PART I

The underwater light field

1 Concepts of hydrologic optics

1.1 Introduction

The purpose of the first part of this book is to describe and explain the behaviour of light in natural waters. The word 'light' in common parlance refers to radiation in that segment of the electromagnetic spectrum – about 400 to $700\,\mu\text{m}$ to which the human eye is sensitive. Our prime concern is not with vision but with photosynthesis. Nevertheless, by a convenient coincidence, the waveband within which plants can photosynthesize corresponds approximately to that of human vision and so we may legitimately refer to the particular kind of solar radiation with which we are concerned simply as 'light'.

Optics is that part of physics which deals with light. Since the behaviour of light is greatly affected by the nature of the medium through which it is passing, there are different branches of optics dealing with different kinds of physical systems. The relations between the different branches of the subject and of optics to fundamental physical theory are outlined diagrammatically in Fig. 1.1. Hydrologic optics is concerned with the behaviour of light in aquatic media. It can be subdivided into limnological and oceanographic optics according to whether fresh, inland or salty, marine waters are under consideration. Hydrologic optics has, however, up to now been mainly oceanographic in its orientation.

1.2 The nature of light

Electromagnetic energy occurs in indivisible units referred to as *quanta* or *photons*. Thus a beam of sunlight in air consists of a continual stream of photons travelling at $3 \times 10^8 \text{ m s}^{-1}$. The actual numbers of quanta

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Fig. 1.1 The relationship between hydrologic optics and other branches of optics (after Preisendorfer, 1976).

involved are very large. In full summer sunlight, for example, 1 m^2 of horizontal surface receives about 10^{21} quanta of visible light per second. Despite its particulate nature, electromagnetic radiation behaves in some circumstances as though it has a wave nature. Every photon has a wavelength, λ , and a frequency, ν . These are related in accordance with

$$\lambda = c/v \tag{1.1}$$

where c is the speed of light. Since c is constant in a given medium, the greater the wavelength the lower the frequency. If c is expressed in m s⁻¹ and v in cycles s⁻¹, then the wavelength, λ , is expressed in metres. For convenience, however, wavelength is more commonly expressed in nanometres, a nanometre (nm) being equal to 10^{-9} m. The energy, ε , in a photon varies with the frequency, and therefore inversely with the wavelength, the relation being

$$\varepsilon = hv = hc/\lambda \tag{1.2}$$

where *h* is Planck's constant and has the value of 6.63×10^{-34} J s. Thus, a photon of wavelength 700 nm from the red end of the photosynthetic spectrum contains only 57% as much energy as a photon of wavelength 400 nm from the blue end of the spectrum. The actual energy in a photon of wavelength λ nm is given by the relation

1.2 The nature of light

$$\varepsilon = (1988/\lambda) \times 10^{-19} \text{ J} \tag{1.3}$$

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A monochromatic radiation flux expressed in quanta s⁻¹ can thus readily be converted to $J s^{-1}$, i.e. to watts (W). Conversely, a radiation flux, Φ , expressed in W, can be converted to quanta s⁻¹ using the relation

quanta
$$s^{-1} = 5.03 \ \Phi \lambda \times 10^{15}$$
 (1.4)

In the case of radiation covering a broad spectral band, such as for example the photosynthetic waveband, a simple conversion from quanta s⁻¹ to W, or *vice versa*, cannot be carried out accurately since the value of λ varies across the spectral band. If the distribution of quanta or energy across the spectrum is known, then conversion can be carried out for a series of relatively narrow wavebands covering the spectral region of interest and the results summed for the whole waveband. Alternatively, an approximate conversion factor, which takes into account the spectral distribution of energy that is likely to occur, may be used. For solar radiation in the 400 to 700 nm band above the water surface, Morel and Smith (1974) found that the factor (*Q/W*) required to convert W to quanta s⁻¹ was 2.77 × 10¹⁸ quanta s⁻¹ W⁻¹ to an accuracy of plus or minus a few per cent, regardless of the meteorological conditions.

As we shall discuss at length in a later section (§6.2) the spectral distribution of solar radiation under water changes markedly with depth. Nevertheless, Morel and Smith found that for a wide range of marine waters the value of Q:W varied by no more than $\pm 10\%$ from a mean of 2.5×10^{18} quanta s⁻¹ W⁻¹. As expected from eqn 1.4, the greater the proportion of long-wavelength (red) light present, the greater the value of Q:W. For yellow inland waters with more of the underwater light in the 550 to 700 nm region (see §6.2), by extrapolating the data of Morel and Smith we arrive at a value of approximately 2.9×10^{18} quanta s⁻¹ W⁻¹ for the value of Q:W.

In any medium, light travels more slowly than it does in a vacuum. The velocity of light in a medium is equal to the velocity of light in a vacuum, divided by the refractive index of the medium. The refractive index of air is 1.00028, which for our purposes is not significantly different from that of a vacuum (exactly 1.0, by definition), and so we may take the velocity of light in air to be equal to that in a vacuum. The refractive index of water, although it varies somewhat with temperature, salt concentration and wavelength of light, may with sufficient accuracy he regarded as equal to 1.33 for all natural waters. Assuming that the velocity of light

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in a vacuum is $3 \times 10^8 \text{ m s}^{-1}$, the velocity in water is therefore about $2.25 \times 10^8 \text{ m s}^{-1}$ The frequency of the radiation remains the same in water but the wavelength diminishes in proportion to the decrease in velocity. When referring to monochromatic radiation, the wavelength we shall attribute to it is that which it has in a vacuum. Because *c* and λ change in parallel, eqns 1.2, 1.3 and 1.4 are as true in water as they are in a vacuum: furthermore, when using eqns 1.3 and 1.4. it is the value of the wavelength in a vacuum which is applicable, even when the calculation is carried out for underwater light.

1.3 The properties defining the radiation field

If we are to understand the ways in which the prevailing light field changes with depth in a water body, then we must first consider what are the essential attributes of a light field in which changes might be anticipated. The definitions of these attributes, in part, follow the report of the Working Groups set up by the International Association for the Physical Sciences of the Ocean (1979), but are also influenced by the more fundamental analyses given by Preisendorfer (1976). A more recent account of the definitions and concepts used in hydrologic optics is that by Mobley (1994).

We shall generally express direction within the light field in terms of the *zenith angle*, θ (the angle between a given light pencil, i.e. a thin parallel beam, and the upward vertical), and the *azimuth angle*, ϕ (the angle between the vertical plane incorporating the light pencil and some other specified vertical plane such as the vertical plane of the Sun). In the case of the upwelling light stream it will sometimes be convenient to express a direction in terms of the *nadir angle*, θ_n (the angle between a given light pencil and the downward vertical). These angular relations are illustrated in Fig. 1.2.

Radiant flux, Φ , is the time rate of flow of radiant energy. It may be expressed in W (J s⁻¹) or quanta s⁻¹.

Radiant intensity, *I*, is a measure of the radiant flux per unit solid angle in a specified direction. The radiant intensity of a source in a given direction is the radiant flux emitted by a point source, or by an element of an extended source, in an infinitesimal cone containing the given direction, divided by that element of solid angle. We can also speak of radiant intensity at a point in space. This, the *field* radiant intensity, is the radiant flux at that point in a specified direction in an infinitesimal cone



1.3 The properties defining the radiation field

Fig. 1.2 The angles defining direction within a light field. The figure shows a downward and an upward pencil of light, both, for simplicity, in the same vertical plane. The downward pencil has zenith angle θ ; the upward pencil has nadir angle θ_n , which is equivalent to a zenith angle of $(180^\circ - \theta_n)$. Assuming the *xy* plane is the vertical plane of the Sun, or other reference vertical plane, then ϕ is the azimuth angle for both light pencils.

containing the given direction, divided by that element of solid angle. *I* has the units W (or quanta s^{-1}) steradian⁻¹.

$$I = d\Phi/d\omega$$

If we consider the radiant flux not only per unit solid angle but also per unit area of a plane at right angles to the direction of flow, then we arrive at the even more useful concept of *radiance*, *L*. Radiance at a point in space is the radiant flux at that point in a given direction per unit solid angle per unit area at right angles to the direction of propagation. The meaning of this *field* radiance is illustrated in Figs. 1.3*a* and *b*. There is also *surface* radiance, which is the radiant flux emitted in a given direction per unit solid angle per unit projected area (apparent unit area, seen from the viewing direction) of a surface: this is illustrated in Fig. 1.3*c*. To indicate that it is a function of direction, i.e. of both zenith and azimuth angle, radiance is commonly written as $L(\theta, \phi)$. The angular structure of a light field is expressed in terms of the variation of radiance with θ and ϕ . Radiance has the units W (or quanta s⁻¹) m⁻² steradian⁻¹.

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Fig. 1.3 Definition of radiance. (a) Field radiance at a point in space. The field radiance at P in the direction D is the radiant flux in the small solid angle surrounding D, passing through the infinitesimal element of area dA at right angles to D divided by the element of solid angle and the element of area. (b) Field radiance at a point in a surface. It is often necessary to consider radiance at a point on a surface, from a specified direction relative to that surface. dS is the area of a small element of surface. $L(\theta, \phi)$ is the radiance incident on dS at zenith angle θ (relative to the normal to the surface) and azimuth angle ϕ : its value is determined by the radiant flux directed at dS within the small solid angle, $d\omega$, centred on the line defined by θ and ϕ . The flux passes perpendicularly across the area dS cos θ , which is the projected area of the element of surface, dS, seen from the direction θ , ϕ . Thus the radiance on a point in a surface, from a given direction, is the radiant flux in the specified direction per unit solid angle per unit projected area of the surface. (c) Surface radiance. In the case of a surface that emits radiation the intensity of the flux leaving the surface in a specified direction is expressed in terms of the surface radiance, which is defined in the same way as the field radiance at a point in a surface except that the radiation is considered to flow away from, rather than on to, the surface.

 $L(\theta, \phi) = \mathrm{d}^2 \Phi / \mathrm{d}S \, \cos\theta \, \mathrm{d}\omega$

Irradiance (at a point of a surface), *E*, is the radiant flux incident on an infinitesimal element of a surface, containing the point under consideration, divided by the area of that element. Less rigorously, it may be defined as the radiant flux per unit area of a surface.^{*} It has the units $W m^{-2}$ or quanta (or photons) s⁻¹ m⁻², or mol quanta (or photons) s⁻¹ m⁻², where 1.0 mol photons is 6.02×10^{23} (Avogadro's number) photons. One mole of photons is sometimes referred to as an *einstein*, but this term is now rarely used.

^{*} Terms such as 'fluence rate' or 'photon fluence rate', sometimes to be found in the plant physiological literature, are superfluous and should not be used.

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$E = d\Phi/dS$

Downward irradiance, E_d , and *upward irradiance*, E_u , are the values of the irradiance on the upper and the lower faces, respectively, of a horizontal plane. Thus, E_d is the irradiance due to the downwelling light stream and E_u is that due to the upwelling light stream.

The relation between irradiance and radiance can be understood with the help of Fig. 1.3b. The radiance in the direction defined by θ and ϕ is L (θ, ϕ) W (or quanta s⁻¹) per unit projected area per steradian (sr). The projected area of the element of surface is dS cos θ and the corresponding element of solid angle is d ω . Therefore the radiant flux on the element of surface within the solid angle d ω is $L(\theta, \phi)dS \cos \theta d\omega$. The area of the element of surface is dS and so the irradiance at that point in the surface where the element is located, due to radiant flux within d ω , is $L(\theta, \phi) \cos \theta d\omega$. The total downward irradiance at that point in the surface is obtained by integrating with respect to solid angle over the whole upper hemisphere

$$E_d = \int_{2\pi} L(\theta, \phi) \cos \theta \, \mathrm{d}\omega \tag{1.5}$$

The total upward irradiance is related to radiance in a similar manner except that allowance must be made for the fact that $\cos \theta$ is negative for values of θ between 90 and 180 °

$$E_u = -\int_{-2\pi} L(\theta, \phi) \cos \theta \, \mathrm{d}\omega \tag{1.6}$$

Alternatively the cosine of the nadir angle, θ_n (see Fig. 1.2), rather than of the zenith angle, may be used

$$E_u = \int_{-2\pi} L(\theta_n, \phi) \cos \theta_n \, \mathrm{d}\omega \tag{1.7}$$

The -2π subscript is simply to indicate that the integration is carried out over the 2π sr solid angle in the *lower* hemisphere.

The *net downward irradiance*, \vec{E} , is the difference between the downward and the upward irradiance

$$\vec{E} = E_d - E_u \tag{1.8}$$

It is related to radiance by the eqn

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$$\vec{E} = \int_{4\pi} L(\theta, \phi) \cos \theta d\omega$$
 (1.9)

which integrates the product of radiance and $\cos \theta$ over all directions: the fact that $\cos \theta$ is negative between 90 and 180° ensures that the contribution of upward irradiance is negative in accordance with eqn 1.8. The net downward irradiance is a measure of the net rate of transfer of energy downwards at that point in the medium, and as we shall see later is a concept that can be used to arrive at some valuable conclusions.

The scalar irradiance, E_0 , is the integral of the radiance distribution at a point over all directions about the point

$$E_o = \int_{4\pi} L(\theta, \phi) \mathrm{d}\omega \tag{1.10}$$

Scalar irradiance is thus a measure of the radiant intensity at a point, which treats radiation from all directions equally. In the case of irradiance, on the other hand, the contribution of the radiation flux at different angles varies in proportion to the cosine of the zenith angle of incidence of the radiation: a phenomenon based on purely geometrical relations (Fig. 1.3, eqn 1.5), and sometimes referred to as the Cosine Law. It is useful to divide the scalar irradiance into a downward and an upward component. The *downward scalar irradiance*, E_{0d} , is the integral of the radiance distribution over the upper hemisphere

$$E_{0d} = \int_{2\pi} L(\theta, \phi) \mathrm{d}\omega \tag{1.11}$$

The *upward scalar irradiance is* defined in a similar manner for the lower hemisphere

$$E_{0u} = \int_{-2\pi} L(\theta, \phi) \mathrm{d}\omega \tag{1.12}$$

Scalar irradiance (total, upward, downward) has the same units as irradiance.

It is always the case in real-life radiation fields that irradiance and scalar irradiance vary markedly with wavelength across the photosynthetic range. This variation has a considerable bearing on the extent to which the radiation field can be used for photosynthesis. It is expressed in terms of the variation in irradiance or scalar irradiance per unit spectral distance (in units of wavelength or frequency, as appropriate) across the spectrum. Typical units would be W (or quanta s^{-1}) m⁻² nm⁻¹.

1.3 The properties defining the radiation field

If we know the radiance distribution over all angles at a particular point in a medium then we have a complete description of the angular structure of the light field. A complete radiance distribution, however, covering all zenith and azimuth angles at reasonably narrow intervals, represents a large amount of data: with 5° angular intervals, for example, the distribution will consist of 1369 separate radiance values. A simpler, but still very useful, way of specifying the angular structure of a light field is in the form of the three average cosines – for downwelling, upwelling and total light – and the irradiance reflectance.

The average cosine for downwelling light, $\overline{\mu}_d$, at a particular point in the radiation field, may be regarded as the average value, in an infinitesimally small volume element at that point in the field, of the cosine of the zenith angle of all the downwelling photons in the volume element. It can be calculated by summing (i.e. integrating) for all elements of solid angle (d ω) comprising the upper hemisphere, the product of the radiance in that element of solid angle and the value of $\cos \theta$ (i.e. $L(\theta, \phi) \cos \theta$), and then dividing by the total radiance originating in that hemisphere. By inspection of eqns 1.5 and 1.11 it can be seen that

$$\overline{\mu}_d = E_d / E_{0d} \tag{1.13}$$

i.e. the average cosine for downwelling light is equal to the downward irradiance divided by the downward scalar irradiance. The average cosine for upwelling light, $\overline{\mu}_u$, may be regarded as the average value of the cosine of the nadir angle of all the upwelling photons at a particular point in the field. By a similar chain of reasoning to the above, we conclude that $\overline{\mu}_u$ is equal to the upward irradiance divided by the upward scalar irradiance

$$\overline{\mu}_u = E_u / E_{0u} \tag{1.14}$$

In the case of the downwelling light stream it is often useful to deal in terms of the reciprocal of the average downward cosine, referred to by Preisendorfer (1961) as the *distribution function* for downwelling light, D_d , which can be shown⁷¹² to be equal to the mean pathlength per vertical metre traversed, of the downward flux of photons per unit horizontal area per second. Thus $D_d = 1/\overline{\mu}_d$. There is, of course, an analogous distribution function for the upwelling light stream, defined by $D_u = 1/\overline{\mu}_u$.

The average cosine, $\overline{\mu}$, for the total light at a particular point in the field may be regarded as the average value, in an infinitesimally small volume element at that point in the field, of the cosine of the zenith angle of all the photons in the volume element. It may be evaluated by integrating the product of radiance and $\cos \theta$ over all directions and dividing by the total

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