

A Physical Introduction to Suspension Dynamics

Understanding the behavior of particles suspended in a fluid has many important applications across a range of fields, including engineering and geophysics.

Comprising two main parts, this book begins with Part I, which describes the well-developed theory of particles in viscous fluids, i.e. microhydrodynamics, particularly for single- and pair-body dynamics. Part II considers many-body dynamics, covering shear flows and sedimentation, bulk flow properties, and collective phenomena. An interlude between the two parts provides the basic statistical techniques needed to employ the results of the first (microscopic) in the second (macroscopic).

The authors introduce theoretical, mathematical concepts through concrete examples, making the material accessible to non-mathematicians. They also include some of the many open questions in the field to encourage further study. Consequently, this is an ideal introduction for students and researchers from other disciplines who are approaching suspension dynamics for the first time.

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Élisabeth Guazzelli, Jeffrey F. Morris, Illustrated by Sylvie Pic

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Contents

| | |
|--|----------------|
| <i>Preface</i> | <i>page ix</i> |
| Prologue | 1 |
| PART I MICROHYDRODYNAMICS | 7 |
| 1 Basic concepts in viscous flow | 9 |
| 1.1 The fluid dynamic equations | 9 |
| 1.2 Scaling arguments and the Stokes approximation | 12 |
| 1.3 Buoyancy and drag | 13 |
| 1.4 Properties of Stokes flow | 16 |
| 1.4.1 Linearity | 16 |
| 1.4.2 Reversibility | 17 |
| 1.4.3 Instantaneity | 20 |
| 1.4.4 And more . . . | 21 |
| Appendix: Three Stokes-flow theorems | 22 |
| A.1 Minimum energy dissipation | 22 |
| A.2 A corollary: Uniqueness | 24 |
| A.3 Reciprocal theorem | 25 |
| Exercises | 26 |
| 2 One sphere in Stokes flow | 28 |
| 2.1 Three single sphere flows: rotation, translation, straining | 28 |
| 2.1.1 Rotation | 30 |
| 2.1.2 Translation | 33 |
| 2.1.3 Straining | 37 |

| | | |
|---|--|------------|
| 2.2 | Hydrodynamic force, torque, and stresslet | 40 |
| 2.2.1 | Force | 40 |
| 2.2.2 | Torque | 41 |
| 2.2.3 | Stresslet | 42 |
| 2.2.4 | Computing the hydrodynamic force | 43 |
| 2.3 | Faxén laws for the sphere | 45 |
| 2.4 | A sphere in simple shear flow | 47 |
| | Exercises | 50 |
| 3 | Toward more sophisticated solution techniques | 53 |
| 3.1 | Point force solution | 53 |
| 3.2 | Point torque and stresslet | 55 |
| 3.3 | Integral representation | 59 |
| 3.4 | Multipole representation | 60 |
| 3.5 | Resistance matrices | 62 |
| 3.6 | Motion of different types of particles | 67 |
| 3.7 | Slender-body theory | 78 |
| 3.8 | Boundary integral method | 80 |
| | Exercises | 82 |
| 4 | Particle pair interactions | 84 |
| 4.1 | A sedimenting pair | 84 |
| 4.2 | A pair in shear | 89 |
| 4.3 | Pair lubrication interactions | 93 |
| 4.4 | Stokesian Dynamics | 99 |
| INTERLUDE: FROM THE MICROSCOPIC TO THE MACROSCOPIC | | 103 |
| 5 | A short presentation of statistical and stochastic concepts | 105 |
| 5.1 | Statistical physics | 105 |
| 5.2 | Averaging concepts | 107 |
| 5.2.1 | Ensemble and other averages | 107 |
| 5.2.2 | Probability distributions | 108 |
| 5.3 | Fluctuational motion | 113 |
| 5.3.1 | Random walks and diffusion | 114 |
| 5.3.2 | Brownian motion | 116 |
| 5.4 | Two routes to diffusive dynamics | 118 |
| 5.4.1 | A macroscopic approach: Stokes–Einstein relation and Smoluchowski equation | 118 |

Contents

vii

| | | |
|---|--|-----|
| | 5.4.2 A microscopic approach: Langevin equation | 120 |
| | 5.5 Chaotic dynamics | 123 |
| PART II TOWARD A DESCRIPTION OF MACROSCOPIC PHENOMENA IN SUSPENSIONS | | |
| | | 125 |
| 6 | Sedimentation | 127 |
| | 6.1 One, two, three . . . spheres | 127 |
| | 6.2 Clusters and clouds | 132 |
| | 6.3 Settling of a suspension of spheres | 135 |
| | 6.4 Influence of the lateral walls of a vessel: Intrinsic convection | 139 |
| | 6.5 Velocity fluctuations and hydrodynamic diffusion | 143 |
| | 6.6 Fronts | 146 |
| | 6.7 Setting of particles in an inclined vessel: Boycott effect | 149 |
| | 6.8 More on polydispersity and anisotropy | 151 |
| 7 | Shear flow | 155 |
| | 7.1 Suspension viscosity | 155 |
| | 7.1.1 Computing the Einstein viscosity | 157 |
| | 7.1.2 First effects of particle interaction on μ_s | 160 |
| | 7.2 Non-Newtonian rheology in suspensions | 161 |
| | 7.2.1 Rate and time dependence of viscosity | 161 |
| | 7.2.2 Normal stresses in suspensions | 164 |
| | 7.2.3 Stress mechanisms | 168 |
| | 7.3 Microstructure of sheared suspensions | 170 |
| | 7.3.1 Concentrated suspension microstructure | 170 |
| | 7.3.2 Smoluchowski theory of suspension microstructure | 173 |
| | 7.4 Constitutive modeling of suspension stress | 180 |
| | 7.5 Irreversible dynamics in shear flow | 182 |
| | 7.5.1 Shear-induced diffusion | 183 |
| | 7.5.2 Shear-induced migration | 184 |
| | 7.6 Orientable particles | 190 |
| 8 | Beyond Stokes flow: Finite inertia | 192 |
| | 8.1 Limit of the Stokes approximation | 193 |
| | 8.1.1 Influence of inertia far from a body | 193 |

| | | |
|-------------------|--|-----|
| 8.1.2 | Oseen solution for a translating sphere | 196 |
| 8.2 | Settling spheres at finite inertia | 198 |
| 8.3 | Migration under dilute conditions in pressure-driven flow | 201 |
| 8.3.1 | Observations | 201 |
| 8.3.2 | Analytical approaches | 204 |
| 8.4 | Particle motion in finite- Re simple-shear flow | 208 |
| 8.5 | Weak-inertia rheology | 211 |
| Epilogue | | 213 |
| <i>References</i> | | 217 |
| <i>Index</i> | | 225 |

Preface

The purpose of this book is to provide an introduction to suspension dynamics, and so we (the authors) thought it would be good to give some historical (as well as personal) perspective on the study of suspensions. Early development of the subject was largely due to two “schools,” one in England and one in the United States. In England, the subject developed from the fluid mechanical tradition at the University of Cambridge, dating from the work of G. G. Stokes and H. Lamb in the mid- and late-1800s. The subject developed in earnest from the work of George Batchelor and collaborators at Cambridge’s Department of Applied Mathematics and Theoretical Physics (DAMTP). In the United States, the development of the discipline took place primarily in chemical engineering departments, largely through the efforts of Andreas Acrivos and a number of his students at the University of California Berkeley, Stanford University, and the City College of New York (CCNY). The authors’ approaches to suspensions owe much to these “schools” of suspension dynamics. Élisabeth Guazzelli was introduced to the subject by Bud Homsy at Stanford University and extended interactions with John Hinch of the University of Cambridge. Jeff Morris received his introduction to suspensions as a doctoral student of John Brady (a student of Acrivos) at the California Institute of Technology.

The development of suspension dynamics in France was strongly influenced by the spirit of P. G. de Gennes, seen in some measure in the book *Hydrodynamique Physique* (translated as *Physical Hydrodynamics*) by É. Guyon, J-P. Hulin, and L. Petit. This book is complementary to the present volume, providing a less specialized examination of topics. In the French school, great importance is placed on experiments in investigating phenomena and development of insight. The interactions of French and American interests in suspensions is seen in the development of the

Stokesian Dynamics simulation technique by Georges Bossis (CNRS and Université de Nice – Sophia Antipolis) and John Brady; the development of understanding of collective phenomena in suspensions has benefitted greatly from the interaction of experimental and numerical investigations. From this interaction have come a number of insights leading to the development of theory and models of suspension behavior.

While the American and English schools have been especially influential to the authors, we would be remiss if we failed to note that numerous others have been influential in the development of suspension dynamics. We cannot hope to be comprehensive, but we note that within the United States, the work of H. A. Brenner (most recently at the Massachusetts Institute of Technology) was extremely influential in the development of low-Reynolds number hydrodynamics, while the work of Yu. A. Buyevich in the former Soviet Union led the way in applying statistical physics concepts to suspension flow, and S. G. Mason of McGill University (Montreal, Canada) and his coworkers performed numerous careful and creative experiments to elucidate phenomena in suspensions.

We owe our introduction to each other to the NATO Advanced Study Institute “Mobile Particulate Systems” (or “MOPS”) held in Cargèse, Corsica in the summer of 1994. This summer school clearly accomplished its goal of promoting discussion and interactions: we have for over a decade been working together to do our little part in advancing the topic of suspension dynamics and establishing its relation to related disciplines in mixture flow and rheology.

Our book grew in part out of lecture notes from courses we have taught at various institutions where we have been affiliated as faculty or visitors (École Polytechnique in Palaiseau, Institut Universitaire des Systèmes Thermiques Industriels – Polytech’Marseille – Aix-Marseille Université, and École Supérieure de Physique et Chimie Industrielle de la Ville de Paris for Guazzelli; Georgia Tech, CCNY, and Brown University for Morris). The students who have taken part in these courses played a valuable role in the development, and we offer our thanks. Colleagues at these various institutions have given much-appreciated support, not only through their encouragement, but also by providing teaching materials, by taking part in lectures, and in many other ways. Likewise, collaborators elsewhere have played an important and valuable part in developing our understanding. And finally, we wish to thank the research students and post-doctoral fellows with whom we have worked: it is through our work with them that it became clear there is a need for an introduction to suspensions.

Preface

xi

As a consequence of our perception that the need was for an accessible introduction to the foundations of the subject, this text does not make any pretensions to cover all topics of interest in suspension dynamics, and does not provide a comprehensive exploration of the topics addressed. The hope is that upon reading the book and working out the examples, a reader will come to understand the underlying principles of the field and be able to move confidently into an exploration of the suspension literature.

We offer our appreciation to our editor, David Tranah of Cambridge University Press, whose suggestions were very valuable in the development of this book. We are deeply indebted to John Hinch for his encouraging comments, and to Jason Butler (University of Florida) and Bud Homsy (now at the University of British Columbia) for their kindness in undertaking a detailed reading of the final draft and making many valuable suggestions and corrections. Thanks finally to Sylvie Pic for creating such beautiful artwork that goes far beyond the usual quality of illustration.

It is probable that several undetected errors remain; we take full responsibility for any mistakes. We will be grateful to readers who discover any inaccuracies and obscurities, or who may have any suggestions, if they would communicate these to us. We close by saying that from time to time we will be posting additional material on the book's web page; please check for updates and extras!

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