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Allen L. Mann, Gabriel Sandu and Merlijn Sevenster

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# Independence-Friendly Logic

A Game-Theoretic Approach

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