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978-0-521-14934-1 - Independence-Friendly Logic: A Game-Theoretic Approach

Allen L. Mann, Gabriel Sandu and Merlijn Sevenster

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