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978-0-521-14856-6 - Theory of p -adic Distributions: Linear and Nonlinear Models

S. Albeverio, A. Yu. Khrennikov and V. M. Shelkovich

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