplifier stages.

In his third chapter Rubiola then combines the elements of the first two chapters to derive models and techniques useful in characterizing phase noise arising in resonator feedback oscillators, adding mathematical formalism to these in the fourth chapter. In the fifth chapter he extends the reader's view to the case of delay-line oscillators such as lasers. In his sixth chapter, Rubiola offers guidance for the instructive "hacking" of existing oscillators, using their external phase spectra and other measurables to estimate their internal configuration. He details cases in which resonator fluctuations mask circuit noise, showing that separately quantifying resonator noise can be fruitful and that device noise figure and resonator Q are not merely arbitrary fitting factors.

It's interesting to consider what lies ahead in this field. The successes of today's consumer wireless products, cellular telephony, WiFi, satellite TV, and GPS, arise directly from the economies of scale of highly integrated circuits. But at the same time this introduces compromises for active-device noise and resonator quality. A measure of the market penetration of multi-signal consumer systems such as cellular telephony and WiFi is that they attract enough users to become interference-limited, often from subscribers much nearer than a distant base station. Hence low phase noise remains essential to preclude an unacceptable decrease of dynamic range, but it must now be achieved within narrower bounds on the available circuit elements.

A search for new understanding and techniques has been spurred by this requirement for low phase noise in oscillators and synthesizers whose primary character is integration and its accompanying minimal cost. This body of knowledge is advancing through a speculative and developmental phase. Today, numerical nonlinear circuit analysis supports additional design variables, such as the timing of the current pulse in nonlinear oscillators, that have become feasible because of the improved capabilities of both semiconductor devices and computers.

The field is alive and well, with emerging players eager to find a role on the stage for their own scenarios. Professionals and students, whether senior or new to the field so ably described by Rubiola, will benefit from his theoretical rigor, experimental viewpoint, and presentation.

David B. Leeson
Stanford University
February 2008

Preface

The importance of oscillators in science and technology can be outlined by two milestones. The *pendulum*, discovered by Galileo Galilei in the sixteenth century, persisted as "the" time-measurement instrument (in conjunction with the Earth's rotation period) until the piezoelectric quartz resonator. Then, it was not by chance that the first *integrated circuit*, built in September 1958 by Jack Kilby at the Bell Laboratories, was a radio-frequency oscillator.

Time, and equivalently frequency, is the most precisely measured physical quantity. The wrist watch, for example, is probably the only cheap artifact whose accuracy exceeds 10^{-5}, while in primary laboratories frequency attains the incredible accuracy of a few parts in 10^{-15}. It is therefore inevitable that virtually all domains of engineering and physics rely on time-and-frequency metrology and thus need reference oscillators. Oscillators are of major importance in a number of applications such as wireless communications, high-speed digital electronics, radars, and space research. An oscillator's random fluctuations, referred to as noise, can be decomposed into amplitude noise and phase noise. The latter, far more important, is related to the precision and accuracy of time-and-frequency measurements, and is of course a limiting factor in applications.

The main fact underlying this book is that an oscillator turns the phase noise of its internal parts into frequency noise. This is a necessary consequence of the Barkhausen condition for stationary oscillation, which states that the loop gain of a feedback oscillator must be unity, with zero phase. It follows that the phase noise, which is the integral of the frequency noise, diverges in the long run. This phenomenon is often referred to as the "Leeson model" after a short article published in 1966 by David B. Leeson [63]. On my part, I prefer the term *Leeson effect* in order to emphasize that the phenomenon is far more general than a simple model. In 2001, in Seattle, Leeson received the W. G. Cady award of the IEEE International Frequency Control Symposium "for clear physical insight and [a] model of the effects of noise on oscillators."

In spring 2004 I had the opportunity to give some informal seminars on noise in oscillators at the NASA/Caltech Jet Propulsion Laboratory. Since then I have given lectures and seminars on noise in industrial contexts, at IEEE symposia, and in universities and government laboratories. The purpose of most of these seminars was to provide a *tutorial*, as opposed to a report on advanced science, addressed to a large-variance audience that included technicians, engineers, Ph.D. students, and senior scientists. Of course, capturing the attention of such a varied audience was a challenging task. The stimulating discussions that followed the seminars convinced me I should write a working document[1] as a preliminary step and then this book. In writing, I have made a serious effort to address the same broad audience.

This work could not have been written without the help of many people. The gratitude I owe to my colleagues and friends who contributed to the rise of the ideas contained in this book is disproportionate to its small size: Rémi Brendel, Giorgio Brida, G. John Dick, Michele Elia, Patrice Féron, Serge Galliou, Vincent Giordano, Charles A. (Chuck) Greenhall, Jacques Groslambert, John L. Hall, Vladimir S. (Vlad) Ilchenko, Laurent Larger, Lutfallah (Lute) Maleki, Andrey B. Matsko, Mark Oxborrow, Stefania Römisch, Anatoliy B. Savchenkov, François Vernotte, Nan Yu.

Among them, I owe special thanks to the following: Lute Maleki for giving me the opportunity of spending four long periods at the NASA/Caltech Jet Propulsion Laboratory, where I worked on noise in photonic oscillators, and for numerous discussions and suggestions; G. John Dick, for giving invaluable ideas and suggestions during numerous and stimulating discussions; Rémi Brendel, Mark Oxborrow, and Stefania Römisch for their personal efforts in reviewing large parts of the manuscript in meticulous detail and for a wealth of suggestions and criticism; Vincent Giordano for supporting my efforts for more than 10 years and for frequent and stimulating discussions.

I wish to thank some manufacturers and their local representatives for kindness and prompt help: Jean-Pierre Aubry from Oscilloquartz; Vincent Candelier from RAKON (formerly CMAC); Art Faverio and Charif Nasrallah from Miteq; Jesse H. Searles from Poseidon Scientific Instruments; and Mark Henderson from Oewaves.

Thanks to my friend Roberto Bergonzo, for the superb picture on the front cover, entitled "The amethyst stairway." For more information about this artist, visit the website http://robertobergonzo.com.

Finally, I wish to thank Julie Lancashire and Sabine Koch, of the Cambridge editorial staff, for their kindness and patience during the long process of writing this book.

How to use this book

Let us first abstract this book in one paragraph. Chapter 1 introduces the language of phase noise and frequency stability. Chapter 2 analyzes phase noise in amplifiers, including flicker and other non-white phenomena. Chapter 3 explains heuristically the physical mechanism of an oscillator and of its noise. Chapter 4 focuses on the mathematics that describe an oscillator and its phase noise. For phase noise, the oscillator turns out to be a linear system. These concepts are extended in Chapter 5 to the delay-line oscillator and to the laser, which is a special case of the latter. Finally, Chapter 6 analyzes in depth a number of oscillators, both laboratory prototypes and commercial products. The analysis of an oscillator's phase noise discloses relevant details about the oscillator.

There are other books about oscillators, though not numerous. They can be divided into three categories: books on radio-frequency and microwave oscillators, which generally focus on the electronics; books about lasers, which privilege atomic physics and classical

[1] E. Rubiola, *The Leeson Effect – Phase Noise in Quasilinear Oscillators*, February 2005, arXiv:physics/0502143, now superseded by the present text.

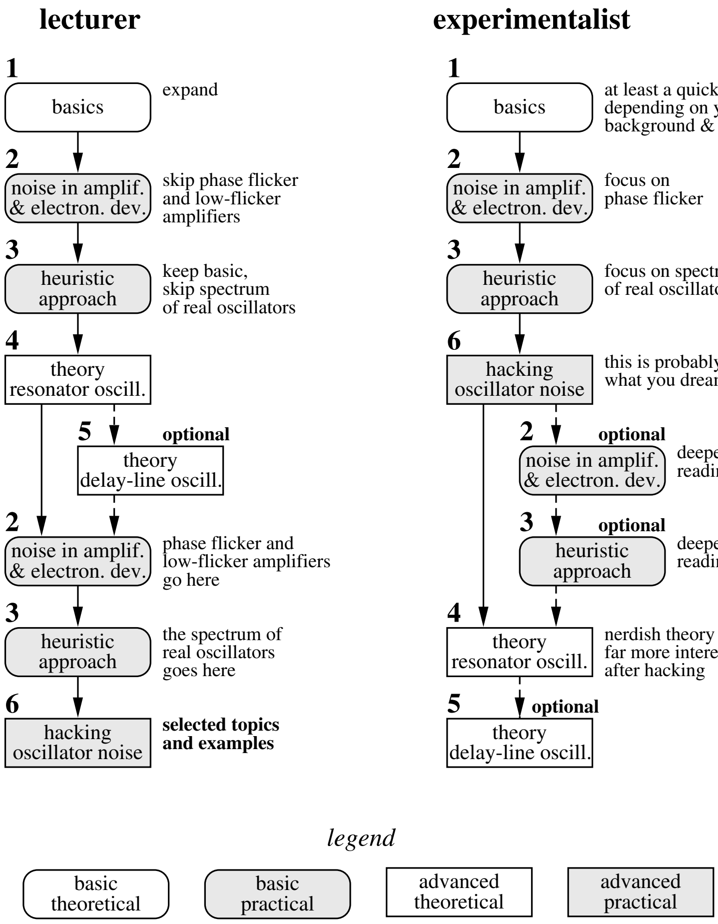

Figure 1 Asymptotic reading paths: on the left, for someone planning lectures on oscillator noise; on the right, for someone currently involved in practical work on oscillators.

optics; books focusing on the relevant mathematical physics. The present text is unique in that we look at the oscillator as a system consisting of more or less complex interacting blocks. Most topics are innovative, and the overlap with other books about oscillators or time-and-frequency metrology is surprisingly small. This may require an additional effort on the part of readers already familiar with the subject area.

The core of this book rises from my experimentalist soul, which later became convinced of the importance of the mathematics. The material was originally thought and drafted in the following (dis)order (see Fig. 1): **3** Heuristic approach, **6** Oscillator hacking, **4** Feedback theory, **5** Delay-line oscillators. The final order of subjects aims at a more understandable presentation. In seminars, I have often presented the material in the 3–6–4–5 order. Yet, the best reading path depends on the reader. Two paths are suggested in Fig. 1 for two "asymptotic" reader types, i.e. a lecturer and experimentalist. When planning to use this book as a supplementary text for a university course, the lecturer should be aware that students often lack the experience to understand and to appreciate Chapter 6 (Oscillator hacking) and other practical issues, while the theory can be more accessible to them. However, some mathematical derivations in Chapters 4 and 5 may require patience on the part of the experimentalist. The sections marked with one or two stars, ⋆ and ⋆⋆, can be skipped at first reading.

Supplementary material

My web page

http://rubiola.org (also http://rubiola.net)

contains material covering various topics about phase noise and amplitude noise. A section of my home page, at the URL

http://rubiola.org/oscillator-noise

has been created for the supplementary material specific to this book. Oscillator noise spectra and slides from my seminars are ready. Other material will be added later.

Cambridge University Press has set up a web page for this book at the URL

www.cambridge.org/rubiola ,

where there is room for supplementary material. It is my intention to make the same material available on my home page and on the Cambridge website. Yet, my web page is under my full control while the other one is managed by Cambridge University Press.

Notation

The following notation list is not exhaustive. Some symbols are not listed because they are introduced in the main text. On occasion a listed symbol may have a different meaning, where there is no risk of ambiguity because the symbol has local scope and the usage is consistent with the general literature.

Uppercase is often used for

- Fourier or Laplace transforms
- constants, when the lower-case symbol is a function of time. For example, in relation to $v(t)$ we have V_{rms}, V_0 (peak)
- quantities conventionally represented with an upper-case symbol.
- in boldface, phasors. Example, $\mathbf{V} = V_{\text{rms}} e^{j\theta}$.

Though ω is the *angular frequency*, for short it is referred to as the frequency. Numerical values are always given in Hz. The symbol ω may be used as a shorthand for $2\pi\nu$ or $2\pi f$. The symbols ν and f always refer to single-sided spectra and ω always refers to two-sided spectra even if only the positive frequencies appear in plots.

Section 1.2 provides additional information about the relevant physical quantities, their meaning, and their usage, and about the variables associated with them.

The list includes some chapter, section, subsection, equation, or figure cross references.

Symbol	Meaning and text references
A	amplifier voltage gain (thus, the power gain is A^2)
b_i	coefficients of the power-law approximation of $S_\varphi(f)$. 1.6.2, (1.70), and Fig. 1.8
$\mathrm{b}(t)$	resonator phase response. 4.4 and (4.62)
$\mathfrak{b}(t)$	resonator impulse response. 4.7
$\mathrm{B}(s)$	resonator phase response, $\mathrm{B}(s) = \mathcal{L}\{\mathrm{b}(t)\}$. 4.4.2, 4.4.3
C	electrical capacitance, farad
$\mathcal{D}$, $\mathcal{D}(s)$	denominator (of a fraction or of a rational function)
E	energy, either physical (J) or mathematical (dimensionless), depending on context
$\mathcal{E}$	electric field, V/m
$\mathbb{E}$	mathematical expectation. 1.3.1 and (1.28)
f	Fourier frequency, Hz. 1.2
f, $f(x)$	generic function. 1.2

f_{c}	amplifier corner frequency, Hz. 2.3.3
f_{L}	Leeson frequency, Hz. 3.2 and (3.21)
F	amplifier noise figure. 2.2 and (2.11)
$\mathcal{F}\{\cdot\}$	Fourier transform operator. (A. 3)
h	Planck's constant, $h = 6.626 \times 10^{-34}$ J s
h_i	coefficients of the power-law approximation of $S_y(f)$. 1.6.3 and (1.73), (1.74)
$h(t)$	impulse response. 1.5.1 and (1.55), (1.56)
$\mathrm{h}(t)$	phase response
$H(s)$	transfer function, $H(s) = \mathcal{L}\{h(t)\}$, also $H(j\omega)$. 1.5.1 and (4.42)
$\mathrm{H}(s)$	phase transfer function, $\mathrm{H}(s) = \mathcal{L}\{\mathrm{h}(t)\}$, also $\mathrm{H}(j\omega)$ 3.2 and 4.5
$i(t)$	current, as a function of time
j	imaginary unit, $j^2 = -1$
k	Boltzmann constant, 1.381×10^{-23} J/K
$k_{(\mathrm{subscript})}$	a constant, k_{d}, k_{o}, k_{L}, etc.
l	harmonic order (in Chapter 5)
ℓ	voltage attenuation or loss (thus, the power loss is ℓ^2)
L	electrical inductance, H
$\mathcal{L}\{\cdot\}$	Laplace transform operator. 1.5.1 and (A.1)
$\mathscr{L}(f)$	single-sideband noise spectrum, dBc/Hz. 1.6.1 and (1.68)
m	integer (in Chapter 5)
m	modulation index (of light intensity)
$n(t)$	random noise, either near-dc or rf–microwave
N	integer
N	noise power spectral density, W/Hz
$\mathcal{N}$, $\mathcal{N}(s)$	numerator (of a fraction or of a rational function)
p	complex variable, replaces s when needed
P, $P(t)$	power, either physical (W) or mathematical (dimensionless), depending on context
q	electron charge, $q = 1.602\times10^{-19}$ C
Q	resonator quality factor. 4.1
R, R_0	resistance, load resistance (often $R_0 = 50\ \Omega$)
R	reflection coefficient. Chapter 5
$R(\tau)$	autocorrelation or correlation function. 1.4.1
s	complex variable, $s = \sigma + j\omega$
$S(f)$	power spectral density (PSD). 1.4.1, 1.4.2
$S_a(f)$	one-sided PSD of the quantity a
$S_\varphi(f)$	one-sided PSD of the random phase $\varphi(t)$. 1.6.1
$S^I(f)$	one-sided PSD. 1.4.2. The variable is could also be ν
$S^{II}(\omega)$	two-sided PSD. (1.4.1).
t	time

T	equivalent noise temperature of a device. 2.2
T	observation or measurement time in truncated signals. 1.4.1
T	period, $T = 1/\nu$
T, T_0	absolute temperature, reference temperature $T_0 = 290$ K
T	transmission coefficient. 5.2
$U(t)$	Heaviside (step) function, $U(t) = \int \delta(t')\,dt'$
$v(t)$	voltage (in theoretical contexts, also a dimensionless signal)
x, $x(t)$	a generic variable
$x(t)$	phase-time fluctuation. 1.2 and (1.17)
$y(t)$	fractional-frequency fluctuation. 1.2 and (1.18)
V, V_0	dc or peak voltage
$V(s)$	Laplace transform of $v(t)$
$\mathbf{V}$	voltage phasor. 1.1
$\alpha(t)$	normalized amplitude noise. 1.1.1
$\beta(s)$	transfer function of the feedback path. 4.2 and Fig. 4.6
$\delta(t)$	Dirac delta function
Δ	difference operator, in $\Delta\nu$, $(\Delta\omega)(t)$, etc.
η	photodetector quantum efficiency. 2.2.3
θ	phase or argument of a complex function $\rho e^{j\theta}$
κ	small phase step. Chapter 4
λ	wavelength
μ	harmonic order in phase space. Chapter 5
ν	frequency (Hz), used for carriers. 1.2
ρ	modulus of a complex function $\rho e^{j\theta}$
ρ	photodetector responsivity, A/W. 2.2.3
σ	real part of the complex variable $s = \sigma + j\omega$
$\sigma_y(\tau)$	Allan deviation, square root of the Allan variance $\sigma_y^2(\tau)$. 1.7
τ	measurement time, in $\sigma_y(\tau)$
τ	resonator relaxation time. 4.1
τ_d, τ_f	delay of a delay line, and group delay of the mode selector filter. Chapter 5
φ, $\varphi(t)$	phase (constant), phase noise. 1.1
$\Phi(s)$	phase noise, $\Phi(s) = \mathcal{L}\{\varphi(t)\}$
χ	dissonance. 4.2 and (4.31)
ψ, $\psi(t)$	amplifier static phase, phase noise. 3.2 and 4.5
$\Psi(s)$	amplifier phase noise, $\Psi(s) = \mathcal{L}\{\psi(t)\}$
ω	angular frequency, carrier or Fourier. 1.1
ω_0	oscillator angular frequency. 1.1
ω_L	Leeson angular frequency
ω_n	resonator natural angular frequency. 1.2
ω_p	resonator free-decay angular pseudo-frequency. 1.2
Ω	replaces ω, when needed
Ω	detuning angular frequency. Chapter 5

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Enrico Rubiola
Frontmatter
More Information

Subscript	Meaning
0	oscillator carrier, in ω_0, P_0, V_0, etc.
i	input. Examples $v_\mathrm{i}(t)$, $\varphi_\mathrm{i}(t)$, $\Phi_\mathrm{i}(s)$
i	current. Example, shot noise $S_i(\omega) = 2q\overline{i}$
l	light
L	Leeson
L	loop
m	main branch
n	resonator natural frequency (ω_n, ν_n)
o	output. Examples $v_\mathrm{o}(t)$, $\varphi_\mathrm{o}(t)$, $\Phi_\mathrm{o}(s)$
p	resonator free-decay pseudofrequency (ω_p, ν_p)
p	pole, as in $s_\mathrm{p} = \sigma_\mathrm{p} + j\omega_\mathrm{p}$ (referring to a complex variable)
p	peak. Example, $V_\mathrm{p} = \sqrt{2}V_\mathrm{rms}$
rms	root mean square
z	zero, as in $s_\mathrm{z} = \sigma_\mathrm{z} + j\omega_\mathrm{z}$ (referring to a complex variable)

Symbol	Meaning
$<\ >$	mean
$<\ >_N$	mean of N values. 1.3.1
$\overline{x}$	time average of x, for example. 1.3.1
$\leftrightarrow$	transform–inverse-transform pair. Example, $x(t) \leftrightarrow X(\omega)$
$*$	convolution. Example, $v_\mathrm{o}(t) = h(t) * v_\mathrm{i}(t)$. 1.5.1
$\asymp$	asymptotically equal