

Index

- Abel means 33
- Abel summability 33, 34
- Abel–Plana formula 63
- Abelian functions 545
- absolute error 73
- acceleration of convergence
- definition 93
 - for sequences 93–94
 - for series 93–94
 - limit-preserving 93
- accumulation point 15
- acoustics
- canonical integrals 792
- additive number theory 644–647
- Dedekind modular function 646
 - Dedekind sum 646
 - discriminant function 646
 - Euler’s pentagonal number theorem 646
 - Goldbach conjecture 644
 - Jacobi’s identities 645
 - notation 638
 - partition function 644
 - unrestricted 645 - Ramanujan’s identity 646
 - Ramanujan’s tau function 646
 - representation by squares 645
 - Waring’s problem 645
- aerodynamics
- Struve functions 298
- affine Weyl groups
- Painlevé equations 732
- Airy functions **194**
- analytic properties 194
 - applications
 - mathematical 208
 - physical 209
 - ship waves 790 - approximations
 - expansions in Chebyshev series 211
 - in terms of elementary functions 211
 - in the complex plane 212 - asymptotic expansions 198–199
 - error bounds 199
 - exponentially-improved 199 - computation 209–210
 - connection formulas 194
 - definitions 194
 - differential equation 194
 - for products 203
 - initial values 194
 - numerically satisfactory solutions 194
 - Riccati form 194 - Dirac delta 38
 - envelope functions 59
 - generalized *see* generalized Airy functions.
 - graphics 195
 - incomplete 208
 - integral identities 787
 - integral representations 196, 203
 - integrals
 - approximations 211, 212
 - asymptotic approximations 202
 - definite 202
 - indefinite 202
 - of products 204
 - repeated 203
 - tables 211 - Laplace transforms 203
 - Maclaurin series 196
 - Mellin transform 203
 - modulus and phase
 - asymptotic expansions 200
 - definitions 199
 - graphs 195
 - identities 200
 - monotonicity 200
 - relation to Bessel functions 199
 - relation to zeros 200 - notation 194
 - products
 - differential equation 203
 - integral representations 203
 - integrals 204
 - Wronskian 203 - relation to umbilics 777
 - relations to other functions
 - Bessel functions 196–197
 - confluent hypergeometric functions . . 197, 328, 338
 - Hankel functions 196–197
 - modified Bessel functions 196–197 - Stieltjes transforms 203
 - tables
 - complex variables 210
 - integrals 211
 - real variables 210
 - zeros 201–202, 211

- Wronskians 194
- zeros
- asymptotic expansions 201
 - computation 76, 210
 - differentiation 200
 - relation to modulus and phase 200
 - tables 201–202, 211
- Airy transform 203
- Airy's equation
- *see* Airy functions, differential equation.
- Aitken's Δ^2 -process
- for sequences 93
 - iterated 93
- Al-Salam–Chihara polynomials 473
- algebraic curves
- Riemann surface 543, 544, 546
- algebraic equations
- parametrization via Jacobian elliptic functions .. 563
 - spherical trigonometry 564
 - uniformization 564
- algebraic Lamé functions 693
- alternant
- determinant 3
- amplitude (am) function **561**
- applications 564
 - approximations
 - small k, k' 562
 - small x 561 - computation 567
 - definition 561
 - Fourier series 562
 - integral representation 561
 - quasi-periodicity 561
 - relation to elliptic integrals 562
 - relation to Gudermannian function 562
 - special values 561
 - tables 567
- analytic continuation 19
- by reflection 19
- analytic function 16
- at infinity 17
 - in a domain 16
 - singularities 19
 - zeros 19
- Anger function *see* Anger–Weber functions.
- Anger–Weber functions **295**
- analytic properties 295
 - asymptotic expansions
 - large argument 297
 - large order 298 - computation 87, 299
 - definitions 295
 - derivatives 297
 - differential equation 295
 - graphics 296
 - incomplete 300
 - integral representations 295
 - integrals 297
 - interrelations 296
 - Maclaurin series 296
 - notation 288
 - order 288
 - recurrence relations 297
 - relations to other functions
 - Fresnel integrals 297
 - Lommel functions 296
 - Struve functions 297 - series expansions
 - power series 296
 - products of Bessel functions 297
 - special values 297
 - sums 297
 - tables 299
- angle between arcs 17
- angular momenta 758
- angular momentum
- generalized hypergeometric functions 418
- angular momentum coupling coefficients
- *see* $3j$ symbols, $6j$ symbols, and $9j$ symbols.
- angular momentum operator
- spherical coordinates 379
- annulus 19
- antenna research
- Lamé functions 694
- Appell functions **412**
- analytic continuation 414
 - applications
 - physical 417 - computation 418
 - definition 412–413
 - integral representations 414
 - integrals 414
 - inverse Laplace transform 414 - notation 412
 - partial differential equations 413
 - relation to Legendre's elliptic integrals 490
 - relation to symmetric elliptic integrals 509
 - relations to hypergeometric functions 414
 - transformations of variables 414–415
 - quadratic 415
 - reduction formulas 414
- approximation techniques
- Chebyshev-series expansions 97
 - least squares 99–100
 - minimax polynomials 96
 - minimax rational functions 97
 - Padé 98–99
 - splines 100

- arc length
 Jacobian elliptic functions 563
- arc(s) 16
 angle between 17
- area of triangle 246
- argument principle *see* phase principle.
- arithmetic Fourier transform 647
- arithmetic mean 3, 13
- arithmetic progression 2
- arithmetic-geometric mean **492**
 hypergeometric function 400
 integral representations 492
 Jacobian elliptic functions 566
 Legendre's elliptic integrals 492–493
 symmetric elliptic integrals 505
- arithmetics
 complex 73
 exact rational 72
 floating-point 72
 interval 72
 level-index 73
- Askey polynomials 475
- Askey scheme for orthogonal polynomials 464
- Askey–Gasper inequality
 Jacobi polynomials 478
- Askey–Wilson class orthogonal polynomials ... 472–474
 as eigenfunctions of a q -difference operator 472
 asymptotic approximations 474
 interrelations with other orthogonal polynomials
 464
 orthogonality properties 472
 representation as q -hypergeometric functions
 472–474
- Askey–Wilson polynomials 472
 asymptotic approximations 474
 relation to q -hypergeometric functions 472–474
- associated Anger–Weber function
 *see* Anger–Weber functions.
- associated Laguerre functions 754
- associated Legendre equation 352, 375
 exponent pairs 352
 numerically satisfactory solutions 352, 375
 singularities 352
 standard solutions 352, 354, 375
- associated Legendre functions **352**
 *see also* Ferrers functions.
 addition theorems 370, 377
 analytic continuation 376
 analytic properties 375
 applications 378–379
 asymptotic approximations
 *see* uniform asymptotic approximations.
 behavior at singularities 361, 375
 computation 379
- connection formulas 362, 375
- continued fractions 364
- cross-products 353
- definitions 353–354, 375
- degree 352
- derivatives 362
 with respect to degree or order 363
- differential equation
 *see* associated Legendre equation.
- expansions in series of 370
- generalized 378
- generating functions 361, 375
- graphics 357–359, 375–376
- Heine's formula 377
- hypergeometric representations 353–354, 375
- integer degree and order 360–361, 375
- integer order 360, 375
- integral representations 363, 377
- integrals
 definite 369
 Laplace transforms 370
 Mellin transforms 370
 products 369
- notation 352
- of the first kind 353
- of the second kind 354
- Olver's 354, 375
- order 352
- orthogonality 369
- principal values (or branches) 375
- recurrence relations 362, 375
- relations to other functions
 elliptic integrals 360
 Gegenbauer function 355
 hypergeometric function 353, 354, 394
 Jacobi function 355
 Legendre polynomials 360
- Rodrigues-type formulas 360
- special values 359, 360
- sums 370–371, 377
- tables 380
- uniform asymptotic approximations
 large degree 366–368, 377
 large order 365–366, 377
- values on the cut 376
- Whipple's formula 362
- Wronskians 352–353, 375
- zeros 368, 377
- associated orthogonal polynomials 474
 corecursive 474
 Jacobi 474
 Legendre 474
- astrophysics
 error functions and Voigt functions 169

- Heun functions and Heun's equation 720
- asymptotic and order symbols 42
 - definition 42
 - differentiation 42
 - integration 42
- asymptotic approximations and expansions... *see also*
 - asymptotic approximations of integrals, asymptotic approximations of sums and sequences, asymptotic solutions of difference equations, asymptotic solutions of differential equations, *and* asymptotic solutions of transcendental equations.
- algebraic operations 42
- cases of failure 52, 66
- differentiation 42
- double asymptotic properties
 - Bessel functions 258
 - Hankel functions 258
 - Kelvin functions 273
 - modified Bessel functions 257
 - parabolic cylinder functions 311
- exponentially-improved expansions 67–69
- generalized 43
- hyperasymptotic expansions 68
- improved accuracy via numerical transformations 69
- integration 42
- logarithms of 42
- null 42
- numerical use of 66, 69
- Poincaré type 42
- powers of 42
- re-expansion of remainder terms 67–69
- reversion of 43
- Stokes phenomenon 67
- substitution of 42
- uniform 43
- uniqueness 42
- via connection formulas 66
- asymptotic approximations of integrals 43–55
 - Bleistein's method 45
 - Chester–Friedman–Ursell method 48
 - coalescing critical points 48
 - coalescing peak and endpoint 45
 - coalescing saddle points 48
 - distributional methods 51–55
 - Fourier integrals 44
 - Haar's method 46
 - integration by parts 43
 - inverse Laplace transforms 46–47
 - Laplace transforms 43
 - Laplace's method 44–45, 47
 - Mellin transform methods 48
 - extensions 49–51
 - method of stationary phase 45
 - extensions 45
 - method of steepest descents 47
 - multidimensional integrals 51
 - Stieltjes transforms 52–53
 - generalized 53
 - Watson's lemma 44, 46
 - generalized 44
- asymptotic approximations of sums and sequences 63–66
 - Abel–Plana formula 63
 - Darboux's method 65–66
 - entire functions 64
 - Euler–Maclaurin formula 63
 - summation by parts 63
- asymptotic scale or sequence 43
- asymptotic solutions of difference equations 61–63
 - characteristic equation 62
 - coincident characteristic values 62
 - Liouville–Green (or WKBJ) type approximations 62
 - transition points 63
 - turning points 63
 - with a parameter 62–63
- asymptotic solutions of differential equations 55–61
 - characteristic equation 56
 - coincident characteristic values 57
 - error-control function 57
 - Fabry's transformation 57
 - irregular singularities of rank 1 56
 - Liouville–Green approximation theorem 57
 - Liouville–Green (or WKBJ) approximations 57
 - numerically satisfactory solutions 58
 - resurgence 57, 68
 - with a parameter 58–61
 - classification of cases 58
 - coalescing transition points 61
 - connection formulas across transition points 61
 - in terms of Airy functions 59
 - in terms of Bessel functions of fixed order 60–61
 - in terms of Bessel functions of variable order 61
 - in terms of elementary functions 59
 - Liouville transformation 58
 - transition points 58
 - turning points 58
- asymptotic solutions of transcendental equations 43
 - Lagrange's formula 43
- atomic photo-ionization
 - Coulomb functions 753
- atomic physics
 - Coulomb functions 754
 - error functions 169
- atomic spectra
 - Coulomb functions 753
- atomic spectroscopy

INDEX	891
$3j, 6j, 9j$ symbols	765
attractive potentials	
Coulomb functions	753, 754
auxiliary functions for Fresnel integrals	
approximations	170
asymptotic expansions	164
computation	169
definitions	160
derivatives	164
integral representations	163
Mellin–Barnes integrals	163
symmetry	162
auxiliary functions for sine and cosine integrals	
analytic continuation	151
approximations	156
asymptotic expansions	153
exponentially-improved	154
Chebyshev-series expansions	157
computation	156
definition	150
integral representations	152
principal values	151
relation to confluent hypergeometric functions	153
tables	156
axially symmetric potential theory	501
Bäcklund transformations	
classical orthogonal polynomials	478
Painlevé transcendents	730–732
backward recursion	85
Bailey's ${}_2F_1(-1)$ sum	
q -analog	426
Bailey's ${}_4F_3(1)$ sum	
q -analogs (first and second)	427
Bailey's ${}_2\psi_2$ transformations	
bilateral q -hypergeometric function	429
Bailey's bilateral summations	
bilateral q -hypergeometric function	427
bandlimited functions	706
Barnes' beta integral	143
Barnes' G -function	
asymptotic expansion	144
definition	144
infinite product	144
integral representation	144
recurrence relation	144
Barnes' integral	
Ferrers functions	369
Bartky's transformation	
Bulirsch's elliptic integrals	487
symmetric elliptic integrals	504
basic elliptic integrals	512
basic hypergeometric functions	<i>see</i> bilateral q -hypergeometric function <i>and</i> q -hypergeometric function.
Basset's integral	
modified Bessel functions	253
Bell numbers	
asymptotic approximations	623
definition	623
generating function	623
recurrence relation	623
table	623
Bernoulli monosplines	597
Bernoulli numbers	588
arithmetic properties	593
asymptotic approximations	593
computation	598
definition	588
degenerate	596
explicit formulas	591
factors	593
finite expansions	590
generalizations	596, 597
generating function	588
identities	591
inequalities	593
integral representations	592
inversion formulas	591
irregular pairs	598
Kummer congruences	593
notation	588
of the second kind	596
recurrence relations	
linear	591
quadratic and higher order	595
relations to	
Eulerian numbers	591
Genocchi numbers	595
Stirling numbers	596
tangent numbers	596
sums	595
tables	589, 598
Bernoulli polynomials	588
applications	
mathematical	597–598
physical	598
asymptotic approximations	593
computation	598
definitions	588
derivative	590
difference equation	589
explicit formulas	591
finite expansions	590
generalized	596, 597
generating function	588
graphs	589
inequalities	593
infinite series expansions	
Fourier	592

- other 592
- integral representations 592
- integrals 594
 - compendia 595
- Laplace transforms 595
- multiplication formulas 590
- notation 588
- recurrence relations
 - linear 591
 - quadratic 595
- relation to Eulerian numbers 591
- relation to Riemann zeta function 591
- representation as sums of powers 589
- special values 590
- sums 595
- symbolic operations 590
- symmetry 589
- tables 589
- zeros
 - complex 594
 - multiple 594
 - real 594
- Bernoulli's lemniscate 515
- Bernstein–Szegő polynomials 474
- Bessel functions **217**
 - *see also* cylinder functions, Hankel functions, Kelvin functions, modified Bessel functions, *and* spherical Bessel functions.
 - addition theorems 246
 - analytic continuation 226
 - applications
 - asymptotic solutions of differential equations 274–275
 - electromagnetic scattering 275
 - Helmholtz equation 275
 - Laplace's equation 275
 - oscillation of chains 275
 - oscillation of plates 276
 - wave equation 275
 - approximations 281
 - asymptotic expansions for large argument .. 228–230
 - error bounds 229–230
 - exponentially-improved 230
 - asymptotic expansions for large order 231–235
 - asymptotic forms 231
 - Debye's expansions 231–232
 - double asymptotic properties 235, 258
 - resurgence properties of coefficients 233
 - transition region 232
 - uniform 232–235
 - branch conventions 218
 - computation 276–277
 - computation by quadrature 83
 - computation by recursion 87
 - connection formulas 222
 - contiguous 235
 - continued fractions 226
 - cross-products 222, 223
 - zeros 238
 - definite integrals 203
 - definitions 217–218
 - derivatives
 - asymptotic expansions for large argument 229
 - asymptotic expansions for large order 231–232
 - explicit forms 222
 - uniform asymptotic expansions for large order 232
 - with respect to order 227–228
 - zeros *see* zeros of Bessel functions.
 - differential equations 217, 226
 - *see also* Bessel's equation.
 - Dirac delta 38
 - envelope functions 61
 - expansions in partial fractions 247
 - expansions in series of 247–248
 - Fourier–Bessel expansion 248
 - generalized 261
 - generating functions 226
 - graphics 218–222
 - incomplete 261
 - inequalities 227
 - infinite integrals 448
 - infinite products 235
 - integral representations
 - along the real line 223–224
 - compendia 226
 - contour integrals 224–225
 - Mellin–Barnes type 225
 - products 225
 - integrals *see also* integrals of Bessel and Hankel functions *and* Hankel transforms.
 - approximations 281
 - computation 277
 - tables 279, 280
 - limiting forms 223
 - minimax rational approximation 98
 - modulus and phase functions
 - asymptotic expansions for large argument 231
 - basic properties 230
 - definitions 230
 - graphics 218
 - relation to zeros 235
 - monotonicity 227
 - multiplication theorem 246
 - notation 217
 - of imaginary argument
 - *see* modified Bessel functions.
 - of imaginary order

- applications 248
- definitions 248
- graphs 221–222
- limiting forms 248
- numerically satisfactory pairs 248
- uniform asymptotic expansions for large order
..... 248
- zeros 248
- of matrix argument **769**
 - applications 773
 - asymptotic approximations 770
 - definitions 769
 - notation 768
 - of the first and second kinds 768
 - properties 769
 - relations to confluent hypergeometric functions of
matrix argument 770
- of the first, second, and third kinds 217–218
- orthogonality 243, 244
- power series 223
- principal branches (or values) 217–218
- recurrence relations 222–223
- relations to other functions
 - Airy functions 196–197
 - confluent hypergeometric functions 228
 - elementary functions 228
 - generalized Airy functions 206
 - generalized hypergeometric functions 228
 - parabolic cylinder functions 228, 315
- sums 246–248
 - addition theorems 246–247
 - compendia 248
 - expansions in series of Bessel functions ... 247–248
 - multiplication theorem 246
- tables 278–279
- Wronskians 222
- zeros *see* zeros of Bessel functions.
- Bessel polynomials 264, 476
 - asymptotic expansions 476
 - definition 476
 - differential equations 476
 - generalized 476
 - orthogonality properties 476
 - recurrence relations 476
 - relations to other functions
 - complex orthogonal polynomials 83
 - confluent hypergeometric functions 476
 - generalized hypergeometric functions 476
 - Jacobi polynomials 476
- Bessel transform *see* Hankel transform.
- Bessel's equation 217
 - inhomogeneous forms 288, 294, 295
 - numerically satisfactory solutions 218
 - singularities 217
 - standard solutions 217–218
- Bessel's inequality
 - Fourier series 13
- Bessel's integral
 - Bessel functions 223
- best uniform polynomial approximation 96
- best uniform rational approximation 97
- beta distribution
 - incomplete beta functions 189
- beta function **142**
 - *see also* incomplete beta functions.
 - applications
 - physical 145–146
 - definition 142
 - integral representations 142
 - multidimensional 143
 - integrals 143
 - multivariate *see* multivariate beta function.
- beta integrals 142–143
- Bickley function 259
 - applications 276
 - approximations 281
- biconfluent Heun equation 718
 - application to Rossby waves 720
- Bieberbach conjecture 417, 479
 - Jacobi polynomials 479
- bifurcation sets 781
 - visualizations 782
- big q -Jacobi polynomials 471
- bilateral basic hypergeometric function
 - *see* bilateral q -hypergeometric function.
- bilateral hypergeometric function 408
- bilateral q -hypergeometric function
 - Bailey's ${}_2\psi_2$ transformations 429
 - Bailey's bilateral summations 427
 - computation 432
 - definition 423
 - notation 420
 - Ramanujan's ${}_1\psi_1$ summation 427
 - special cases 427–428
 - transformations 432
- bilateral series 408
- bilinear transformation 17
 - cross ratio 17
 - $SL(2, \mathbb{Z})$ 579
- binary number system 72
- binary quadratic sieve
 - number theory 648
- Binet's formula
 - gamma function 140
- binomial coefficients
 - definitions 619
 - generating functions 619
 - identities 619

- limiting form 619
- recurrence relations 619
- relation to lattice paths 619
- tables 619, 635
- binomial expansion 108
- binomial theorem 2
- binomials 2
- black holes
 - Heun functions 720
- Bohr radius
 - Coulomb functions 754
- Bohr-Mollerup theorem
 - gamma function 138
 - q -gamma function 145
- Boole summation formula 597
- Borel summability 33
- Borel transform theory
 - applications to asymptotic expansions 68
- Bose–Einstein condensates
 - Lamé functions 694
- Bose–Einstein integrals
 - computation 614
 - definition 611
 - relation to polylogarithms 612
- Bose–Einstein phase transition 614
- bound-state problems
 - hydrogenic 754
 - Whittaker functions 754
- boundary points 11, 15
- boundary-value methods or problems
 - difference equations 86, 87
 - ordinary differential equations 88
 - parabolic cylinder functions 317
- bounded variation 6
- Boussinesq equation
 - Painlevé transcendents 739
- box
 - plane partitions 629
- branch
 - of multivalued function 20, 104
 - construction 20
 - example 20
- branch cut 104
- branch point 20
 - movable 724
- Bromwich integral 83
- Bulirsch’s elliptic integrals 487
 - computation 518
 - first, second, and third kinds 486
 - notation 486
 - relation to symmetric elliptic integrals 508
- calculus
 - complex variable 14–18
 - one variable 4–7
 - two or more variables 7–9
- calculus of finite differences 597
- canonical integrals 776
 - applications
 - acoustics 792
 - caustics 791
 - integrals with coalescing critical points 789–790
 - optics 791
 - quantum mechanics 791
 - asymptotic approximations 789–790
 - computation 792
 - convergent series 787
 - definitions 776
 - differential equations 788
 - integral identities 787–788
 - notation 776
 - relations to other functions
 - Airy function 777
 - Pearcey integral 777
 - special cases 777
 - symmetries 777
 - visualizations of modulus 778–779
 - visualizations of phase 780–781
 - zeros 785–787
- cardinal function 77
- cardinal monosplines 597–598
- cardinal spline functions 597
- Carmichael numbers
 - number theory 644
- Casimir forces
 - Bernoulli polynomials 598
- Casimir–Polder effect
 - Riemann zeta function 614
- Catalan numbers
 - definitions 620
 - generating function 621
 - identities 623
 - limiting forms 621
 - recurrence relations 621
 - relation to lattice paths 620
 - table 621
- Catalan’s constant
 - Riemann zeta function 610
- Cauchy determinant 4
- Cauchy principal values
 - integrals 6
- Cauchy’s integral formula 16
 - for derivatives 16
- Cauchy’s theorem 16
- Cauchy–Riemann equations 16
- Cauchy–Schwarz inequalities for sums and integrals
 - 12, 13
- caustics
 - Airy functions 209

- canonical integrals 791
 - Cayley's identity for Schwarzian derivatives 27
 - central differences in imaginary direction 436
 - Cesàro means 33
 - Cesàro summability 33, 34
 - chain rule
 - for derivatives 5, 7
 - characteristic equation
 - difference equations 62
 - differential equations 56
 - characteristics
 - Riemann theta functions 539
 - characters
 - number theory
 - Dirichlet 642
 - induced modulus 642
 - orthogonality relation 642
 - primitive 642
 - principal 642
 - quadratic Jacobi symbol 642
 - quadratic Legendre symbol 642
 - real 642
 - Charlier polynomials
 - *see* Hahn class orthogonal polynomials.
 - Chebyshev ψ -function 613
 - Chebyshev polynomials **438**
 - .. *see also* Chebyshev-series expansions *and* classical orthogonal polynomials.
 - applications
 - approximation theory 478
 - solutions of differential equations 478
 - computation 479
 - continued fractions 450
 - definition 439
 - derivatives 447
 - differential equations 445
 - dilated 437
 - expansions in series of 96, 459, 461
 - explicit representations 442–443
 - generating functions 449
 - graphs 440
 - inequalities 450
 - integral representations 448
 - integrals 458
 - interrelations with other classical orthogonal polynomials 444–445
 - leading coefficients 439
 - linearization formula 460
 - local maxima and minima 451
 - normalization 439
 - notation 436
 - of the first, second, third, and fourth kinds 439
 - orthogonality properties
 - with respect to integration 96, 439
 - with respect to summation 97, 440
 - recurrence relations 96, 446
 - relations to other functions
 - hypergeometric function 394
 - Jacobi polynomials 444
 - trigonometric functions 442
 - Rodrigues formula 442
 - scaled 478
 - shifted 437, 439
 - special values 444
 - symmetry 444
 - tables 480
 - of coefficients 440
 - upper bounds 451
 - weight functions 439
 - zeros 438, 440
- Chebyshev-series expansions
- complex variables 97
 - computation of coefficients 97
 - relation to minimax polynomials 97
 - summation 97
- chemical reactions
- $3j, 6j, 9j$ symbols 765
- chi-square distribution function
- incomplete gamma functions 189
- Chinese remainder theorem
- number theory 647
- Christoffel coefficients (or numbers) *see* Gauss quadrature, Christoffel coefficients (or numbers)
- Christoffel-Darboux formula
- classical orthogonal polynomials 438
 - confluent form 438
- Chu–Vandermonde identity
- hypergeometric function 387
- circular trigonometric functions
- *see* trigonometric functions.
- classical dynamics
- Jacobian elliptic functions 566
 - Weierstrass elliptic functions 582
- classical orthogonal polynomials **438**
- addition theorems 459
 - applications
 - approximation theory 478
 - Bieberbach conjecture 479
 - integrable systems 478
 - numerical solution of differential equations 478
 - physical 479
 - quadrature 478
 - quantum mechanics 479
 - Radon transform 479
 - random matrix theory 479
 - Riemann–Hilbert problems 479
 - asymptotic approximations 451–454
 - computation 479

- connection formulas 460
 contiguous relations 446
 continued fractions 450
 definitions 438–439
 derivatives 446–447
 differential equations 445
 expansions in series of 459–461
 explicit representations 442–443
 Fourier transforms 456–457
 generating functions 449
 in two or more variables 477
 inequalities
 local maxima and minima 450–451
 Turan-type 450
 upper bounds 450
 integral representations 447–448
 for products 455
 integrals 455–459
 compendia 459
 interrelations
 limiting forms 445
 linear 444
 quadratic 445
 with other orthogonal polynomials 464
 Laplace transforms 457
 leading coefficients 439
 limiting forms
 Mehler–Heine type formulas 449
 linearization formulas 460
 local maxima and minima 450–451
 Mellin transforms 457
 multiplication theorems 460
 normalization 439
 notations 436
 orthogonality properties 439, 443
 parameter constraints 439, 443
 Poisson kernels 461
 recurrence relations 446
 relations to other functions
 confluent hypergeometric functions 442
 generalized hypergeometric functions 442
 hypergeometric function 393–394, 442
 sums 459–461
 Bateman-type 461
 compendia 461
 tables 480
 of coefficients 440
 upper bounds 450
 weight functions 439
 zeros
 asymptotic approximations 454–455
 distribution 438
 inequalities 454
 classical theta functions *see* theta functions.
- Clausen's integral 611
 tables 614
 Clebsch–Gordan coefficients *see* $3j$ symbols.
 relation to generalized hypergeometric functions
 418
 Clenshaw's algorithm
 Chebyshev series 97
 classical orthogonal polynomials 480
 Clenshaw–Curtis quadrature formula 79, 82
 comparison with Gauss quadrature 80
 closed point set 11, 15
 closure
 of interval 6
 of point sets in complex plane 15
 coalescing saddle points 789–790
 coaxial circles
 symmetric elliptic integrals 516
 coding theory
 combinatorics 635
 Krawtchouk and q -Racah polynomials 479
 cofactor *see* determinants.
 coherent states
 generalized
 confluent hypergeometric functions 346
 cols *see* saddle points.
 combinatorial design 635
 combinatorics **618**
 applications 635
 mathematical 635
 physical 635
 generalized hypergeometric functions 417
 hypergeometric identities 400
 Painlevé transcendents 739
 compact set 18
 complementary error function *see* error functions.
 complementary exponential integral
 *see* exponential integrals.
 completely multiplicative functions 640
 complex numbers
 arithmetic operations 15
 complex conjugates 15
 DeMoivre's theorem 15
 imaginary part 14
 modulus 15
 phase 15
 polar representation 14
 powers 15
 real part 14
 triangle inequality 15
 complex physical systems
 incomplete gamma functions 189
 complex tori
 theta functions 533
 computer arithmetic

- generalized exponentials and logarithms 131
- computer-aided design
 Cornu's spiral 169
- conductor
 generalized Bernoulli polynomials 597
- confluent Heun equation 717
 applications 720
 properties of solutions 718
 special cases 717
- confluent hypergeometric functions . . *see also* Kummer
 functions *and* Whittaker functions.
 of matrix argument **770**
 asymptotic approximations 771
 computation 773
 definition 770
 first kind 768
 Laguerre form 770
 notation 768
 properties 770
 relations to Bessel functions of matrix argument
 770
 second kind 768
 relations to other functions
 Airy functions 197
 Bessel and Hankel functions 228
 classical orthogonal polynomials 442
 Coulomb functions 742, 748
 error functions 164
 exponential integrals 153
 generalized Bessel polynomials 476
 generalized exponential integral 186
 Hahn class orthogonal polynomials 466
 modified Bessel functions 255
 parabolic cylinder functions 308, 315
 repeated integrals of error functions 167
 sine and cosine integrals 153
- conformal mapping 16–17
 generalized hypergeometric functions 417
 hypergeometric function 399
 Jacobian elliptic functions 564
 modular functions 581
 symmetric elliptic integrals 515
 Weierstrass elliptic functions 581
- congruence of rational numbers 593
- conical functions **372**
 applications 379
 asymptotic approximations
 large degree 374
 large order 374
 behavior at singularities 373
 connection formulas 372
 definitions 372
 degree 352
 differential equation 372
 generalized Mehler–Fock transformation 373
 graphics 373
 integral representation 373
 integrals with respect to degree 375
 notation 352, 372
 order 352
 tables 380
 trigonometric expansion 373
 Wronskians 372
 zeros 375
- connected point set 15
- constants
 roots of 23
- continued fractions 24–25
 applications 25
 approximants 24
 canonical denominator (or numerator) 24
 contraction 25
 convergence 25
 convergents 24
 existence of 25
 determinant formula 24
 equivalent 24
 even part 25
 extension 25
 fractional transformations 25
 J-fraction 95
 Jacobi fraction 95
 associated 95
 notation 24
 numerical evaluation
 backward recurrence 95
 forward recurrence 95
 forward series recurrence 96
 odd part 25
 Pringsheim's theorem 25
 quotient-difference algorithm 95
 recurrence relations 24
 relation to power series 94, 95
 S-fraction 95
 series 24
 Stieltjes fraction 95
 Van Vleck's theorem 25
- continuous dual Hahn polynomials
 *see* Wilson class orthogonal polynomials.
- continuous dynamical systems and mappings
 Painlevé transcendents 739
- continuous function
 at a point 4, 7, 15
 notation 4
 of two variables 7, 15
 on a point set 7
 on a region 15
 on an interval 4

- on the left (or right).....4
- piecewise4, 7
- removable discontinuity4
- sectionally4
- simple discontinuity4
- continuous Hahn polynomials
 - *see* Hahn class orthogonal polynomials.
- continuous q -Hermite polynomials473
- continuous q^{-1} -Hermite polynomials473
 - asymptotic approximations to zeros474
- continuous q -ultraspherical polynomials473
- contour16
 - simple16
 - simple closed16
- convergence
 - acceleration *see* acceleration of convergence.
 - cubic90
 - geometric90
 - linear90
 - local90
 - of the p th order90
 - quadratic90
- convex functions7
- coordinate systems
 - cylindrical7
 - ellipsoidal582, 693
 - elliptic720
 - elliptical677–678
 - oblate spheroidal705
 - parabolic cylinder317
 - paraboloid of revolution317
 - paraboloidal346, 678
 - polar7
 - projective581
 - prolate spheroidal704
 - spherical (or spherical polar)8
 - sphero-conal693
 - toroidal371, 379
- Cornu's spiral168
 - applications169
 - connection with Fresnel integrals168
- cosecant function *see* trigonometric functions.
- cosine function *see* trigonometric functions.
- cosine integrals150
 - analytic continuation151
 - applications155
 - approximations156
 - asymptotic expansions153
 - exponentially-improved154
 - auxiliary functions... *see* auxiliary functions for sine and cosine integrals.
 - Chebyshev-series expansions156–157
 - computation155
 - definition150
 - expansion in spherical Bessel functions153
 - generalized188–189
 - graphics151
 - hyperbolic analog150
 - analytic continuation151
 - integral representations152
 - integrals154
 - Laplace transform154
 - notation150
 - power series151
 - principal value150
 - relations to exponential integrals151
 - sums154
 - tables156
 - value at infinity150
 - zeros154
 - asymptotic expansion154
 - computation156
- cosmology
 - confluent hypergeometric functions346
 - incomplete beta functions189
- cotangent function *see* trigonometric functions.
- Coulomb excitation of nuclei753
- Coulomb field754
- Coulomb functions
 - Dirac delta38
- Coulomb functions: variables ρ, η 742
 - analytic properties742
 - applications753–755
 - asymptotic expansions
 - large η 747
 - large ρ 747
 - uniform expansions747–748
 - case $\eta = 0$ 744
 - complex variable and parameters748, 754
 - computation755
 - continued fractions745
 - conversions between variables and parameters...754
 - cross-product742
 - definitions742
 - derivatives744
 - expansions in Airy functions747
 - expansions in Bessel functions746
 - expansions in modified Bessel functions746
 - expansions in spherical Bessel functions745
 - functions $F_\ell(\eta, \rho), G_\ell(\eta, \rho), H_\ell^\pm(\eta, \rho)$ 742
 - graphics743–744
 - integral representations745
 - limiting forms
 - large ℓ 744
 - large $|\eta|$ 746
 - large ρ 746, 747
 - small $|\eta|$ 744
 - small ρ 744

- normalizing constant 742
- phase shift (or phase) 742, 756
- power-series expansions in ρ 745
- recurrence relations 744
- relations to other functions
- confluent hypergeometric functions 742
 - Coulomb functions with variables r, ϵ 751
 - Whittaker functions 742
- scaling of variables and parameters 753, 754
- tables 755
- transition region 747
- WKB approximations 755
- Wronskians 742
- Coulomb functions: variables r, ϵ **748**
- analytic properties 748
 - applications 753–755
 - asymptotic approximations and expansions for large $|r|$ 753
 - asymptotic expansions as $\epsilon \rightarrow 0$ 753
 - uniform 753
 - case $\epsilon = 0$ 752
 - complex variables and parameters 754
 - computation 755
 - conversions between variables and parameters 754
 - definitions 748
 - derivatives 752
 - expansions in Airy functions 753
 - expansions in Bessel functions 752, 753
 - expansions in modified Bessel functions 752, 753
 - functions $f(\epsilon, \ell; r), h(\epsilon, \ell; r)$ 748
 - functions $s(\epsilon, \ell; r), c(\epsilon, \ell; r)$ 748
 - graphics 749–750
 - integral representations for Dirac delta 749
 - limiting forms for large ℓ 752
 - power-series expansions in ϵ 752
 - power-series expansions in r 752
 - recurrence relations 752
 - relations to other functions
 - confluent hypergeometric functions 748
 - Coulomb functions with variables ρ, η 751
 - Whittaker functions 748, 751
 - scaling of variables and parameters 753, 754
 - tables 755
 - Wronskians 749
- Coulomb phase shift 145, 742, 755, 756
- Coulomb potential barriers 754
- Coulomb potentials 753–754
- q -hypergeometric function 432
- Coulomb radial functions
- *see* Coulomb functions: variables ρ, η .
- Coulomb spheroidal functions 704
- as confluent Heun functions 717
- Coulomb wave equation
- irregular solutions 742, 748
 - regular solutions 742, 748
 - singularities 742, 748
 - turning points 742, 748, 754
- Coulomb wave functions .. *see* Coulomb functions: variables ρ, η and Coulomb functions: variables r, ϵ .
- counting techniques 634
- critical phenomena
- elliptic integrals 517
 - hypergeometric function 400
- critical points **781**
- coalescing 789–790
- cross ratio 17
- cryptography 647
- Weierstrass elliptic functions 582
- curvature
- for disks and squares 84–85
- cubic equation 23
- resolvent 23
- cubic equations
- solutions as trigonometric and hyperbolic functions 131
- curve
- piecewise differentiable 11
 - simple closed 11
- cusp bifurcation set
- formula 781
 - picture 782
- cusp canonical integral **776**, 785
- zeros 785
 - table 786
- cusp catastrophe **776**, 784
- cuspsoids
- normal forms 776
- cut 20
- domain 20
 - neighborhood 20
- cycle 618
- Riemann surface 543
- cyclic identities
- Jacobian elliptic functions 558
- cyclotomic fields
- Bernoulli and Euler polynomials 598
- cylinder functions
- addition theorems 246
 - definition 218
 - derivatives 222
 - differential equations 217, 226
 - integrals 240–241
 - multiplication theorem 246
 - recurrence relations 222
 - zeros *see* zeros of cylinder functions.
- cylindrical coordinates 7
- cylindrical polar coordinates
- *see* cylindrical coordinates.

- Darboux's method
 asymptotic approximations of sums and sequences
 65–66
- Dawson's integral **160**
 applications 169
 approximations 170
 computation 169
 definition 160
 generalized 166
 graphics 161
 integral representation 162
 notation 160
 relation to error functions 162
 relation to parabolic cylinder functions 308
 tables 169
- de Branges–Wilson beta integral 143
- De Moivre's theorem
 trigonometric functions 118
- Dedekind modular function 646
 functional equation 646
- Dedekind sums
 number theory 646
- Dedekind's eta function *see* modular functions.
- Dedekind's modular function... *see* modular functions.
- del operator 10
- Dellano numbers
 definition 621
 generating functions 623
 recurrence relation 623
 relation to lattice paths 621
 table 621
- delta sequence 37
- delta wing equation
 Lamé polynomials 694
- derivatives
 chain rule 5, 7
 definition 5, 7
 distributional 36
 Faà di Bruno's formula 5
 Jacobian 9
 L'Hôpital's rule 5
 left-hand 14
 Leibniz's formula 5
 mean value theorem 5
 notation 5, 7
 of distribution 35
 partial 7
 right-hand 14
- Descartes' rule of signs (for polynomials) 22
- determinants
 alternants 3
 Cauchy 4
 circulant 4
 cofactor 3
- definition 3
- Hadamard's inequality 3
- Hankel 93, 595
- inequalities 3
- infinite
 convergence 4
 Hill's type 4
- Krattenthaler's formula 4
- minor 3
- notation 3
- persymmetric 595
- properties 3
- Vandermonde 3
- diatomic molecules
 hypergeometric function 400
- difference equations
 asymptotic solutions
 .. *see* asymptotic solutions of difference equations.
 distinguished solutions 85
 minimal solutions 85
 numerical solution 85–88
 backward recursion method 85, 86
 boundary-value methods 86, 87
 homogeneous equations 85–86
 inhomogeneous equations 86
 normalizing factor 86
 stability 85
 recessive solutions 85
- difference operators 436
 backward 436
 central in imaginary direction 436
 forward 436
- differentiable functions 5, 15
- differential equations
 asymptotic solutions *see* asymptotic solutions of
 differential equations.
 change of variables
 elimination of first derivative 26
 Liouville transformation 26
 point at infinity 26
 classification of singularities 56, 409
 closed-form solutions 27
 dominant solutions 57
 Fuchs–Frobenius theory 55
 homogeneous 26, 88
 indices differing by an integer 56
 indicial equation 56
 inhomogeneous 26, 88
 solution by variation of parameters 26
 irregular singularity 56
 nonhomogeneous *see* inhomogeneous.
 numerical solution
 boundary-value problems 88
 eigenfunctions 89

- eigenvalues 89
- initial-value problems 88
- Runge–Kutta method 89–90
- stability 88
- Sturm–Liouville eigenvalue problems 89
- Taylor-series methods 88–89
- numerically satisfactory solutions 58
- of arbitrary order 409
- ordinary point 55, 409
- rank of singularity 56
- recessive solutions 57
- regular singularity 56
- Schwarzian derivative 27
- solutions
 - existence 25
 - fundamental pair 26
 - in series of Chebyshev polynomials 478, 480
 - in series of classical orthogonal polynomials ... 479
 - linearly independent 26
 - products 27
 - Wronskian 26
- subdominant solutions *see* recessive solutions.
- with a parameter 26
- differentiation
 - Cauchy–Riemann equations 16
 - numerical
 - analytic functions 77
 - Lagrange’s formula for equally-spaced nodes ... 77
 - partial derivatives 78
 - of integrals 8, 21
 - partial 7
- diffraction catastrophes **777**, 789
- notation 776
- scaling laws 785
- diffraction of light
 - Fresnel integrals and Cornu’s spiral 161, 169
- diffraction problems
 - Mathieu functions 678
- diffusion equations
 - theta functions 533
- diffusion problems
 - Mathieu functions 678
- digamma function *see* psi function.
- dilogarithms
 - analytic properties 610
 - approximations 615
 - computation 614
 - definition 610
 - graphics 611
 - principal branch (or value) 610
 - tables 614
- Dirac delta 37–38
- delta sequences 37–38
- integral representations
 - Airy functions 38
 - Bessel functions 38
 - Coulomb functions 38
 - Fourier 37–38
 - spherical Bessel functions 38
- mathematical definitions 38
- series representations
 - Fourier 38
 - Hermite polynomials 38
 - Laguerre polynomials 38
 - Legendre polynomials 38
 - spherical harmonics 38
- Dirac delta distribution 53
- Dirac delta function *see* Dirac delta.
- Dirac equation
 - Coulomb functions 754
- Dirichlet L -functions
 - analytic properties 612
 - definition 612
 - functional equation 612
 - infinite products 612
 - tables 614
 - zeros 613
- Dirichlet characters 642
- Gauss sum 643
- Dirichlet problem
 - with toroidal symmetry 379
- Dirichlet product (or convolution) 641
- Dirichlet series 602, 640
- generating function 640
- Dirichlet’s divisor problem
 - number theory 643
- Dirichlet’s theorem
 - prime numbers in arithmetic progression 643
- discontinuity 4
- discrete Fourier transform 99
- discrete q -Hermite I and II polynomials 471
- discriminant
 - of a polynomial 22
- discriminant function
 - number theory 646
 - functional equation 646
- disk
 - around infinity 16
 - open 15
- disk polynomials 477
- dislocation theory
 - Heun functions 720
- distribution function
 - Painlevé transcendents 739
- distribution functions
 - connection with
 - incomplete beta functions 189
 - incomplete gamma functions 189

- distributional derivative 36
- distributions 35–37
- convergence 35, 36
 - convolutions 54
 - derivatives 35
 - Dirac delta 36
 - distributional derivative 36
 - Fourier transforms 37
 - Heaviside function 36
 - linear functionals 35
 - of derivatives 52
 - regular 35
 - regularization 55
 - several variables 36–37
 - singular 35
 - support 35
 - tempered *see* tempered distributions., 52
 - test function space 35
 - test functions 35
 - convergence 35
- divergence theorem
- .. *see* Gauss's theorem for vector-valued functions.
- divergent integrals 51
- regularization 55
- divided differences
- definition 76
 - integral representation 76
- divisor function
- number theory 638
- Dixon's ${}_3F_2(1)$ sum
- q -analog 426
- Dixon's sum
- F. H. Jackson's q -analog 426
- domain 15
- closed 15
 - cut 20
 - simply-connected 25
- dominated convergence theorem
- infinite series 18
- double gamma function *see* Barnes' G -function.
- double integrals 8–9
- change of order of integration 9
 - change of variables 9
 - infinite 9
- double sequence 18
- convergence 18
- double series 18
- convergence 18
- doubly-confluent Heun equation 717
- Dougall's ${}_7F_6(1)$ sum
- F. H. Jackson's q -analog 427
- Dougall's bilateral sum 387
- Dougall's expansion
- associated Legendre functions 371
- dual Hahn polynomials
- *see* Wilson class orthogonal polynomials.
- Duffing's equation
- Jacobian elliptic functions 565
- dynamical systems
- Mathieu functions 679
 - Painlevé transcendents 739
- Dyson's integral
- gamma function 144
- ecological systems
- incomplete gamma functions 189
- Einstein summation convention for vectors 10
- Eisenstein convention 577
- Eisenstein series
- Jacobian elliptic functions 559
- electric particle field
- Stieltjes electrostatic interpretation 719
- electromagnetic scattering
- Bessel functions and spherical Bessel functions .. 275
- electromagnetic theory
- sine and cosine integrals 155
- electromagnetic waves
- Mathieu functions 678
- electron-ion collisions
- Coulomb functions 753
- electronic structure of heavy elements
- Coulomb functions 754
- electrostatics
- Jacobian elliptic functions 566
 - zeros of classical orthogonal polynomials 479
- elementary functions *see* exponential function, hyperbolic functions, inverse hyperbolic functions, inverse trigonometric functions, Lambert W -function, logarithm function, power function, *and* trigonometric functions.
- relation to R_C -function 495
- elementary particle physics
- conical functions 379
- ellipse
- elliptic integrals 514
- ellipse arc length
- Jacobian elliptic functions 563
- ellipsoid
- capacity 516
 - depolarization 497, 516
 - potential 516
 - self-energy 516
 - surface area 515
 - triaxial 515
- ellipsoidal coordinates 693
- Weierstrass elliptic functions 582
- ellipsoidal harmonics
- Lamé polynomials 694
- ellipsoidal wave equation *see* Lamé wave equation.

- elliptic coordinates 720
- elliptic crack and punch problems
Lamé polynomials 694
- elliptic curves 564
addition law 581
definition 581
Jacobi–Abel addition theorem 564
Jacobian normal form 564
Mordell’s theorem 582
tables 582
Weierstrass elliptic functions 581
- elliptic functions .. *see also* Jacobian elliptic functions
and Weierstrass elliptic functions.
general 571
representation as Weierstrass elliptic functions
..... 576
Weierstrass *see* Weierstrass elliptic functions.
- elliptic integrals .. *see* basic elliptic integrals, Bulirsch’s
elliptic integrals, general elliptic integrals, general-
izations of elliptic integrals, Legendre’s elliptic in-
tegrals, *and* symmetric elliptic integrals.
complete
quasiconformal mapping 399
relations to other functions
associated Legendre functions 360
Ferrers functions 360
Weierstrass elliptic functions 576
- elliptic modular function *see* modular functions.
- elliptic umbilic bifurcation set
formula 781
picture 782
- elliptic umbilic canonical integral **776**
asymptotic approximations 789–790
convergent series 787
differential equations 788
formulas for Stokes set 783
integral identity 788
pictures of modulus 779
pictures of phase 780
scaling laws 785
zeros 786
- elliptic umbilic catastrophe **776**, 785
- elliptical coordinates
Mathieu functions 677–678
- entire functions 16
asymptotic expansions 64
Liouville’s theorem 16
- enumerative topology
Painlevé transcendents 739
- epsilon function *see* Jacobi’s epsilon function.
- equation of Ince .. *see* Hill’s equation, equation of Ince.
- equiconvergent 699
- Erlang loss function
incomplete gamma functions 189
- error-control function
differential equations 57
- error functions **160**
applications
asymptotic approximation of integrals 168
physics 169
statistics 169
Stokes phenomenon 168
approximations 170
asymptotic expansions 164
exponentially-improved 164
computation 83–84, 169
continued fractions 163
definitions 160
derivatives 163
expansions in spherical Bessel functions 162
generalized 166
graphics 160, 161
inequalities 163
integral representations 162–163
integrals
Fourier transform 166
Laplace transforms 166
interrelations 162
inverse functions 166
asymptotic expansions 166
computation 169
power-series expansions 166
notation 160
power-series expansions 162
relations to other functions
confluent hypergeometric functions .. 164, 328, 338
Dawson’s integral 162
Fresnel integrals 162
generalized exponential integrals 164
incomplete gamma functions 164
parabolic cylinder functions 308
probability functions 160
Voigt functions 167
repeated integrals of ... *see* repeated integrals of the
complementary error function.
sums 166
tables 169, 170
values at infinity 160
zeros 165
asymptotic expansions 165
tables 165, 170
- error measures
absolute error 73
complex arithmetic 73
mollified error 73
relative error 73
relative precision 73
- error term 43

- essential singularity 19
 *see also* isolated essential singularity.
- eta function *see* Dedekind's eta function.
- Euler–Maclaurin formula 63
 extensions 63
 generalization 597
- Euler numbers **588**
 arithmetic properties 593
 asymptotic approximations 593
 computation 598
 definition 588
 explicit formulas 591
 factors 593
 finite expansions 590
 generalizations 596, 597
 generating function 588
 identities 591
 inequalities 593
 integral representation 592
 inversion formulas 591
 Kummer congruences 593
 notation 588
 recurrence relations
 linear 591
 quadratic and higher order 595
 sums 595
 tables 589, 598
- Euler polynomials **588**
 applications
 mathematical 597–598
 physical 598
 asymptotic approximations 593
 computation 598
 definition 588
 derivative 590
 difference equation 589
 explicit formulas 591
 finite expansions 590
 generalized 596, 597
 generating function 588
 graphs 589
 inequalities 593
 infinite series expansions
 Fourier 592
 other 592
 integral representations 592
 integrals 594
 compendia 595
 Laplace transforms 595
 multiplication formulas 590
 notation 588
 recurrence relations
 linear 591
 quadratic 595
- representations as sums of powers 589
 special values 590
 sums 595
 symbolic operations 590
 symmetry 589
 tables 589
 zeros
 complex 594
 multiple 594
 real 594
- Euler product
 number theory 640
- Euler splines 597
- Euler sums
 Riemann zeta function 613
 reciprocity law 614
- Euler's beta integral 142
- Euler's constant 136
 integral representations 140
- Euler's homogeneity relation
 symmetric elliptic integrals 501
- Euler's integral
 gamma function 136
- Euler's pentagonal number theorem
 number theory 646
- Euler's sums
 q -hypergeometric function 423, 424
- Euler's totient
 number theory 638
- Euler's transformation
 applied to asymptotic expansions 69
 of series 93
- Euler–Fermat theorem
 number theory 638, 647
- Euler–Poisson differential equations 501
- Euler–Poisson–Darboux equation
 symmetric elliptic integrals 501
- Euler–Tricomi equation
 Airy functions 209
- Eulerian numbers
 definition 632
 generating functions 632
 identities 632
 notation 632
 relation to Bernoulli numbers 591
 relation to permutations 632
 special values 633
 table 632
- evolution equations
 Lamé polynomials 694
- exact rational arithmetic 72
- exponential function **105**
 analytic properties 105
 approximations 132

- Chebyshev-series expansions 132
- computation 131
- conformal maps 106
- continued fractions 110
- definition 105
- derivatives 109
- differential equation 109
- generalized 111
- graphics
- complex argument 107
- real argument 106
- identities 109
- inequalities 108
- integrals 110
- limits 107
- notation 104
- periodicity 105
- power series 105
- special values 107
- sums 110
- tables 132
- zeros 105
- exponential growth 28
- exponential integrals **150**
- analytic continuation 151
- applications 155
- approximations 156
- asymptotic expansions 153
- exponentially-improved 153
- re-expansion of remainder term 153
- Chebyshev-series expansions 156, 157
- computation 155
- continued fraction 153
- definition 150
- expansion in inverse factorials 153
- expansions in modified spherical Bessel functions
- 153
- generalized 185
- graphics 151
- inequalities 152
- integral representations 152
- integrals 154
- interrelations 150, 151
- Laplace transform 154
- notation 150
- power series 151
- principal value 150
- relations to other functions
- confluent hypergeometric functions 153
- incomplete gamma function 153
- logarithmic integral 150
- sine and cosine integrals 151
- small argument 51
- tables 156
- zeros 154
- extended complex plane 16
- Faà di Bruno's formula
- for derivatives 5
- Fabry's transformation
- differential equations 57
- factorials (rising or falling) 618
- factorization
- of integers 648
- via Weierstrass elliptic functions 582
- Faddeeva function 169
- fast Fourier transform 100
- Fay's trisecant identity
- Riemann theta functions with characteristics ... 544
- generalizations 544
- Fejér kernel
- Fourier integral 34
- Fourier series 33
- Fermat numbers
- number theory 648
- Fermat's last theorem
- Bernoulli and Euler numbers and polynomials ... 598
- Fermi–Dirac integrals
- approximations 615
- computation 614
- definition 611
- relation to polylogarithms 612
- tables 614
- uniform asymptotic approximation 612
- Ferrers board 633
- Ferrers function
- of the first kind
- integral equation for Lamé functions 689
- Ferrers functions **352**
- addition theorems 370
- analytic continuation 376
- applications
- spherical harmonics 378–379
- spheroidal harmonics 378
- asymptotic approximations
- *see* uniform asymptotic approximations.
- behavior at singularities 361–362
- computation 379
- connection formulas 362
- cross-products 352
- definitions 353–354
- degree 352
- derivatives 362
- with respect to degree or order 363
- differential equation
- *see* associated Legendre equation.
- generating functions 361
- graphics 355–357
- integer degree and order 360–361

- integer order 360
- integral representations 363
- integrals
 - definite 369
 - indefinite 368
 - Laplace transforms 370
 - Mellin transforms 370
 - orthogonality properties 369
- notation 352
- of the first kind 353
- of the second kind 353
- order 352
- orthogonality 369
- recurrence relations 362
- reflection formulas 361
- relations to other functions
 - elliptic integrals 360
 - hypergeometric function 353, 354, 394
 - Legendre polynomials 360
 - ultraspherical polynomials 448
- Rodrigues-type formulas 360
- special values 359–360
- sums 370–371
- tables 380
- trigonometric expansions 364
- uniform asymptotic approximations
 - large degree 366–368
 - large order 365–366
- Wronskians 352–353
- zeros 368
- Ferrers graph 626
- Feynman diagrams
 - Appell functions 417
- Feynman path integrals
 - theta functions 534
- Fibonacci numbers 596, 629
- fine structure constant
 - Coulomb functions 753
- finite Fourier series
 - number theory 643
- fixed point 90
- floating-point arithmetic
 - bits 72
 - format width 72
 - significant 72
 - double precision 72
 - exponent 72
 - fractional part 72
 - IEEE standard 72
 - machine epsilon 72
 - machine number 72
 - machine precision 72
 - overflow 72
 - rounding
 - by chopping 72
 - down 72
 - symmetric 72
 - to nearest machine number 72
 - significant 72
 - single precision 72
 - underflow 72
- Floquet solutions
 - Hill's equation 675
 - Mathieu's equation 653
- Floquet's theorem
 - Hill's equation 674
 - Mathieu's equation 653
- fluid dynamics
 - elliptic integrals 517
 - Legendre polynomials 479
 - Riemann theta functions 545
 - Struve functions 298
- fold canonical integral **776**, 785
 - bifurcation set 781
 - differential equation 788
 - integral identity 787
 - relation to Airy function 777
 - zeros 785
- fold catastrophe **776**, 785
- Fourier cosine and sine transforms
 - definition 27
 - generalized 400
 - inversion 28
 - Parseval's formula 28
 - tables 31, 32
- Fourier integral
 - asymptotic expansions 44, 45
 - Dirac delta 38
 - Fejér kernel 34
 - Poisson kernel 34
 - summability 34
- Fourier series 13–14
 - Bessel's inequality 13
 - coefficients 13
 - compendia 14
 - convergence 14
 - definition 13
 - differentiation 14
 - Dirac delta 38
 - Fejér kernel 33
 - finite
 - number theory 643
 - integration 14
 - Parseval's formula 14
 - Poisson kernel 34
 - Poisson's summation formula 14
 - properties 13
 - summability 33–34

- uniqueness 13
- Fourier transform 27–28
 - convergence 27
 - convolution 27
 - definitions 27
 - discrete 99
 - distributions 37
 - fast 100
 - group
 - hypergeometric function 400
 - inversion 27
 - Parseval's formula 27
 - tables 30, 32
 - tempered distributions 37
 - uniqueness 27
- Fourier–Bessel expansion
 - Bessel functions 248
 - computation 278
- Fourier-series expansions
 - nonuniformity of convergence 155
 - piecewise continuous functions 155
- fractals 92
- fractional derivatives 35
- fractional integrals 35
 - asymptotic expansions 53–55
 - definition 53
- fractional linear transformation
 - *see* bilinear transformation.
- Fresnel integrals **160**
 - applications
 - Cornu's spiral 168
 - interference patterns 161
 - physics and astronomy 169
 - probability theory 169
 - statistics 169
 - approximations 170
 - asymptotic expansions 164
 - exponentially-improved 164
 - auxiliary functions
 - *see* auxiliary functions for Fresnel integrals.
 - computation 169
 - definition 160
 - expansions in spherical Bessel functions 162
 - graphics 161
 - integrals
 - Laplace transforms 166
 - interrelations 162
 - notation 160
 - power-series expansions 162
 - relations to other functions
 - Anger–Weber functions 297
 - auxiliary functions 160, 162
 - confluent hypergeometric functions 164
 - error functions 162
 - generalized hypergeometric functions 164
 - symmetry 161
 - tables 169
 - values at infinity 160
 - zeros 165
 - asymptotic expansions 165
 - tables 165
- Freud weight function 475
- Frobenius' identity
 - Riemann theta functions with characteristics 544
- Fuchsian equation
 - classification of parameters 718
 - definitions 718
 - normal form 718
 - polynomial solutions 718
 - relation to Heun's equation 718
- functions
 - analytic *see* analytic function.
 - analytically continued 19
 - continuous *see* continuous function.
 - continuously differentiable 5, 7
 - convex 7
 - decreasing 4
 - defined by contour integrals 21
 - differentiable 5
 - entire *see* entire functions.
 - harmonic 16
 - holomorphic *see* analytic function.
 - increasing 4
 - inverse 21
 - limits *see* limits of functions.
 - many-valued *see* multivalued function.
 - meromorphic 19
 - monotonic 4
 - multivalued *see* multivalued function.
 - nondecreasing 4
 - nonincreasing 4
 - of a complex variable 18–22
 - of bounded variation 6
 - of compact support 35
 - of matrix argument
 - *see* functions of matrix argument.
 - strictly decreasing 4
 - strictly increasing 4
 - strictly monotonic 4
 - support of 35
 - vector-valued 10–11
- functions of matrix argument **768**
 - Laplace transform 768
 - orthogonal invariance 771
- fundamental theorem of arithmetic 638
- fundamental theorem of calculus 6
- gamma distribution
 - incomplete gamma functions 189

- gamma function **136**
 *see also* incomplete gamma functions.
 analytic properties 136
 applications
 mathematical 145
 physical 145–146
 approximations
 Chebyshev series 147
 complex variables 147
 rational 146, 147
 asymptotic expansions 140–142
 error bounds 141
 exponentially-improved 141
 for ratios 141
 Bohr-Mollerup theorem 138
 computation 146
 continued fraction 140
 definition 136
 duplication formula 138
 Euler's integral 136
 extrema
 asymptotic approximation 138
 table of 138
 Gauss's multiplication formula 138
 graphics 136–137
 inequalities 138
 infinite products 139
 integral representations 139–140, 143–144, 188
 for derivatives 140
 multidimensional 143–144
 logarithm
 continued fraction 140
 convexity 136, 138
 graphics 136
 integral representations 140
 Taylor series 139
 maxima and minima 138
 multiplication formulas 138
 multivariate *see* multivariate gamma function.
 notation 136
 reciprocal
 analytic properties 136
 graphics 136, 137
 Maclaurin series 139
 zeros 136
 recurrence relation 138
 reflection formula 138
 relations to hypergeometric function 387
 scaled 185
 special values 137
 tables 146
 Gaunt coefficient
 $3j$ symbol 761
 Gaunt's integral
 $3j$ symbol 761
 Gauss quadrature 80–83
 Christoffel coefficients (or numbers) 80
 comparison with Clenshaw–Curtis formula 80
 eigenvalue/eigenvector characterization 82
 for contour integrals 83
 Gauss–Chebyshev formula 80
 Gauss–Hermite formula 81
 Gauss–Jacobi formula 80
 Gauss–Laguerre formula 80–81
 generalized 80
 Gauss–Legendre formula 80
 logarithmic weight function 81–82
 nodes 80
 tables 80–83
 remainder terms 80
 weight functions 80
 tables 80–83
 Gauss series
 hypergeometric function 384
 convergence 384
 Gauss sums
 number theory
 Dirichlet character 643
 separable 643
 theta functions 532
 Gauss's ${}_2F_1(-1)$ sum
 q -analog 426
 Gauss's theorem for vector-valued functions 12
 Gauss–Christoffel quadrature ... *see* Gauss quadrature.
 Gaussian
 nonperiodic 533
 Gaussian elimination 73–74
 back substitution 73
 forward elimination 73
 iterative refinement 74
 multipliers 73
 partial pivoting 73
 pivot (or pivot element) 73
 residual vector 74
 triangular decomposition 73
 tridiagonal systems 74
 Gaussian hypergeometric function
 *see also* hypergeometric function.
 of matrix argument **771**
 applications 773
 asymptotic approximations 772
 basic properties 771
 case $m = 2$ 771
 computation 773
 confluent form 771
 definition 771
 Gauss formula 771
 integral representation 771

- Jacobi form..... 771
- notation..... 768
- partial differential equations..... 771–772
- reflection formula..... 771
- transformations of parameters..... 771
- Gaussian noise
 - Lambert W -function..... 131
- Gaussian polynomials
 - definition..... 627
 - tables..... 635
- Gaussian probability functions..... 160
- Gaussian unitary ensemble
 - Painlevé transcendents..... 739
- Gegenbauer function
 - definition..... 394
 - relation to associated Legendre functions..... 355
 - relation to hypergeometric function..... 394
- Gegenbauer polynomials
 - *see* ultraspherical polynomials *and also* classical orthogonal polynomials.
- Gegenbauer's addition theorem
 - Bessel functions..... 247
 - modified Bessel functions..... 261
- general elliptic integrals..... **486**
 - reduction to basic elliptic integrals..... 512
 - reduction to Legendre's elliptic integrals.... 496–497
 - reduction to symmetric elliptic integrals.... 512–514
- general orthogonal polynomials..... **437**
 - computation..... 479
 - difference operators..... 437
 - monic..... 438
 - on finite point sets..... 437
 - on intervals..... 437
 - orthonormal..... 438
 - recurrence relations..... 438
 - sums of products..... 438
 - weight functions..... 437
 - x -difference operators..... 437
 - zeros..... 438
- generalizations of elliptic integrals..... 516
- generalized Airy functions
 - from differential equation..... 206–207
 - asymptotic approximations..... 206
 - definitions..... 206, 207
 - relation to Bessel functions..... 206
 - relation to modified Bessel functions..... 206
 - tables..... 211
 - from integral representations..... 207–208
 - connection formulas..... 208
 - definitions..... 207
 - difference equation..... 208
 - differential equation..... 208
- generalized exponential integral..... **185**
 - analytic continuation..... 187
- applications
 - mathematical..... 189
 - physical..... 189
- approximations..... 191
- asymptotic expansions
 - exponentially-improved..... 187
 - large parameter..... 187
 - large variable..... 187
- Chebyshev-series expansions..... 191
- computation..... 190
- continued fraction..... 187
- definition..... 185
- derivatives..... 186
- further generalizations..... 187
- graphics..... 185–186
- inequalities..... 187
- integral representations..... 185
 - Mellin–Barnes type..... 185
- integrals..... 187
- notation..... 174
- of large argument..... 66
- principal values..... 185
- recurrence relation..... 186
- relations to other functions
 - confluent hypergeometric functions..... 186
 - error functions..... 164
 - incomplete gamma functions..... 185
 - series expansions..... 186
 - special values..... 186
 - tables..... 190
- generalized exponentials..... 111
- generalized functions
 - distributions..... 55
- generalized hypergeometric differential equation... 409
 - confluence of singularities..... 410
 - connection formula..... 410
 - fundamental solutions..... 409
 - singularities..... 409
- generalized hypergeometric function ${}_0F_2$
 - definition..... 404, 408
 - of large argument..... 64
- generalized hypergeometric functions..... **404**
 - analytic continuation..... 408
 - analytic properties..... 404, 405, 408
 - applications
 - mathematical..... 417
 - physical..... 417
 - approximations..... 418
 - argument unity..... 405
 - as functions of parameters..... 405
 - asymptotic expansions
 - formal series..... 411
 - large parameters..... 412
 - large variable..... 411

- small variable 408
- balanced 405
- bilateral series 408
 - Dougall's bilateral sum 408
- computation 418
- contiguous balanced series 407
- contiguous functions 405
- contiguous relations 407
- continued fractions 407
- definitions 404, 408
- derivatives 405
- differential equation *see* generalized hypergeometric differential equation.
- Dixon's well-poised sum 406
- Dougall's bilateral sum 408
- Dougall's very well-poised sum 406
- Džrbasjan's sum 406
- expansions in series of 410
- extensions of Kummer's relations 407
- identities 407
- integral representations 408
- integrals 408
 - inverse Laplace transform 408
 - Laplace transform 408
- k -balanced 405
- Kummer-type transformations 407, 409
- monodromy 417
- notation 404
- of matrix argument **772**
 - applications 773
 - computation 773
 - confluence 772
 - convergence properties 772
 - definition 772
 - Euler integral 773
 - expansion in zonal polynomials 772
 - general properties 772
 - invariance 773
 - Kummer transformation 772
 - Laplace transform 773
 - Mellin–Barnes integrals 773
 - notation 768
 - Pfaff–Saalschutz formula 772
 - relations to other functions 772
 - Thomae transformation 772
 - ${}_3F_2$ case 772
 - value at $\mathbf{T} = \mathbf{0}$ 772
- Pfaff–Saalschutz balanced sum 406
- polynomial cases 404
- principal branch (value) 404
- products 412
- recurrence relations 407
- relations to other functions
 - associated Jacobi polynomials 474
 - Bessel functions 228
 - classical orthogonal polynomials 442
 - Fresnel integrals 164
 - generalized Bessel polynomials 476
 - Hahn class orthogonal polynomials 463
 - Kummer functions 328
 - Meijer G -function 416
 - modified Bessel functions 255
 - orthogonal polynomials *and* other functions 409
 - $3j, 6j, 9j$ symbols 407, 418
 - Wilson class orthogonal polynomials 468
- Rogers–Dougall very well-poised sum 406
- Saalschützian 405
- terminating 404
- transformation of variable 408
 - cubic 409
 - quadratic 408
- very well-poised 405
- Watson's sum 406
- well-poised 405
- Whipple's sum 406
- Whipple's transformation 407
- with two variables 412–415
- zeros 410
- generalized hypergeometric series 404
- generalized integrals
 - asymptotic expansions 52
- generalized logarithms 73, 111
 - applications 131
- generalized precision 73
- generalized sine and cosine integrals **188**
 - analytic properties 188
 - asymptotic expansions for large variable 189
 - auxiliary functions 189
 - asymptotic expansions for large variable 189
 - integral representations 189
 - computation 190
 - definitions
 - general values 188
 - principal values 188
 - expansions in series of spherical Bessel functions 188
 - integral representations 188
 - interrelations 188
 - notation 174
 - power-series expansions 188
 - relation to sine and cosine integrals 188
 - special values 188
- Genocchi numbers 595
 - table 596
- genus
 - Riemann surface 543
- geometric mean 3, 13
- geometric progression (or series) 2
- geophysics

- spherical harmonics 379
- Gibbs phenomenon
 - sine integral 154
- Glaisher's constant 63, 144
- Glaisher's notation
 - Jacobian elliptic functions 550
- Goldbach conjecture
 - number theory 644
- Goodwin–Staton integral
 - asymptotic expansion 164
 - computation 169
 - definition 160
 - relations to Dawson's integral and exponential integral 162
- Graf's addition theorem
 - Bessel functions 247
 - modified Bessel functions 261
- Gram–Schmidt procedure
 - for least squares approximation 99
- graph theory
 - combinatorics 635
- gravitational radiation
 - Coulomb functions 755
- Green's theorem for vector-valued functions
 - three dimensions 12
 - two dimensions 11
- group representations
 - orthogonal polynomials 479
- group theory
 - hypergeometric function 400
- Gudermannian function 121
 - inverse 121
 - relation to R_C -function 495
 - relation to amplitude (am) function 561
 - tables 132
- Haar measure 769
- Hadamard's inequality for determinants 3
- Hahn class orthogonal polynomials 462–467
 - asymptotic approximations 466–467
 - computation 479
 - definitions 462
 - difference equations on variable 465
 - differences 465
 - dualities 463
 - generating functions 466
 - interrelations with other orthogonal polynomials 463, 464
 - leading coefficients 462
 - limit relations 463
 - normalizations 462
 - notation 436
 - orthogonality properties 462
 - recurrence relations 464
 - relations to confluent hypergeometric functions and generalized hypergeometric functions 328, 463
 - relations to hypergeometric function 394, 463
 - Rodrigues formula 462
 - special cases 463
 - weight functions 462
- Hahn polynomials
 - *see* Hahn class orthogonal polynomials.
- Hamiltonian systems
 - chaotic
 - Lamé functions 694
 - handle
 - Riemann surface 543
- Hankel functions **217**
 - addition theorems 246–247
 - analytic continuation 226
 - approximations 281
 - asymptotic expansions for large argument .. 229–230
 - error bounds 229–230
 - exponentially-improved 230
 - asymptotic expansions for large order 231–235
 - asymptotic forms 231
 - double asymptotic properties 235, 258
 - transition region 232
 - uniform 232–235
 - branch conventions 218
 - computation 276–277
 - connection formulas 222
 - cross-product 222
 - definitions 217
 - derivatives 222
 - asymptotic expansions for large argument 229
 - asymptotic expansions for large order 231–232
 - uniform asymptotic expansions for large order 233
 - zeros 238
 - differential equations 217
 - *see also* Bessel's equation.
 - graphics 220–221
 - incomplete 262
 - integral representations
 - along real line 224
 - compendia 226
 - contour integrals 224
 - integrals
 - *see* integrals of Bessel and Hankel functions.
 - limiting forms 217, 223
 - multiplication theorem 246
 - notation 217
 - power series 223
 - principal branches (or values) 217
 - recurrence relations 222
 - relations to other functions
 - Airy functions 196–197

- confluent hypergeometric functions 228
- elementary functions 228
- Wronskians 222
- zeros 238
 - computation 277
 - tables 279
 - with respect to order (ν -zeros) 240
- Hankel transform 246
 - computation 277
- Hankel's expansions
 - for Bessel and Hankel functions 228–229
 - for modified Bessel functions 255
- Hankel's integrals
 - Bessel functions and Hankel functions 226
- Hankel's inversion theorem
 - Bessel functions 246
- Hankel's loop integral
 - gamma function 139
- harmonic analysis
 - hypergeometric function 399
- harmonic functions 16
 - maximum modulus 20
 - mean value property 16
 - Poisson integral 16
- harmonic mean 3, 13
- harmonic oscillators
 - Hermite polynomials 479
 - q -hypergeometric function 432
- harmonic trapping potentials
 - parabolic cylinder functions 317
- heat conduction in liquids
 - Rayleigh function 276
- heat theory
 - conical functions 379
- Heaviside function 36, 54
 - derivative 36
- Heine's formula
 - associated Legendre functions 377
- Heine's integral
 - Legendre functions 364
- Helmholtz equation
 - $3j, 6j, 9j$ symbols 765
 - associated Legendre functions 379
 - Bessel functions and modified Bessel functions 275
 - parabolic cylinder functions 317
 - paraboloidal coordinates 678
- Hermite polynomials **438**
 - *see also* classical orthogonal polynomials.
 - addition theorem 460
 - applications
 - integrable systems 478
 - random matrix theory 479
 - Schrödinger equation 479
 - asymptotic approximations 453
 - computation 479
 - continued fractions 450
 - definitions 439
 - derivatives 447
 - differential equations 445
 - Dirac delta 38
 - expansions in series of 459–461
 - explicit representations 442–443
 - Fourier transforms 457
 - generating functions 450
 - graphs 441
 - inequalities 450, 451
 - Turan-type 450
 - integral representations 447, 448
 - integrals 455, 457–459
 - indefinite 455
 - Nicholson-type 455
 - interrelations with other orthogonal polynomials
 - 444–445, 463, 464
 - Laplace transform 457
 - leading coefficients 439
 - limiting forms as trigonometric functions 449
 - linearization formulas 461
 - local maxima and minima 451
 - Mellin transform 458
 - monic 81, 441
 - multiplication theorem 460
 - normalizations 439
 - notation 436
 - orthogonality properties 439
 - Poisson kernels 461
 - recurrence relations 446
 - relations to other functions
 - confluent hypergeometric functions 328, 338, 449
 - derivatives of the error function 163
 - generalized hypergeometric functions 443
 - parabolic cylinder functions 308
 - repeated integrals of the complementary error function
 - 167
 - Rodrigues formula 442
 - special values 444
 - symmetry 444
 - tables 480
 - of coefficients 440
 - of zeros 81
 - upper bounds 450
 - zeros 438, 455
 - asymptotic behavior 455
 - tables 81
- Hermite–Darboux method
 - Heun functions 713
- Hermitian matrices
 - Gaussian unitary ensemble
 - limiting distribution of eigenvalues 739

- Heun equation *see* Heun's equation.
- Heun functions **710**
- applications
 - mathematical 719–720
 - physical 720
 - asymptotic approximations 718
 - computation 720
 - definition 712
 - differential equation *see* Heun's equation.
 - expansions in series of
 - hypergeometric functions 716–717
 - orthogonal polynomials 717
 - integral equations and representations 714–716
 - notation 710
 - orthogonality
 - double 714
 - single 714
 - relations to hypergeometric function 713
 - relations to Lamé functions 713
- Heun polynomials **712**
- applications 719
 - definitions 712
 - integral equations and representations 715
 - notation 712
 - orthogonality 714
 - products 719
- Heun's equation **710**
- accessory parameter 710
 - asymptotic approximations 718
 - applications 718
 - mathematical 719–720
 - physical 720
 - asymptotic approximations
 - eigenvalues of accessory parameters 718
 - solutions near irregular singularities 718
 - solutions of confluent forms 718
 - solutions with coalescing singularities 718
 - automorphisms 711–713
 - composite 711
 - F -homotopic transformations 711
 - homographic transformations 711
 - basic solutions
 - equivalent expressions 712
 - Fuchs–Frobenius 711
 - biconfluent 718
 - classification of parameters 710
 - computation of solutions 720
 - confluent forms 717–718
 - asymptotic approximations 718
 - integral equations 716
 - special cases 717
 - doubly-confluent 717
 - doubly-periodic forms
 - Jacobi's elliptic 710
 - Weierstrass's 710
 - eigenvalues of accessory parameter 712
 - expansions of solutions in series of
 - hypergeometric functions 716–717
 - orthogonal polynomials 717
 - exponent parameters 710
 - integral equations 714–716
 - integral representation of solutions 714–716
 - kernel functions 714, 716
 - separation constant 714
 - inversion problem 719
 - Jacobi's elliptic form 710
 - Liouvillean solutions 713
 - monodromy group 719
 - normal form 710
 - parameters
 - classification 710
 - path-multiplicative solutions 712
 - biorthogonality 714
 - expansions in series of hypergeometric functions
 - 717
 - relation to Fuchsian equation 718
 - relation to Lamé's equation 685
 - separation of variables 720
 - singularities 710
 - singularity parameter 710
 - solutions analytic at three singularities
 - *see* Heun polynomials.
 - solutions analytic at two singularities
 - *see* Heun functions.
 - solutions via quadratures 713
 - triconfluent 718
 - trigonometric form 710
 - uniformization problem 719
 - Weierstrass's form 710
- Heun's operator 714
- hexadecimal number system 72
- high-frequency scattering
 - parabolic cylinder functions 317
- higher-order $3nj$ symbols 765
- highway design
 - Cornu's spiral 169
- Hilbert space
 - interrelation between bases
 - Heun polynomial products 719–720
 - $L^2_p(Q)$ orthonormal basis 719
- Hilbert transform
 - computation 84
 - definition 29
 - Fourier transform of 29
 - inequalities 29
 - inversion 29
- Hill's equation **674**
- *see also* Whittaker–Hill equation.

- antiperiodic solutions 675
- basic solutions 674
- characteristic equation 675
- characteristic exponents 675
- definition 674
- discriminant 675
- eigenfunctions 676
- eigenvalues 675
- equation of Ince 676
 - Fourier-series solutions 676
 - polynomial solutions *see* Ince polynomials.
- expansions in series of eigenfunctions 676
- Floquet solutions 675
- Floquet's theorem 674
- periodic solutions 675
- pseudoperiodic solutions 674
- real case 675
- separation constants 677, 678
- symmetric case 675
- Hölder's inequalities for sums and integrals 12, 13
- holomorphic function *see* analytic function.
- homogeneous harmonic polynomials 379
- homographic transformation
 - *see* bilinear transformation.
- Horner's scheme for polynomials 22
 - extended 22
- Hurwitz criterion for stable polynomials 23
- Hurwitz system
 - Riemann surface 546
- Hurwitz zeta function **607**
 - analytic properties 607
 - asymptotic expansions for large parameter 610
 - computation 614
 - definition 607
 - derivatives 608
 - asymptotic expansions for large parameter . . . 610
 - graphics 607–608
 - integral representations 609
 - integrals 610
 - relations to other functions
 - Lerch's transcendent 612
 - periodic zeta function 612
 - polylogarithms 611
 - Riemann zeta function 607
 - representations by Euler–Maclaurin formula 608
 - series representations 608, 610
 - special values 608
 - sums 610
 - tables 614
- hydrodynamics
 - Jacobian elliptic functions 566
- hyperasymptotic expansions 68
- hyperbola
 - elliptic integrals 514
- hyperbolic cosecant function . . *see* hyperbolic functions.
- hyperbolic cosine function . . . *see* hyperbolic functions.
- hyperbolic cotangent function
 - *see* hyperbolic functions.
- hyperbolic functions **123**
 - addition formulas 125
 - analytic properties 123
 - approximations 132
 - computation 132
 - conformal maps 124
 - continued fractions 129
 - definitions 123
 - derivatives 125
 - differential equations 125
 - elementary properties 124
 - graphics
 - complex argument 124
 - real argument 123–124
 - identities 125
 - inequalities 125
 - infinite products 126
 - integrals
 - definite 130
 - indefinite 130
 - inverse *see* inverse hyperbolic functions.
 - Laurent series 125
 - limits 125
 - Maclaurin series 125
 - moduli 126
 - multiples of argument 126
 - notation 104
 - partial fractions 126
 - periodicity 123
 - poles 123
 - real and imaginary parts 126
 - relations to trigonometric functions 123
 - special values 125
 - squares and products 126
 - sums 130
 - tables 132
 - zeros 123
- hyperbolic secant function . . . *see* hyperbolic functions.
- hyperbolic sine function *see* hyperbolic functions.
- hyperbolic tangent function . . *see* hyperbolic functions.
- hyperbolic trigonometric functions
 - *see* hyperbolic functions.
- hyperbolic umbilic bifurcation set
 - formula 781
 - picture 782
- hyperbolic umbilic canonical integral **776**
 - asymptotic approximations 789–790
 - convergent series 787
 - differential equations 788
 - formulas for Stokes set 783

- integral identity 787
- pictures of modulus 779
- pictures of phase 781
- scaling laws 785
- zeros 787
- hyperbolic umbilic catastrophe **776**, 785
- hyperelliptic functions 566
- hyperelliptic integrals 498
- hypergeometric differential equation **394**
 - equivalent equation for contiguous functions 388
 - fundamental solutions 394–395
 - Kummer's solutions 395
 - singularities 395
- hypergeometric equation
 - *see* hypergeometric differential equation.
- hypergeometric function **384**
 - *see also* Gaussian hypergeometric function.
 - analytic properties 384
 - applications
 - mathematical 399
 - physical 400
 - asymptotic approximations
 - large a (or b) and c 397, 398
 - large a and b 397, 398
 - large a or b 398
 - large a , b , and c 398
 - large c 396–398
 - large variable 396
 - branch points 384
 - computation 400
 - contiguous 388
 - continued fractions 389
 - definition 384
 - derivatives 387–388
 - Fourier transforms 398
 - graphics 385–386
 - Hankel transforms 398
 - integral representations 388–389
 - Mellin–Barnes type 388–389
 - integrals 326, 327, 337, 398–399
 - compendia 398–399
 - Laplace transforms 398
 - Maclaurin series 384
 - Mellin transform 398
 - multivariate 498
 - notation 384
 - Olver's 384
 - polynomial cases 385
 - principal value (or branch) 384
 - products
 - series expansions 399
 - recurrence relations 388
 - relations to other functions
 - associated Legendre functions 353, 354, 394
 - classical orthogonal polynomials 442
 - elementary functions 386
 - Ferrers functions 353, 354, 394
 - gamma function 387
 - Gegenbauer function 394
 - Hahn class orthogonal polynomials 463
 - Heun functions 713
 - incomplete beta functions 183
 - Jacobi function 394
 - orthogonal polynomials 393–394
 - Painlevé transcendents 399
 - Pollaczek polynomials 476
 - psi function 387
 - symmetric elliptic integrals 509
 - Szegő–Askey polynomials 475
 - Wilson class orthogonal polynomials 469
 - singularities 384
 - special cases
 - argument ± 1 387
 - argument a fraction 387
 - arguments $e^{\pm i\pi/3}$ 387, 400
 - elementary functions 386–387
 - sums 399
 - compendia 399
 - transformation of variable
 - cubic 393
 - linear 390–391, 400
 - quadratic 391–393
 - with two variables *see* Appell functions.
 - Wronskians 395
 - zeros 398
- hypergeometric functions of matrix argument
 - *see* confluent hypergeometric functions of matrix argument, Gaussian hypergeometric functions of matrix argument, *and* generalized hypergeometric functions of matrix argument.
- hypergeometric R -function **498**
 - derivative 500
 - differential equation 501
 - elliptic cases 498
 - integral representations 498
- implicit function theorem 7
- Ince polynomials 676
 - normalization 676
 - zeros 676
- Ince's equation ... *see* Hill's equation, equation of Ince.
- Ince's theorem *see* Theorem of Ince.
- incomplete Airy functions 208
- incomplete beta functions **183**
 - applications
 - physical 189
 - statistical 189
 - asymptotic expansions for large parameters
 - general case 185

- inverse function 185
 - symmetric case 184
- basic properties 183
- continued fraction 184
- historical profile 183
- integral representation 183
- inverse function 185
- notation 174
- recurrence relations 183
- relation to hypergeometric function 183
- sums 184
- tables 190
- incomplete gamma functions **174**
 - analytic continuation 174
 - applications
 - mathematical 189
 - physical 189
 - statistical 189
 - approximations 191
 - asymptotic approximations and expansions
 - exponentially-improved 179, 181
 - for inverse function 182
 - large variable and/or large parameter
 - 179–180, 182
 - uniform for large parameter 181, 182
 - basic properties 174
 - Chebyshev-series expansions 191
 - computation 190
 - continued fraction 179
 - definitions
 - general values 174
 - principal values 174
 - derivatives 178
 - differential equations 174
 - expansions in series of
 - Bessel functions 178
 - Laguerre polynomials 178
 - modified spherical Bessel functions 178
 - generalizations 183
 - graphics
 - complex argument 176
 - real variables 175–176
 - inequalities 179
 - integral representations
 - along real line 177
 - compendia 178
 - contour integrals 177
 - Mellin–Barnes type 177
 - integrals 182
 - monotonicity properties 176
 - normalized 174
 - notation 174
 - of imaginary argument 177
 - Padé approximant 179
 - power-series expansions 178
 - principal values 174
 - recurrence relations 178
 - relations to other functions
 - confluent hypergeometric functions .. 177, 328, 338
 - error functions 164
 - exponential integrals 153
 - generalized exponential integral 185
 - incomplete Riemann zeta function 189
 - special values 176
 - sums 183
 - tables 190
 - zeros 182
- incomplete Riemann zeta function 189
 - asymptotic approximations 189
 - expansions in series of incomplete gamma functions
 - 189
 - zeros 189
- inductance
 - symmetric elliptic integrals 516
- inequalities
 - means 13
 - sums and integrals
 - Cauchy–Schwarz 12, 13
 - Hölder’s 12, 13
 - Jensen’s 13
 - Minkowski’s 12, 13
- infinite partial fractions 22
 - Mittag-Leffler’s expansion 22
- infinite products
 - convergence 21
 - absolute 21
 - uniform 21
 - M*-test for uniform convergence 21
 - relation to infinite partial fractions 22
 - Weierstrass product 22
- infinite sequences
 - convergence 17
 - absolute 17
 - pointwise 17
 - uniform 17
 - double 18
 - convergence 18
 - relation to infinite double series 18
- infinite series
 - *see also* power series.
 - convergence 17
 - absolute 17
 - pointwise 17
 - uniform 17
 - Weierstrass *M*-test 17
 - divergent 17
 - dominated convergence theorem 18
 - double 18

- doubly-infinite 17
- summability methods 33–34
- term-by-term integration 18
- inhomogeneous Airy functions.... *see* Scorer functions.
- initial-value problems
 - Mathieu functions..... 679
- integrable differential equations
 - Riemann theta functions 545–546
- integrable equations
 - *see* integrable differential equations.
- integral equations
 - Painlevé transcendents 729
- integral transforms **27**
 - *see also* Fourier cosine and sine transforms, Fourier transform, Jacobi transform, Hankel (or Bessel) transform, Hilbert transform, Kontorovich–Lebedev transform, Laplace transform, Mellin transform, spherical Bessel transform, and Stieltjes transform.
 - compendia 32
 - in terms of parabolic cylinder functions 317
 - in terms of Whittaker functions 344
- integrals
 - asymptotic approximations
 - *see* asymptotic approximations of integrals.
 - Cauchy principal values 6
 - change of variables 6
 - convergence 6
 - absolute 6
 - uniform 8, 21
 - convolution product 53
 - definite 5
 - differentiation 6, 8
 - double *see* double integrals.
 - fundamental theorem of calculus 6
 - generalized 52
 - indefinite 5
 - infinite 6, 9, 16
 - Jensen’s inequality 13
 - line 11
 - mean value theorems
 - first 6
 - second 6
 - multiple 8
 - over parametrized surface 12
 - path 11
 - repeated 6
 - square-integrable 6
 - summability methods 34–35
 - tables 5
 - with coalescing saddle points 789–790
- integrals of Bessel and Hankel functions
 - compendia 246
 - convolutions 242
 - fractional 243
 - Hankel (or Bessel) transform 246
 - indefinite 240–241
 - orthogonal properties 243, 244
 - over finite intervals 241–243
 - over infinite intervals 243–246, 326
 - products 241–246
 - triple 245
 - trigonometric arguments 241
- integrals of modified Bessel functions
 - compendia 261
 - computation 277
 - fractional 259
 - indefinite 258
 - Kontorovich–Lebedev transform 260
 - over finite intervals 258
 - over infinite intervals 205, 258–260, 326, 337
 - products 260
 - tables 279
- integration 5, 16
 - by parts 5
 - numerical *see* cubature, Gauss quadrature, Monte-Carlo methods, and quadrature.
 - term by term 18
- interaction potentials
 - hypergeometric function 400
- interior Dirichlet problem
 - for oblate spheroids 706
 - for prolate spheroids 705
- interior points 15
- interpolation 75–77, 91
 - *see also* Lagrange interpolation.
 - based on Chebyshev points 77
 - based on Sinc functions 77
 - bivariate 77
 - convergence properties 77
 - Hermite 77
 - inverse 76
 - inverse linear 91
 - linear 76
 - rational 77
 - spline 77
 - trigonometric 77
- interval
 - closure 6
- interval arithmetic 72
- inverse function 21
 - Lagrange inversion theorem 21
 - extended 21
- inverse Gudermannian function 121
 - relation to Legendre’s elliptic integrals 491
 - relation to R_C -function 491
- inverse hyperbolic functions **127**
 - addition formulas 129

- analytic properties 127
- approximations 132
- branch cuts 127
- branch points 127
- Chebyshev-series expansions 132
- computation 132
- conformal maps 124
- continued fractions 129
- definitions 127
- derivatives 129
- fundamental property 128
- general values 127
- graphics
 - complex argument 124
 - real argument 123–124
- integrals 129
- interrelations 128
- logarithmic forms 128
- notation 104
- power series 129
- principal values 127
- reflection formulas 128
- tables 132
- values on the cuts 128
- inverse incomplete beta function 185
- inverse incomplete gamma function 182
- inverse Jacobian elliptic functions **561**
 - applications 563
 - as Legendre's elliptic integrals 561
 - as symmetric elliptic integrals 561
 - computation 567
 - definitions 561
 - equivalent forms 561
 - normal forms 561
 - notation 561
 - power-series expansions 561
 - principal values 561
- inverse Laplace transforms
 - asymptotic expansions 46–47
- inverse trigonometric functions **118**
 - addition formulas 121
 - analytic properties 119
 - approximations 132
 - branch cuts 119
 - branch points 119
 - Chebyshev-series expansions 132
 - computation 132
 - conformal maps 113
 - continued fractions 121
 - definitions 118
 - derivatives 121
 - fundamental property 120
 - general values 118
 - graphics
 - complex argument 113–115
 - real argument 112
 - integrals 122
 - interrelations 120
 - logarithmic forms 119–120
 - notation 104
 - power series 121
 - principal values 119
 - real and imaginary parts 120
 - reflection formulas 119
 - special values 120
 - sums 123
 - tables 132
 - values on the cuts 119–120
- Ising model
 - Appell functions 417
 - combinatorics 635
 - generalized hypergeometric functions 417
 - Painlevé transcendents 739
- isolated essential singularity 19
 - *see also* essential singularity.
 - movable 724
- isolated singularity 19
- iterative methods
 - Bairstow's method (for zeros of polynomials) 91
 - bisection method 91
 - convergence
 - cubic 90
 - geometric 90
 - linear 90
 - local 90
 - logarithmic 94
 - of the p th order 90
 - quadratic 90
 - eigenvalue methods 91
 - fixed-point methods 92
 - Halley's rule 92
 - Newton's rule (or method) 90
 - regula falsi 91
 - secant method 91
 - Steffensen's method 91
- Jacobi fraction (J -fraction) 95
- Jacobi function
 - applications 399
 - definition 394
 - relations to other functions
 - associated Legendre functions 355
 - conical functions 379
 - hypergeometric function 394
- Jacobi polynomials **438**
 - *see also* classical orthogonal polynomials.
 - applications
 - Bieberbach conjecture 479
 - associated 474

- asymptotic approximations 451–452
- Bateman-type sums 461
- computation 479
- definition 439
- derivatives 446
- differential equations 445
- expansions in series of 459–461
- Fourier transform 456
- generating functions 449
- graphs 440
- inequalities 450, 451
 - Szegő–Szász 451
 - Turan-type 450
- integral representations 447, 448
- integrals 455–457, 459
 - fractional 456
 - indefinite 455
- interrelations with other orthogonal polynomials 444–445, 463, 464
- Laplace transform 457
- leading coefficients 439
- limiting form
 - as Bessel functions 449
 - as Bessel polynomials 476
- limits to monomials 444
- local maxima and minima 450–451
- Mellin transform 457
- monic 80
- normalization 439
- notation 436
- orthogonality properties 439
- parameter constraint 439, 443
- recurrence relations 446
- relations to other functions
 - hypergeometric function 393, 442
 - orthogonal polynomials on the triangle 478
- Rodrigues formula 442
- shifted 437
- special values 444
- symmetry 444
- tables of coefficients 440
- upper bounds 450
- weight function 439
- zeros 438, 454
 - asymptotic approximations 454
- Jacobi symbol
 - number theory 642
- Jacobi transform 379, 394
 - inversion 394
- Jacobi’s amplitude function *see* amplitude (am) function.
- Jacobi’s epsilon function **562**
 - applications 563
 - computation 567
- definition 562
- graphs 563
- integral representations 562
- quasi-addition formula 562
- quasi-periodicity 562
- relation to Legendre’s elliptic integrals 562
- relation to theta functions 562
- tables 567
- Jacobi’s identities
 - number theory 645
- Jacobi’s imaginary transformation 556
- Jacobi’s inversion problem for elliptic functions 532
- Jacobi’s nome
 - power-series expansion 490
- Jacobi’s theta functions *see* theta functions.
- Jacobi’s triple product 529
 - q -version 427
- Jacobi’s zeta function **562**
 - computation 567
 - definition 562
 - graphs 563
 - quasi-addition formula 562
 - tables 567
- Jacobi–Abel addition theorem
 - Jacobian elliptic functions 564
- Jacobi-type polynomials 477
- Jacobian 9
- Jacobian elliptic functions **550**
 - addition theorems 557
 - analytic properties 550, 563
 - applications
 - mathematical 563–564
 - physical 564–566
 - change of modulus 563
 - computation 566–567
 - congruent points 553
 - coperiodic 553
 - copolar 553
 - cyclic identities
 - notation 558
 - points 558
 - rank 558
 - simultaneously permuted 558
 - definitions 550
 - derivatives 560
 - differential equations
 - first-order 560
 - second-order 560
 - double argument 556
 - Eisenstein series 559
 - elementary identities 556
 - equianharmonic case 555
 - expansions in doubly-infinite partial fractions ... 559
 - Fourier series 559

- for squares 559
 - fundamental unit cell 554
 - Glaisher's notation 550, 554
 - graphical interpretation via Glaisher's notation . . 554
 - graphics
 - complex modulus 552–553
 - complex variable 552
 - real variable 550–552
 - half argument 556
 - hyperbolic series for squares 559
 - integrals
 - definite 561
 - indefinite 560
 - of squares 562
 - interrelations 508
 - inverse *see* inverse Jacobian elliptic functions.
 - Jacobi's imaginary transformation 556
 - Landen transformations
 - ascending 557, 563, 566
 - descending 556, 563, 566
 - generalized 557
 - theta functions 531
 - lattice 554
 - computation 566
 - lemniscatic case 555
 - limiting forms as $k \rightarrow 0$ or $k \rightarrow 1$ 555
 - Maclaurin series
 - in k, k' 559
 - in z 558
 - modulus **550**
 - change of 563
 - complex 552, 553, 563
 - limiting values 555
 - outside the interval $[0, 1]$ 563
 - purely imaginary 563
 - real 563
 - nome 550
 - notation 550
 - periods 550, 553–554
 - poles 553–554
 - poristic polygon constructions 557
 - principal 550
 - relations to other functions
 - symmetric elliptic integrals 508
 - theta functions 550
 - Weierstrass elliptic functions 575
 - rotation of argument 556
 - special values of the variable 554–555, 557
 - subsidiary 550
 - sums of squares 556
 - tables 567
 - translation of variable 554
 - trigonometric series expansions 559
 - zeros 553
- Jensen's inequality for integrals 13
 - Jonquière's function *see* polylogarithms.
 - Jordan curve theorem 16
 - Jordan's function
 - number theory 638
 - Jordan's inequality
 - sine function 116
 - Julia sets 92
 - Kadomtsev–Petviashvili equation
 - Riemann theta functions 545
 - Kapteyn's inequality
 - Bessel functions 227
 - Kelvin functions **267**
 - applications 276
 - approximations 281
 - asymptotic expansions for large argument 271
 - cross-products and sums of squares 271
 - exponentially-small contributions 271
 - asymptotic expansions for large order *see* uniform asymptotic expansions for large order.
 - computation 276–277
 - cross-products 269
 - definitions 267
 - derivatives 269
 - with respect to order 269
 - differential equations 268
 - expansions in series of Bessel functions 270
 - graphs 268
 - integral representations 269
 - integrals
 - compendia 274
 - definite 274
 - indefinite 274
 - Laplace transforms 274
 - modulus and phase functions
 - asymptotic expansions for large argument 272
 - definitions 272
 - properties 272
 - notation 217
 - orders $\pm \frac{1}{2}$ 268
 - power series 269–270
 - compendia 270
 - cross-products and sums of squares 270
 - recurrence relations 269
 - reflection formulas for arguments and orders 268
 - uniform asymptotic expansions for large order 273
 - double asymptotic property 273
 - exponentially-small contributions 273
 - zeros
 - asymptotic approximations for large zeros 273
 - computation 277
 - tables 281
 - Kelvin's ship-wave pattern 790–791
 - kernel equations

- Heun's equation 715, 716
- kernel functions
- Heun's equation 715, 716
- Klein's complete invariant *see* modular functions.
- Klein–Gordon equation
- Coulomb functions 754
- Kontorovich–Lebedev transform
- modified Bessel functions 260
 - computation 278
- Korteweg–de Vries equation
- Airy functions 209
 - Jacobian elliptic functions 565
 - Lamé polynomials 694
 - Painlevé transcendents 738
 - Riemann theta functions 545
 - Weierstrass elliptic functions 582
- Kovacic's algorithm 713, 718
- KP equation *see* Kadomtsev–Petviashvili equation.
- Krattenthaler's formula for determinants 4
- Krawtchouk polynomials
- *see also* Hahn class orthogonal polynomials.
 - applications
 - coding theory 479
 - relation to hypergeometric function 394
- Kummer congruences
- Bernoulli and Euler numbers 593
- Kummer functions **322**
- *see also* confluent hypergeometric functions.
 - addition theorems 333
 - analytic continuation 323
 - analytical properties 322
 - applications
 - physical 346 - approximations 347
 - asymptotic approximations for large parameters
 - large a 330–331
 - large b 330
 - uniform 330–331 - asymptotic expansions for large argument .. 328–329
 - error bounds 329
 - exponentially-improved 329
 - hyerasymptotic 329 - Chebyshev-series expansions 347
 - computation 346–347
 - connection formulas 325
 - continued fractions 327
 - definitions 322
 - derivatives 325–326
 - differential equation *see* Kummer's equation
 - integer parameters 322–323
 - integral representations
 - along the real line 326
 - contour integrals 326–327
 - Mellin–Barnes type 327 - integrals
 - along the real line 326
 - compendia 333
 - Fourier transforms 332
 - Hankel transforms 332–333
 - indefinite 332
 - Laplace transforms 332
 - Mellin transforms 332 - interrelations 322, 325
 - Kummer's transformations 325
 - limiting forms
 - as $z \rightarrow 0$ 323
 - as $z \rightarrow \infty$ 323 - Maclaurin series 322
 - multiplication theorems 334
 - notation 322
 - polynomial cases 322, 323
 - principal branches (or values) 322
 - products 333
 - recurrence relations 325
 - relations to other functions
 - Airy functions 328
 - elementary functions 327
 - error functions 328
 - generalized hypergeometric functions 328
 - incomplete gamma functions 328
 - modified Bessel functions 328
 - orthogonal polynomials 328
 - parabolic cylinder functions 328
 - Whittaker functions 334 - series expansions
 - addition theorems 333
 - in modified Bessel functions 333
 - Maclaurin 322
 - multiplication theorems 334 - tables 347
 - Wronskians 324
 - zeros
 - asymptotic approximations 331
 - distribution 331
 - inequalities 331
 - number of 331
- Kummer's equation 322
- equivalent form 325
 - fundamental solutions 323–324
 - numerically satisfactory solutions 323–324
 - relation to hypergeometric differential equation .. 322
 - relation to Whittaker's equation 334
 - standard solutions 322
- Kummer's transformations
- for ${}_3F_2$ hypergeometric functions of matrix argument
 - 772 - for confluent hypergeometric functions 325
- L'Hôpital's rule for derivatives 5

- Lagrange interpolation 75–76
 abscissas 75
 coefficients 75
 equally-spaced nodes 75–76
 error term 75
 formula 75
 Newton's interpolation formula 76
 nodal polynomials 75
 nodes 75
 polynomial 75
 remainder terms 75–76
 via divided differences 76
- Lagrange inversion theorem 21
 extended 21
- Lagrange's formula for reversion of series 43
- Laguerre functions
 associated 754
- Laguerre polynomials **438**
 *see also* classical orthogonal polynomials.
 addition theorem 460
 applications
 Schrödinger equation 479
 asymptotic approximations 452–453
 computation 479
 continued fraction 450
 derivatives 447
 differential equations 445
 Dirac delta 38
 expansions in series of 459, 460
 explicit representations 442–443
 Fourier transforms 457
 generalized 436
 generating functions 449
 graphics 441
 inequalities 450, 451
 Turan-type 450
 integral representations 447, 448
 integrals 455–457
 fractional 456
 indefinite 455
 interrelations with other orthogonal polynomials
 445, 463, 464
 Laplace transform 457
 leading coefficients 439
 limiting form as a Bessel function 449
 limits to monomials 444
 local maxima and minima 451
 Mellin transform 458
 monic 80
 multiplication theorem 460
 normalization 439
 notation 436
 orthogonality properties 439
 parameter constraint 439, 443
- Poisson kernels 461
 recurrence relations 446
 relation to confluent hypergeometric functions
 328, 338, 443, 448
 Rodrigues formula 442
 tables 480
 of coefficients 440
 of zeros 81
 tables of zeros 81
 upper bounds 450
 value at $z = 0$ 443
 weight function 439
 zeros 438, 454
 asymptotic behavior 454
 tables 81
- Lamé functions **684**
 algebraic 693
 applications
 conformal mapping 694
 physical 694
 rotation group 694
 sphero-conal coordinates 693
 asymptotic expansions 689
 computation 694
 definition 685
 differential equation *see* Lamé's equation.
 eigenvalues
 asymptotic expansions 689
 coalescence 685
 computation 694
 continued-fraction equation 685
 definition 685
 distribution 685
 graphics 686–687
 interlacing 685
 limiting forms 688
 notation 684
 parity 685
 periods 685
 power-series expansions 686
 special cases 688
 tables 694
 Fourier series 688–689
 graphics 687–688
 integral equations 689
 limiting forms 688
 normalization 686
 notation 684
 order 685
 orthogonality 686
 parity 686
 period 686
 relations to Heun functions 713
 relations to Lamé polynomials 689, 690

- special cases 688
- with imaginary periods 690
- with real periods 685
- zeros 685
- Lamé polynomials **690**
 - algebraic form 691
 - applications
 - ellipsoidal harmonics 694
 - physical 691, 694
 - spherical harmonics 694
 - asymptotic expansions 693
 - Chebyshev series 693
 - coefficients 694
 - computation 694
 - definition 690
 - eigenvalues
 - asymptotic expansions 693
 - computation 694
 - graphics 691
 - elliptic-function form 690
 - explicit formulas 693
 - Fourier series 692
 - graphics 692
 - notation 684, 690, 691
 - orthogonality 692
 - periodicity 690
 - relation to Lamé functions 689, 690
 - tables 694
 - zeros 690
 - computation 694
 - electrostatic interpretation 691
- Lamé wave equation 690
- Lamé's equation **684**
 - algebraic form 684
 - eigenfunctions 686
 - eigenvalues *see* Lamé functions, eigenvalues.
 - Jacobian elliptic-function form 684
 - other forms 684–685
 - relation to Heun's equation 685
 - second solution 693
 - singularities 684
 - stability 690
 - trigonometric form 684
 - Weierstrass elliptic-function form 685
- Lamé–Wangerin functions 693
- Lambert W -function **111**
 - applications 131
 - asymptotic expansions 111
 - computation 132
 - definition 111
 - graphs 111
 - integral representations 111
 - notation 111
 - principal branch 111
 - other branches 111
 - properties 111
- Lambert series
 - number theory 641
- Lanczos tridiagonalization of a symmetric matrix ... 75
- Lanczos vectors 75
- Landen transformations
 - Jacobian elliptic functions 556, 557, 563, 566
 - theta functions 531
- Laplace equation
 - $3j, 6j, 9j$ symbols 765
- Laplace transform
 - analyticity 28
 - asymptotic expansions for large parameters
 - 43, 44, 46
 - asymptotic expansions for small parameters 51
 - convergence 28
 - convolution 28
 - definition 28
 - derivatives 28
 - differentiation 28
 - for functions of matrix argument 768
 - analytic properties 768
 - convolution theorem 768
 - definition 768
 - inversion formula 768
 - integration 28
 - inversion 28
 - notation 28
 - numerical inversion 83–84, 99
 - of periodic functions 28
 - tables 32
 - translation 28
 - uniqueness 29
- Laplace's equation
 - Bessel functions 275
 - for elliptical cones 694
 - spherical coordinates 379
 - symmetric elliptic integrals 501
 - toroidal coordinates 379
- Laplace's method for asymptotic expansions of integrals
 - 44, 47
- Laplacian 7
 - cylindrical coordinates 7
 - ellipsoidal coordinates 583
 - numerical approximations 78
 - oblate spheroidal coordinates 706
 - parabolic cylinder coordinates 317
 - polar coordinates 7
 - prolate spheroidal coordinates 705
 - spherical coordinates 8
- lattice
 - for elliptic functions
 - *see* Weierstrass elliptic functions, lattice.

- lattice models of critical phenomena
 elliptic integrals..... 517
- lattice parameter
 theta functions..... 524
- lattice paths..... 618–623
 definition..... 618
 k -dimensional..... 618
- lattice walks
 Appell functions..... 417
 generalized hypergeometric functions..... 417
- Laurent series..... 19
 asymptotic approximations for coefficients..... 65
- Lauricella's function
 relation to symmetric elliptic integrals..... 509
- Lax pairs
 classical orthogonal polynomials..... 478
 Painlevé transcendents..... 728
- layered materials
 elliptic integrals..... 517
- least squares approximations..... 99–100
 conditioning..... 99
 normal equations..... 99
 orthogonal functions with respect to weighted summation..... 99
- Lebesgue constants..... 13, 97
 asymptotic behavior..... 13
- Legendre functions..... 352
 ... *see also* associated Legendre functions *and* Ferrers functions.
 complex degree..... 379
- Legendre functions on the cut... *see* Ferrers functions.
- Legendre polynomials..... **438**
 *see also* classical orthogonal polynomials.
 addition theorem..... 459
 applications
 Schrödinger equation..... 479
 associated..... 474
 asymptotic approximations..... 452
 computation..... 479
 continued fraction..... 450
 definition..... 439
 differential equation..... 445
 Dirac delta..... 38
 expansions in series of..... 459, 461
 explicit representations..... 442–443
 Fourier transforms..... 456
 generating functions..... 449
 graphs..... 441
 inequalities..... 450
 Turan-type..... 450
 integral representations..... 447, 448
 for products..... 455
 integrals..... 455, 458
 Nicholson-type..... 455
- interrelations with other orthogonal polynomials
 444
- large degree..... 65
- leading coefficients..... 439
- Mellin transforms..... 458
- monic..... 80
- normalization..... 439
- notation..... 436
- orthogonality properties..... 439
- recurrence relations..... 446
- relations to other functions
 associated Legendre functions..... 360
 Ferrers functions..... 360
 hypergeometric function..... 394
 $3j$ symbols..... 760–761
- Rodrigues formula..... 442
- shifted..... 436, 439
- special values..... 444
- symmetry..... 444
- tables..... 480
 of coefficients..... 440
 of zeros..... 80
- value at argument zero..... 285
- weight function..... 439
- zeros..... 438, 454
 tables..... 80
- Legendre symbol
 prime numbers..... 642
- Legendre's elliptic integrals..... **486**
 addition theorem..... 495
 applications
 mathematical..... 514–516
 physical..... 517
 approximations (except asymptotic)..... 519
 arithmetic-geometric mean..... 492
 asymptotic approximations..... 495
 change of amplitude..... 492
 change of modulus..... 492
 change of parameter..... 492
 circular cases..... 487, 492
 complete..... 487
 computation..... 517–518
 connection formulas..... 491
 derivatives..... 490
 differential equations..... 490
 duplication formulas..... 495
 first, second, and third kinds..... 486
 Gauss transformation..... 493
 graphics..... 488–489
 hyperbolic cases..... 487, 492
 imaginary-argument transformations..... 492
 imaginary-modulus transformations..... 492
 incomplete..... 486
 inequalities

- complete integrals 494
- incomplete integrals 494
- integration
 - with respect to amplitude 496
 - with respect to modulus 496
- Landen transformations
 - ascending 493
 - descending 493
- Laplace transforms 496
- limiting values 491
- notation 486
- power-series expansions 490
- quadratic transformations 492
- reciprocal-modulus transformation 492
- reduction of general elliptic integrals 496–497
- relations to other functions
 - am function 562
 - Appell functions 490
 - inverse Gudermannian function 491
 - inverse Jacobian elliptic functions 561
 - Jacobi's epsilon function 562
 - Jacobi's zeta function 562
 - Jacobian elliptic functions 494
 - symmetric elliptic integrals 507, 508
 - theta functions 494
 - Weierstrass elliptic functions 494
- special cases 491
- tables 518–519
- Legendre's equation 352
 - standard solutions 352
- Legendre's relation
 - Legendre's elliptic integrals 492
- Legendre's relation for the hypergeometric function
 - generalized 399
- Leibniz's formula for derivatives 5
- lemniscate arc length 563
- lemniscate constants 502, 503
- lengths of plane curves
 - Bernoulli's lemniscate 515
 - ellipse 514
 - hyperbola 514
- Lerch's transcendent
 - definition 612
 - properties 612
 - relation to Hurwitz zeta function 612
 - relation to polylogarithms 612
- level-index arithmetic 73
- Levi-Civita symbol for vectors 10
- Levin's transformations
 - application to asymptotic expansions 69
 - for sequences 94
- Lie algebras
 - q -series 432
- light absorption
 - Voigt functions 169
- limit points (or limiting points) 15
- limits of functions
 - of a complex variable 15
 - of one variable 4
 - of two complex variables 15
 - of two variables 7
- line broadening function 167
- linear algebra 73–75
 - *see also* Gaussian elimination.
 - condition numbers 74, 75
 - conditioning of linear systems 74
 - error bounds 74
 - a posteriori* 74
 - norms
 - Euclidean 74
 - of arbitrary order 74
 - of matrices 74
 - of vectors 74
- linear functional 35
- linear transformation 17
- Liouville transformation for differential equations
 - 26, 58
- Liouville's function
 - number theory 639
- Liouville's theorem for entire functions 16
- Liouville–Green (or WKBJ) approximation 57
 - for difference equations 62
- little q -Jacobi polynomials 471
- local maxima and minima 450
- locally analytic 724
- locally integrable 48
- logarithm function **104**
 - analytic properties 104
 - approximations 132
 - branch cut 104
 - Briggs 105
 - Chebyshev-series expansions 132
 - common 105
 - computation 131
 - conformal maps 106
 - continued fractions 109
 - definition 104
 - derivatives 108
 - differential equations 109
 - general base 105
 - general value 104
 - generalized 111
 - graphics
 - complex argument 107
 - real argument 106
 - hyperbolic 105
 - identities 109
 - inequalities 108

- integrals 110
- limits 107
- Napierian 105
- natural 105
- notation 104
- power series 108
- principal value 104
- real and imaginary parts 104
- special values 107
- sums 110
- tables 132
- values on the cut 104
- zeros 104
- logarithmic integral **150**
 - asymptotic expansion 153
 - definition 150
 - graph 155
 - notation 150
 - number-theoretic significance 155
 - relation to exponential integrals 150
- Lommel functions **294**
 - asymptotic expansions for large argument 295
 - computation 299
 - definitions 294–295
 - differential equation 294
 - integral representations 295
 - integrals 295
 - notation 288
 - power series 294
 - reflection formulas 295
 - relation to Anger–Weber functions 296
 - series expansions
 - Bessel functions 295
 - power series 294
- Lucas numbers 596
- M*-test for uniform convergence
 - infinite products 21
 - infinite series 17
- magic squares
 - number theory 648
- magnetic monopoles
 - Riemann theta functions 545
- Mangoldt's function
 - number theory 639
- many-body systems
 - confluent hypergeometric functions 346
- many-valued function *see* multivalued function.
- mathematical constants 100
- Mathieu functions **652, 664**
 - *see also* Mathieu's equation, modified Mathieu functions, *and* radial Mathieu functions.
 - analytic properties 653, 661, 665
 - antiperiodicity 654
 - applications
 - mathematical 677–678
 - physical 678–679
 - asymptotic expansions for large q
 - *see also* uniform asymptotic approximations for large parameters.
 - Goldstein's 662
 - Sips' 661
 - computation 679–680
 - connection formulas 665
 - definitions 664
 - differential equation 652
 - expansions in series of 664, 667
 - Fourier coefficients
 - asymptotic forms for small q 657, 666
 - asymptotic forms of higher coefficients 657
 - normalization 657, 666
 - recurrence relations 656, 666
 - reflection properties in q 657
 - tables 680
 - values at $q = 0$ 657
 - Fourier series 653, 656, 666
 - graphics 655–656, 665
 - integral equations
 - compendia 663
 - variable boundaries 663
 - with Bessel-function kernels 663
 - with elementary kernels 663, 672
 - integral representations 672
 - compendia 674
 - integrals
 - compendia 674
 - of products 674
 - of products with Bessel functions 673–674
 - irreducibility 661
 - limiting forms as order tends to integers 665
 - normalization 654, 664
 - notation 652
 - of integer order **654**
 - of noninteger order **664**
 - orthogonality 654, 664
 - parity 654
 - periodicity 654, 664
 - power series in q 660, 666
 - pseudoperiodicity 653, 664
 - reflection properties in ν 664
 - reflection properties in q 654, 664, 665
 - reflection properties in z 664
 - relations to other functions
 - basic solutions of Mathieu's equation 654
 - confluent Heun functions 717
 - modified Mathieu functions 667
 - tables 680
 - uniform asymptotic approximations for large parameters

- Barrett's 662
 Dunster's 662–663
 values at $q = 0$ 654
 Wronskians 658
 zeros 663
 tables 680
- Mathieu's equation **652**
 algebraic form 652
 basic solutions 653
 relation to eigenfunctions 654
 characteristic equation 653
 characteristic exponents 653
 computation 679
 eigenfunctions *see* Mathieu functions.
 eigenvalues (or characteristic values) 653
 analytic continuation 661
 analytic properties 661
 asymptotic expansions for large q 661, 666
 branch points 661
 characteristic curves 667
 computation 679
 continued-fraction equations 659, 666
 distribution 654, 664
 exceptional values 661
 graphics 654, 665
 normal values 661, 664
 notation 652, 653, 664
 power-series expansions in q 659–660, 666
 reflection properties in ν 664
 reflection properties in q 654, 664
 tables 680
- Floquet solutions 653
 computation 679
 Fourier-series expansions 653
 uniqueness 653
- Floquet's theorem 653
- parameters
 definition 652
 stability chart 667
 stable pairs 667
 stable regions 667
 unstable pairs 667
- second solutions
 antiperiodicity 657
 definitions 657
 expansions in Mathieu functions 658
 Fourier series 658
 graphics 658
 normalization 657
 notation 652
 periodicity 657
 reflection properties in q 658
 values at $q = 0$ 658
 singularities 652
- standard form 652
 Theorem of Ince 653, 657
 transformations 653
- matrix
 *see also* linear algebra.
 augmented 73
 characteristic polynomial 74
 condition number 74
 eigenvalues 74–75
 characteristic polynomial 74
 computation 75
 condition numbers 75
 conditioning 75
 multiplicity 74
- eigenvectors
 left 74
 normalized 74
 right 74
- equivalent 542
 factorization 73
 Jacobi 82
 nondefective 74
 norms 74
 Riemann 538
 symmetric
 tridiagonalization 75
 symplectic 541
 triangular decomposition 73
 tridiagonal 74
- maximum 7
 local 5, 8
- maximum-modulus principle
 analytic functions 20
 harmonic functions 20
 Schwarz's lemma 20
- McKean and Moll's theta functions 524
- McMahon's asymptotic expansions
 zeros of Bessel functions 236
 error bounds 236
- mean value property for harmonic functions 16
- mean value theorems
 differentiable functions 5
 integrals 6
- means *see* Abel
 means, arithmetic mean, Cesàro means, geometric
 mean, harmonic mean, *and* weighted means.
- measure 437
 theory 437
- Mehler functions *see* conical functions.
- Mehler–Dirichlet formula
 Ferrers functions 363
- Mehler–Fock transformation 373, 379
 generalized 373, 379
- Mehler–Sonine integrals

- Bessel and Hankel functions 224
- Meijer G -function **415**
- approximations 418
- asymptotic expansions 417
- differential equation 417
- identities 416
- integral representations 415
- integrals 416
- notation 415
- relation to generalized hypergeometric function
 415–416
- special cases 416
- sums 416
- Meixner polynomials
 *see* Hahn class orthogonal polynomials.
 relation to hypergeometric function 394
- Meixner–Pollaczek polynomials
 *see* Hahn class orthogonal polynomials.
 relation to hypergeometric function 394
- Mellin transform
- analytic properties 48
- analyticity 29
- convergence 29
- convolution 29
- convolution integrals 49
- definition 29, 48
- inversion 29, 48
- notation 29
- Parseval-type formulas 29
- tables 32
- Mellin–Barnes integrals 145
- meromorphic function 19
- Mersenne numbers
 number theory 648
- Mersenne prime
 number theory 644
- method of stationary phase
 asymptotic approximations of integrals 45
- metric coefficients
- for oblate spheroidal coordinates 705
- for prolate spheroidal coordinates 704
- Mill’s ratio for complementary error function 163
- inequalities 163
- Miller’s algorithm
 difference equations 85–87
- minimax polynomial approximations 96
- computation of coefficients 96
- minimax rational approximations 97
- computation of coefficients 98
- type 97
- weight function 97
- minimum 7
- local 5, 8
- Minkowski’s inequalities for sums and series 12, 13
- minor *see* determinants.
- Mittag-Leffler function 261
- Mittag-Leffler’s expansion
 infinite partial fractions 22
- Möbius transformation *see* bilinear transformation.
- Möbius function
 number theory 639
- Möbius inversion formulas
 number theory 641
- modified Bessel functions **248**
- addition theorems 260
- analytic continuation 253
- applications
 asymptotic solutions of differential equations .. 274
- wave equation 275
- approximations 281
- asymptotic expansions for large argument .. 255–256
- error bounds 255, 256
- exponentially-improved 256
- for derivatives with respect to order 255
- for products 255
- asymptotic expansions for large order 256–258
- asymptotic forms 256
- double asymptotic properties 257–258
- in inverse factorial series 257
- uniform 256–257
- branch conventions 249
- computation 276–277
- connection formulas 251
- continued fractions 253
- cross-products 251
- definitions 248
- derivatives
 asymptotic expansions for large argument 255
- explicit forms 252
- uniform asymptotic expansions for large order
 256–257
- derivatives with respect to order 254
- asymptotic expansion for large argument 255
- differential equations 248, 254
 *see also* modified Bessel’s equation.
- generating function 254
- graphics 249
- hyperasymptotic expansions 276
- incomplete 262
- inequalities 254
- integral representations
 along real line 252–253
- compendia 253
- contour integrals 253
- Mellin–Barnes type 253
- products 253
- integrals .. *see* integrals of modified Bessel functions.
- limiting forms 252

- monotonicity 254
- multiplication theorem 260
- notation 217
- of imaginary order
 - approximations 281
 - computation 278
 - definitions 261
 - graphs 250, 251
 - limiting forms 261
 - numerically satisfactory pairs 261
 - tables 280
 - uniform asymptotic expansions for large order
 - 261
 - zeros 261
- power series 252
- principal branches (or values) 249
- recurrence relations 251
- relations to other functions
 - Airy functions 197
 - confluent hypergeometric functions . . 255, 328, 338
 - elementary functions 254
 - generalized Airy functions 206
 - generalized hypergeometric functions 255
 - parabolic cylinder functions 255, 308
- sums
 - addition theorems 260
 - compendia 261
 - expansions in series of 261
 - multiplication theorem 260
- tables 279
- Wronskians 251
- zeros 258
 - computation 277
 - tables 280
- modified Bessel's equation 248
 - inhomogeneous forms 288, 295
 - numerically satisfactory solutions 249
 - singularities 248
 - standard solutions 249
- modified Korteweg–de Vries equation
 - Painlevé transcendents 738
- modified Mathieu functions **667**
 - *see also* radial Mathieu functions.
 - addition theorems 672
 - analytic continuation 668
 - applications
 - mathematical 677
 - physical 678–679
 - asymptotic approximations
 - *see also* uniform asymptotic approximations for large parameters.
 - for large $\Re z$ 667, 672
 - for large q 672
 - computation 680
 - connection formulas 667, 669
 - definitions 667
 - differential equation 667
 - expansions in series of
 - Bessel functions 670
 - cross-products of Bessel functions and modified Bessel functions 671
 - graphics 669
 - integral representations 672–674
 - compendia 674
 - of cross-products 674
 - integrals 672
 - compendia 674
 - joining factors 652, 669
 - tables 680
 - notation 652
 - relation to Mathieu functions 667
 - shift of variable 668
 - tables 680
 - uniform asymptotic approximations for large parameters 662, 672
 - Wronskians 668
 - zeros
 - tables 680
- modified Mathieu's equation **667**
 - algebraic form 667
- modified spherical Bessel functions
 - *see* spherical Bessel functions.
- modified Struve functions *see* Struve functions and modified Struve functions.
- modified Struve's equation *see* Struve functions and modified Struve functions, differential equations.
- modular equations
 - modular functions 582
- modular functions **579**
 - analytic properties 579
 - applications
 - mathematical 581–582
 - physical 582–583
 - computation 583
 - cuspidal form 579
 - definitions 579
 - elementary properties 580
 - general 579
 - graphics 579
 - infinite products 580
 - interrelations 581
 - Laurent series 580
 - level 579
 - modular form 579
 - modular transformations 580
 - notation 570, 579
 - power series 580
 - relations to theta functions 525, 532, 579

- special values 580
- modular theorems
 - generalized elliptic integrals 516
- molecular spectra
 - Coulomb functions 753
- molecular spectroscopy
 - $3j, 6j, 9j$ symbols 765
- mollified error 73
- moment functionals 476
- monic polynomial 22, 80
- monodromy groups
 - Heun functions 719
 - hypergeometric function 400
- monosplines
 - Bernoulli 597
 - cardinal 597
- monotonicity 4
- Monte Carlo sampling 189
- Monte-Carlo methods
 - for multidimensional integrals 84
- Mordell's theorem 582
 - elliptic curves 581
- Motzkin numbers
 - definition 621
 - generating function 623
 - identities 623
 - recurrence relation 623
 - relation to lattice paths 621
 - table 622
- multidimensional theta functions
 - *see* Riemann theta functions *and* Riemann theta functions with characteristics.
- multinomial coefficients
 - definitions 620
 - generating function 620
 - recurrence relation 620
 - relation to lattice paths 620
 - table 620
- multiple orthogonal polynomials 477
- multiplicative functions 640
- multiplicative number theory 638–644
 - completely multiplicative functions 640
 - Dirichlet series 640
 - Euler product 640
 - fundamental theorem of arithmetic 638
 - multiplicative functions 640
 - notation 638
 - primitive roots 638
- multivalued function 20
 - branch 20, 104
 - branch cut 104
 - principal value 104
 - closed definition 104
- multivariate beta function
 - definition 768
 - notation 768
 - properties 769
- multivariate gamma function
 - definition 768
 - notation 768
 - properties 769
- multivariate hypergeometric function 498
- mutual inductance of coaxial circles
 - elliptic integrals 516
- n -dimensional sphere
 - gamma function 145
- Nörlund polynomials 596
- nanotubes
 - Struve functions 298
- Narayana numbers
 - definition 622
 - generating function 623
 - identity 623
 - relation to lattice paths 622
 - table 622
- negative definite
 - Taylor series 8
- neighborhood 7, 15
 - cut 20
 - of infinity 16
 - punctured 19
- Neumann's addition theorem
 - Bessel functions 246
 - modified Bessel functions 260
- Neumann's expansion
 - Bessel functions 247
- Neumann's integral
 - Bessel functions 224
 - Legendre functions 364
- Neumann's polynomial
 - Bessel functions 247
- Neumann-type expansions
 - modified Bessel functions 261
- Neville's theta functions 524
 - relations to Jacobian elliptic functions 550
- Newton's interpolation formula 76
- Newton's rule (or method) 90
 - convergence 90
- Nicholson's integral
 - Bessel functions 225
- Nicholson-type integral
 - parabolic cylinder functions 313
- $9j$ symbols **763**
 - addition theorem 764
 - applications 765
 - approximations for large parameters 764
 - computation 765
 - definition 763

- generating functions 764
- graphical method 765
- notation 758
- orthogonality 764
- recursion relations 764
- representation as
 - finite sum of $6j$ symbols 763
 - finite sum of $3j$ symbols 763
 - generalized hypergeometric functions 764
- special case 764
- sum rule 764
- summation convention 760
- sums 764
- symmetry 764
- tables 765
- zeros 765
- nodal polynomials 75
- nodes 79, 80
- nome
 - Jacobi's 490
 - Jacobian elliptic functions 550
 - theta functions 524
 - Weierstrass elliptic functions 570
- nonlinear equations
 - fixed points 90
 - numerical solutions
 - iterative methods 90–92
 - systems 92
- nonlinear evolution equations
 - Weierstrass elliptic functions 582
- nonlinear harmonic oscillator
 - Painlevé equations 725
- nonlinear ordinary differential equations
 - Jacobian elliptic functions 565
- nonlinear partial differential equations
 - Jacobian elliptic functions 565
- normal probability functions 160
- Novikov's conjecture
 - Riemann theta functions 546
- nuclear physics
 - Coulomb functions 754
- nuclear structure
 - $3j, 6j, 9j$ symbols 765
- number theory *see also* additive number theory, multiplicative number theory, *and* prime numbers.
 - Bernoulli and Euler numbers and polynomials 598
 - generalized hypergeometric functions 417
 - Jacobian elliptic functions 564
 - modular functions 582
 - theta functions 533
 - Weierstrass elliptic functions 582
- number-theoretic functions 638–643
 - completely multiplicative 640
 - computation 649
- Dirichlet character 642
 - induced modulus 642
 - Legendre symbol 642
 - primitive 642
 - principal 642
- Dirichlet divisor problem 643
- divisor function 638
- divisor sums 641
- inversion formulas 641
- Lambert series 641
- Möbius inversion 641, 647
 - pairs 641
- multiplicative 640
- orthogonality 642
- periodic 642
- Ramanujan's sum 642
- tables 649
- numerical differentiation
 - *see* differentiation, numerical.
- oblate spheroidal coordinates 705
 - Laplacian 706
 - metric coefficients 705
- Olver's algorithm
 - difference equations 86–87
- Olver's associated Legendre function 354, 375
- Olver's confluent hypergeometric function 322
- Olver's hypergeometric function 353, 384
- OP's *see* orthogonal polynomials.
- open disks around infinity 16
- open point set 11, 15
- optical diffraction
 - Struve functions 298
- optics
 - canonical integrals 791
- Orr–Sommerfeld equation
 - Airy functions 209
- orthogonal matrix polynomials 477
- orthogonal polynomials
 - complex 83
 - Painlevé transcendents 739
 - relations to confluent hypergeometric functions
 - 328, 338
 - relations to hypergeometric function 393–394
- orthogonal polynomials associated with root systems
 - 478
- orthogonal polynomials on the triangle 478
- orthogonal polynomials on the unit circle
 - *see* polynomials orthogonal on the unit circle.
- orthogonal polynomials with Freud weights 475
- oscillations of chains
 - Bessel functions 275
- oscillations of plates
 - Bessel functions 276
- \wp -function *see* Weierstrass elliptic functions.

- packing analysis
incomplete beta functions 189
- Padé approximations 98–99
computation of coefficients 98
convergence 98
Padé table 98
- Painlevé equations **724**
..... *see also* Painlevé transcendents.
affine Weyl groups 732
alternative forms 724
Bäcklund transformations 730–732
coalescence cascade 725
compatibility conditions 728–729
elementary solutions 732–735
elliptic form 725
graphs of solutions 726–728
Hamiltonian structure 729–730
interrelations 730–732
isomonodromy problems 728
compatibility condition 728
Lax pair 728
rational solutions 732–734
renormalizations 724
special function solutions 735–736
Airy functions 735
Bessel functions 735
Hermite polynomials 735
hypergeometric function 399, 736
parabolic cylinder functions 735
Whittaker functions 736
symmetric forms 725
- Painlevé property 724
applications 739
- Painlevé transcendents **724**
..... *see also* Painlevé equations.
applications 738
Boussinesq equation 739
combinatorics 739
enumerative topology 739
integrable continuous dynamical systems 739
integral equations 729
Ising model 739
Korteweg–de Vries equation 738
modified Korteweg–de Vries equation 738
orthogonal polynomials 739
partial differential equations 738–739
quantum gravity 739
sine-Gordon equation 739
statistical physics 739
string theory 739
asymptotic approximations 736–738
complex variables 738
real variables 736–738
Bäcklund transformations 730–732
computation 740
differential equations for 724
graphs 726–728
Hamiltonians 729–730
Lax pair 728
notation 724, 730–732
- parabolic cylinder functions **304, 314**
addition theorems 313
applications
mathematical 317
physical 317
approximations 318
asymptotic expansions for large parameter .. *see* uni-
form asymptotic expansions for large parameter.
asymptotic expansions for large variable 309, 315
exponentially-improved 309, 317
computation 318
connection formulas 304, 315
continued fraction 308
definitions 304, 305, 314
derivatives 309
differential equations 304
numerically satisfactory solutions 304, 314
standard solutions 304
envelope functions 367
expansions in Chebyshev series 318
generalized 317
graphics
complex variables 306
real variables 305–306, 314
Hermite polynomial case 304, 308
integral representations
along the real line 307, 315
compendia 308
contour integrals 307
Mellin–Barnes type 308
integral transforms 317
integrals 313
asymptotic methods 317
compendia 313
Nicholson-type 313
modulus and phase functions 305, 316
notation 304
orthogonality 317
power-series expansions 307, 315
recurrence relations 309
reflection formulas 304
relations to other functions
Bessel functions 228, 315
confluent hypergeometric functions
..... 308, 315, 328, 338
error and related functions 308
Hermite polynomials 308
modified Bessel functions 255, 308

- probability functions 308
- repeated integrals of the complementary error function 167
- sums 313
- tables 318
- uniform asymptotic expansions for large parameter 309–312, 315–316
 - double asymptotic property 311
 - in terms of Airy functions 311–312, 316
 - in terms of elementary functions 310–311, 316
 - modified expansions in terms of Airy functions 312
 - modified expansions in terms of elementary functions 311
- values at $z = 0$ 304, 314
- Wronskians 304, 314
- zeros
 - asymptotic expansions for large parameter 313
 - asymptotic expansions for large variable 312, 317
 - distribution 312
- paraboloidal coordinates
 - wave equation 346
 - Whittaker–Hill equation 678
- paraboloidal wave functions 677
 - asymptotic behavior for large variable 677
 - orthogonality properties 677
 - reflection properties 677
- parallelepiped
 - volume 10
- parallelogram
 - area 10
- parametrization of algebraic equations
 - Jacobian elliptic functions 563
- parametrized surfaces
 - area 11
 - integral over 12
 - of revolution 12
 - orientation 12
 - smooth 11
 - sphere 11
 - tangent vector 11
- paraxial wave equation 788
- Parseval’s formula
 - Fourier cosine and sine transforms 28
 - Fourier series 14
 - Fourier transform 27
- Parseval-type formulas
 - Mellin transform 29, 49
- partial derivative 7
- partial differential equations
 - nonlinear
 - Weierstrass elliptic functions 582
 - Painlevé transcendents 738
 - spectral methods 479
- partial differentiation 7
- partial fractions 2
 - *see also* infinite partial fractions.
- particle scattering
 - Coulomb functions 753
- partition *see* partition function.
- partition function
 - asymptotic expansion 646
 - calculation 646
 - divisibility 646
 - generating function 646
 - hadronic matter 146
 - parts 645
 - Ramanujan congruences 646
 - unrestricted 645
- partitional shifted factorial 769
- partitions 618–620, 624–631, 769
 - applications 635
 - compositions 628
 - conjugate 626
 - definition 618
 - of a set 618–620, 624–626
 - of integers 618, 626–628
 - parts 618
 - plane *see* plane partitions.
 - restricted *see* restricted integer partitions.
 - tables 619, 629, 635
 - weight of 769
- path
 - integrals of vector-valued functions 11
 - length 11
- PCFs *see* parabolic cylinder functions.
- Pearcey integral 777
 - asymptotic approximations 789–790
 - convergent series 787
 - definition 777
 - differential equation 788
 - formula for Stokes set 783
 - integral identities 787
 - picture of Stokes set 784
 - pictures of modulus 778
 - pictures of phase 780
 - scaling laws 785
 - zeros 785
 - table 786
- pendulum
 - amplitude (am) function 564
 - Jacobian elliptic functions 564
 - Mathieu functions 679
- pentagonal numbers
 - number theory 646
- periodic Bernoulli functions 588
- periodic Euler functions 588
- periodic zeta function

- relation to Hurwitz zeta function 612
- relation to polylogarithms 612
- permutations 618, 631–634
 - adjacent transposition 631
 - cycle notation 631
 - definition 618
 - derangement 631
 - derangement number 631
 - descent 632
 - even or odd 631
 - excedance 632
 - weak 632
 - fixed points 631
 - generating function 632
 - greater index 632
 - identities 632
 - inversion numbers 631–634
 - major index 632, 634
 - matrix notation 633
 - multiset 634
 - order notation 632
 - restricted position 633
 - sign 631, 633
 - special values 633
 - transpositions 631
 - twelfold way 634
- Pfaff–Saalschutz formula
 - ${}_3F_2$ functions of matrix argument 772
- phase principle 20, 92
- photon scattering
 - hypergeometric function 400
- pi
 - computation to high precision via elliptic integrals 516
- Picard’s theorem 19
- Picard–Fuchs equations
 - generalized hypergeometric functions 417
- piecewise continuous 4
- piecewise differentiable curve 11
- pion-nucleon scattering
 - Coulomb functions 754
- pionic atoms
 - Coulomb functions 754
- plane algebraic curves *see* algebraic curves.
- plane curves
 - elliptic integrals 514–515
 - Jacobian elliptic functions 563
- plane partitions
 - applications 635
 - complementary 630
 - definitions 629
 - descending 630
 - generating functions 630
 - limiting form 631
 - recurrence relation 631
 - strict shifted 630
 - symmetric 629
 - table 629
- plane polar coordinates *see* polar coordinates.
- plasma dispersion function 169
- plasma waves
 - error functions 169
- plasmas
 - hypergeometric function 400
- Pochhammer double-loop contour 326, 389, 714
- Pochhammer’s integral
 - beta function 142
 - Heun functions 714
- Pochhammer’s symbol 136
- point sets in complex plane
 - closed 15
 - closure 15
 - compact 18
 - connected 15
 - domain 15
 - exterior 16
 - interior 16
 - open 15
 - region 15
- points in complex plane
 - accumulation 15
 - at infinity 16
 - boundary 15
 - interior 15
 - limit (or limiting) 15
- Poisson identity
 - discrete analog 532
 - Gauss sum 532
- Poisson integral 16, 34
 - conjugate 34
 - harmonic functions 16
- Poisson kernel 33
 - Fourier integral 34
 - Fourier series 33
- Poisson’s equation
 - in channel-like geometries 379
- Poisson’s integral
 - Bessel functions 224
- Poisson’s summation formula
 - Fourier series 14
- polar coordinates 7
- polar representation
 - complex numbers 14
- pole 19
 - movable 724
 - multiplicity 19
 - order 19
- Pollaczek polynomials 476

- definition 476
 expansions in series of 477
 orthogonality properties 477
 relation to hypergeometric function 476
 relations to other orthogonal polynomials 477
polygamma functions **144**
 computation 146
 continued fractions 144
 definition 144
 properties 144
 special values 144
 sums 144
 tables 146
polylogarithms **611**
 analytic properties 611
 computation 614
 definitions 611
 integral representations 611
 relations to other functions
 Fermi–Dirac integrals 612
 Lerch’s transcendent 612
 periodic zeta function 612
 Riemann zeta function 611
 series expansions 611
 tables 614
polynomials
 characteristic 74
 deflation 91
 discriminant 22
 monic 22, 80
 nodal 75
 stable *see* stable polynomials.
 Wilkinson’s 92
 zeros *see* zeros of polynomials.
 zonal *see* zonal polynomials.
polynomials orthogonal on the unit circle 475–476
 biorthogonal 476
 connection with orthogonal polynomials on the line
 475
 definition 475
 recurrence relations 475
 special cases 475
population biology
 incomplete gamma functions 189
poristic polygon constructions of Poncelet
 Jacobian elliptic functions 557
positive definite
 Taylor series 8
potential theory
 conical functions 379
 symmetric elliptic integrals 501, 516
power function **105**
 analytic properties 105
 branch cut 104
 definition 105
 derivatives 109
 general bases 105
 general value 105
 identities 109
 limits 107
 modulus 105
 notation 105
 phase 105
 principal value 105
 special values 107
power series
 addition 17
 convergence 17
 circle of 17
 radius of 17
 differentiation 18
 multiplication 17
 of logarithms 18
 of powers 18
 of reciprocals 18
 subtraction 17
primality testing
 Weierstrass elliptic functions 582
prime number theorem 638, 643, 644
 equivalent statement 613
prime numbers
 applications 647
 asymptotic formula 644
 computation 648–649
 counting 648
 cryptography 647
 distribution 613, 638
 asymptotic estimate 638
 Euler–Fermat theorem 638
 in arithmetic progressions
 Dirichlet’s theorem 613, 643
 Jacobi symbol 642
 largest known 644
 Legendre symbol 642
 Mersenne prime 644, 648
 prime number theorem 638, 643, 644
 quadratic reciprocity law 642
 relation to logarithmic integral 155
 tables 639, 649
primes *see* prime numbers.
primitive Dirichlet characters
 relation to generalized Bernoulli polynomials 597
principal branches *see* principal values.
principal values 104
 *see also* Cauchy principal values.
 closed definition 104
principle of the argument *see* phase principle.
Pringsheim’s theorem for continued fractions 25

- probability distribution
 symmetric elliptic integrals 515
- probability functions 160, 167, 308
 Gaussian 160
 normal 160
 relations to other functions
 error functions 160
 parabolic cylinder functions 308
 repeated integrals of the complementary error function 167, 308
- problème des ménages 633
- projective coordinates 581
- projective quantum numbers
 $3j$ symbols 758
- prolate spheroidal coordinates 704
 Laplacian 705
 metric coefficients 704
- Prym's functions 174
- pseudoperiodic solutions
 of Hill's equation 674
 of Mathieu's equation 653, 664
- pseudoprime test 644
- pseudorandom numbers 648
- psi function **136**
 analytic properties 136
 applications
 mathematical 145
 approximations
 Chebyshev series 147
 complex variable 147
 rational 146, 147
 asymptotic expansion 140
 computation 146
 continued fractions 140
 definition 136
 expansions in partial fractions 139
 graphics 136, 137
 inequalities 138
 integral representations 140
 multiplication formula 138
 notation 136
 recurrence relation 138
 reflection formula 138
 relation to hypergeometric function 387
 special values 137
 tables 146
 Taylor series 139
 zeros 136
 asymptotic approximation 138
 table of 138
- public key codes 647
- punctured neighborhood 19
- q -beta function 145
- q -factorials 145
- q -gamma function 145
- q -Appell functions 423
 transformations 430
- q -Bernoulli polynomials 422
- q -binomial coefficient 421, 627
- q -binomial series 423
- q -binomial theorem 421, 424
- q -calculus 420–422
- q -cosine function 422
- q -derivatives 421
- q -differential equations 425
- q -Dyson conjecture 431
- q -elementary functions 422, 432
- q -Euler numbers 422
- q -exponential function 422
 applications 432
- q -hypergeometric function **420**
 Andrews–Askey sum 424, 426
 Andrews' q -Dyson conjecture 431
 applications
 mathematical 432
 physical 432
 Bailey chain 430
 Bailey lemma
 strong 430
 weak 430
 Bailey pairs 430
 Bailey transform 430
 Bailey's ${}_2F_1(-1)$ sum
 q -analog 426
 Bailey's ${}_4F_3(1)$ sum
 q -analogs (first and second) 427
 Bailey's transformation of very-well-poised ${}_8\phi_7$ 429
 Bailey–Daum q -Kummer sum 424
 balanced series 423
 bibasic sums and series 429
 bilateral *see* bilateral q -hypergeometric function.
 Cauchy's sum 424
 Chu–Vandermonde sums (first and second)
 q -analogs 424
 computation 432
 constant term identities 431
 contiguous relations (Heine's) 425
 continued fractions 426
 definition 423
 differential equations 425
 Dixon's ${}_3F_2(1)$ sum
 q -analog 426
 Dixon's sum
 F. H. Jackson's q -analog 426
 Dougall's ${}_7F_6(1)$ sum
 F. H. Jackson's q -analog 427
 Euler's sums (first, second, third) 423, 424
 F. H. Jackson's transformations 428

- Fine's transformations (first, second, third) 424
- Gauss's ${}_2F_1(-1)$ sum
 q -analog 426
- generalizations 432
- Heine's transformations (first, second, third) 424
- idem function 420, 429
- integral representations 426
- integrals 431
- k -balanced series 423
- mixed base Heine-type transformations 429
- nearly-poised 423
- notation 420
- q -Pfaff–Saalschütz sum 426
- q -Saalschütz sum
nonterminating form 426
- q -Sheppard identity 428
- quintuple product identity 427
- Ramanujan's integrals 431
- relations to other functions
Askey–Wilson class orthogonal polynomials
. 472–474
- q -Hahn class orthogonal polynomials 470–472
- Rogers–Fine identity 424
- Saalschützian series 423
- Sears' balanced ${}_4\phi_3$ transformation 428
- special cases 426
- three-term ${}_2\phi_1$ transformation 425
- transformations 428
- Vandermonde sum
nonterminating q -version 425
- very-well-poised 423
- well-poised 423
- Zeilberger–Bressoud theorem 431
- q -Hahn class orthogonal polynomials 470–472
as eigenvalues of q -difference operator 470
- asymptotic approximations 474
- orthogonality properties 470
- relation to q -hypergeometric function 470–472
- q -Hahn polynomials 470
- q -hypergeometric orthogonal polynomials 470
- q -integrals 422
- q -Laguerre polynomials 471
applications 432
- asymptotic approximations to zeros 474
- q -Leibniz rule 421
- q -multinomial coefficient 634
- q -Pochhammer symbol 436
- q -product 436
- q -Racah polynomials 474
applications
coding theory 479
- relation to q -hypergeometric function 474
- q -series
classification 423
- q -sine function 422
- q -Stirling numbers 422
- q^{-1} -Al-Salam–Chihara polynomials 473
- quadratic characters
number theory 642
- quadratic equations 23
- quadratic reciprocity law
number theory 642
- quadrature 78–84
contour integrals 83–84
- interpolatory rules (or formulas)
. *see also* Gauss quadrature.
Clenshaw–Curtis 79
- closed 79
- error term 79
- Fejér's 79
- midpoint 79
- Newton–Cotes 79
- nodes 79
- open 79
- weight function 79
- oscillatory integrals
Clenshaw–Curtis formula (extended) 82
- Filon's rule 82
- Longman's method 82
- multidimensional 82
- Romberg integration 79
- Simpson's rule
composite 78, 79
- elementary 79
- steepest-descent paths 83–84
- trapezoidal rule
composite 78, 79, 84
- elementary 78, 79
- improved 78
- via classical orthogonal polynomials 478
- quantum chemistry
generalized exponential integral 190
- incomplete gamma functions 189
- quantum chromo-dynamics
hypergeometric function 400
- quantum field theory
modular functions 582
- Riemann zeta function 614
- quantum gravity
Painlevé transcendents 739
- quantum groups
 q -series 432
- quantum mechanics
associated Legendre functions 379
- canonical integrals 791
- classical orthogonal polynomials 479
- Heun functions 720
- Mathieu functions 679

- nonrelativistic
 - gamma function 145
 - parabolic cylinder functions 317
 - Struve functions 298
 - Whittaker functions 346
- quantum probability distributions
 - Euler polynomials 598
- quantum scattering
 - hypergeometric function 400
- quantum spin models
 - Painlevé transcendents 739
- quantum spins
 - Heun's equation 720
- quantum systems
 - Heun's equation 720
- quantum wave-packets
 - theta functions 534
- quark-gluon plasma
 - Bernoulli polynomials 598
- quartic equations 23
- quartic oscillator
 - Jacobian elliptic functions 565
- quasiconformal mapping
 - complete elliptic integrals 399
 - hypergeometric function 399
- queueing theory
 - incomplete gamma functions 189
- quintic equations
 - modular functions 582
- quotient-difference algorithm 95
- rhombus rule 95
- stability 95
- quotient-difference scheme 95
- Raabe's theorem
 - Bernoulli polynomials 590
- Racah polynomials 407
 - *see* Wilson class orthogonal polynomials.
- radial Mathieu functions **668**
 - *see also* modified Mathieu functions.
 - definitions 668
 - expansions in series of Bessel functions 670
 - expansions in series of cross-products of Bessel functions and modified Bessel functions 671–672
 - graphics 669
 - integral representations 672–674
 - compendia 674
 - of cross-products 674
 - joining factors 652, 669
 - notation 652
 - relation to modified Mathieu functions 668
 - shift of variable 668
- radial spheroidal wave functions **703**
 - applications 706
 - asymptotic behavior for large variable 703
 - computation 708
 - connection formulas 703
 - connection with spheroidal wave functions 704
 - definitions 703
 - graphics 703
 - integral representation 704
 - tables 708
 - Wronskian 703
- radiative equilibrium
 - generalized exponential integral 190
- Radon transform
 - classical orthogonal polynomials 479
- railroad track design
 - Cornu's spiral 169
- rainbow
 - Airy functions 209
- Ramanujan's ${}_1\psi_1$ summation
 - bilateral q -hypergeometric function 427
- Ramanujan's beta integral 143
- Ramanujan's cubic transformation
 - hypergeometric function 393
- Ramanujan's partition identity
 - number theory 646
- Ramanujan's sum
 - number theory 643
- Ramanujan's tau function
 - number theory 646–647
- random graphs
 - generalized hypergeometric functions 417
- random matrix theory
 - Hermite polynomials 479
 - Painlevé transcendents 739
- random walks 417
- rational arithmetics 72
 - exact 72
- rational functions
 - summation 145
- Rayleigh function 240
 - applications 276
- R_C -function **487**
 - asymptotic approximations 496
 - limiting values 491
 - relation to elementary functions 495
 - relation to Gudermannian function 495
 - relation to inverse Gudermannian function 491
 - special values 491
- reduced Planck's constant 379, 479, 753
- reduced residue system
 - number theory 638
- reductions of partial differential equations
 - Painlevé transcendents 738
- Regge poles
 - Coulomb functions 754
- Regge symmetries

INDEX

939

- 6j symbols 762
- 3j symbols 759
- region 15
- regularization
 - distributional methods 55
- relative error 73
- relative precision 73
- relativistic Coulomb equations 754
- relaxation times for proteins
 - incomplete gamma functions 189
- Remez's second algorithm
 - minimax rational approximations 98
- removable singularity 19
- repeated integrals of the complementary error function
 - **167**
 - applications 169
 - asymptotic expansions 167
 - computation 169
 - continued fractions 167
 - definition 167
 - derivatives 167
 - differential equation 167
 - graphics 167
 - power-series expansion 167
 - recurrence relations 167
 - relations to other functions
 - confluent hypergeometric functions 167
 - Hermite polynomials 167
 - parabolic cylinder functions 167, 308
 - probability functions 167, 308
 - scaled 167
 - tables 169
- representation theory
 - partitions 635
- repulsive potentials
 - Coulomb functions 753, 754
- residue 19
 - theorem 19
- resistive MHD instability theory
 - Struve functions 298
- resolvent cubic equation 23
- resonances
 - Coulomb functions 754
- restricted integer partitions
 - Bessel-function expansion 628
 - conjugate 626
 - generating functions 627
 - identities 628
 - limiting form 627, 628
 - notation 626, 627
 - recurrence relations 627, 628
 - relation to lattice paths 626
 - tables 626, 627
- resurgence
 - asymptotic solutions of differential equations 57
 - reversion of series 43
- Riccati–Bessel functions 240
 - zeros 240
- Riemann hypothesis 606
 - equivalent statements 613, 614, 644
- Riemann identity
 - Riemann theta functions 542
 - Riemann theta functions with characteristics 542
- Riemann matrix 538
 - computation 546
- Riemann surface **543**
 - connection with Riemann theta functions 543, 546
 - cycles 543
 - definition 543
 - genus 543
 - handle 543
 - holomorphic differentials 543
 - hyperelliptic 544
 - intersection indices 538, 543
 - prime form 544
 - representation via Hurwitz system 546
 - representation via plane algebraic curve 546
 - representation via Schottky group 546
- Riemann theta functions **538**
 - analytic properties 538
 - applications 543–546
 - components 538
 - computation 546
 - definition 538
 - dimension 538
 - genus 538
 - graphics 539–541
 - modular group 542
 - modular transformations 541–542
 - notation 538
 - period lattice 539
 - products 542
 - quasi-periodicity 539
 - relation to classical theta functions 539
 - Riemann identity 542
 - scaled 538, 546
 - symmetry 539
- Riemann theta functions with characteristics **539**
 - addition formulas 543
 - applications
 - Abelian functions 545
 - characteristics 539
 - half-period 539
 - modular transformations 542
 - notation 538
 - quasi-periodicity 539
 - Riemann identity 542
 - symmetry 539

- Riemann zeta function **602**
- analytic properties 602
 - applications
 - mathematical 613
 - physical 614
 - approximations 615
 - asymptotic 606
 - Chebyshev series 615
 - computation 614
 - connection with incomplete gamma functions 189
 - critical line 606
 - critical strip 606
 - definition 602
 - derivatives 602
 - integer arguments 605
 - series expansions 602
 - Euler-product representation 640
 - graphics 603
 - incomplete 189
 - infinite products 602
 - integer argument 605
 - integral representations
 - along the real line 604
 - contour integrals 605
 - integrals 606
 - notation 602
 - recursion formulas 605
 - reflection formulas 603
 - relations to other functions
 - Bernoulli and Euler numbers and polynomials 598, 605
 - Hurwitz zeta function 607
 - polylogarithms 611
 - representations by Euler–Maclaurin formula 602
 - series expansions 602
 - sums 606
 - tables 614
 - zeros
 - computation 614
 - counting 607, 614
 - distribution 606
 - on critical line or strip 606, 614
 - relation to quantum eigenvalues 614
 - Riemann hypothesis 606
 - trivial 606
- Riemann’s ξ -function 603
- approximations 615
- Riemann’s differential equation
- general form 396
 - reduction to hypergeometric differential equation 396
 - singularities 396
 - solutions
 - P -symbol notation 396
 - transformations 396
- Riemann–Hilbert problems
 - classical orthogonal polynomials 479
- Riemann–Lebesgue lemma 14
- Riemann–Siegel formula 607
- coefficients 614
- Riemann’s P -symbol 396
- ring functions *see* toroidal functions.
- Ritt’s theorem
 - differentiation of asymptotic approximations 42
- robot trajectory planning
 - Cornu’s spiral 169
- Rodrigues formulas
 - classical orthogonal polynomials 442
 - Hahn class orthogonal polynomials 462
- Rogers polynomials
 - *see* continuous q -ultraspherical polynomials.
- Rogers–Ramanujan identities 422, 430
- constant term 431
 - partitions 628
- Rogers–Szegő polynomials 475
- rolling of ships
 - Mathieu functions 679
- rook polynomial 633
- roots
 - of equations 90
- Rossby waves
 - biconfluent Heun functions 720
- rotation matrices
 - relation to $3j$ symbols 761
- Rouché’s theorem 20, 92
- round-robin tournaments 648
- Runge–Kutta methods
 - ordinary differential equations 89–90
- Rutherford scattering
 - Coulomb functions 753
 - gamma function 145
- Rydberg constant
 - Coulomb functions 754
- S -matrix scattering
 - Coulomb functions 754
- saddle points 47
- coalescing 48, 789–790
- sampling expansions
 - parabolic cylinder functions 317
- scaled gamma function 185
- scaled Riemann theta functions
 - computation 546
 - definition 538
- scaled spheroidal wave functions 706–707
- bandlimited 706
 - extremal properties 707
 - Fourier transform 706
 - integral equation 706

INDEX	941
orthogonality	706
scaling laws	
for diffraction catastrophes	785
scattering problems	
associated Legendre functions	379
Coulomb functions	753–755
scattering theory	
Mathieu functions	679
Schlöffli's integrals	
Bessel functions	224, 225
Schlöffli–Sommerfeld integrals	
Bessel and Hankel functions	224
Schlöffli-type integrals	
Kelvin functions	269
Schottky group	
Riemann surface	546
Schottky problem	
Riemann surface	545
Schröder numbers	
definition	622
generating function	623
relation to lattice paths	622
table	622
Schrödinger equation	
Airy functions	209
Coulomb functions	753–755
nonlinear	
Jacobian elliptic functions	565
Riemann theta functions	545
q -deformed quantum mechanical	432
solutions in terms of classical orthogonal polynomials	479
theta functions	534
Schwarz reflection principle	19
Schwarz's lemma	20
Schwarzian derivative	27
Scorer functions	204
analytic properties	204
applications	209
approximations	
expansions in Chebyshev series	212
asymptotic expansions	205
computation	210
computation by quadrature	84
connection formulas	205
definition	204
differential equation	204
initial values	204
numerically satisfactory solutions	204
standard solutions	204
graphs	204
integral representations	204–205
integrals	
asymptotic expansions	206
tables	211
Maclaurin series	205
notation	194
tables	211
zeros	206
computation	210
tables	211
secant function	<i>see</i> trigonometric functions.
sectorial harmonics	378
Selberg integrals	
generalized elliptic integrals	516
Selberg-type integrals	
gamma function	143
separable Gauss sum	
number theory	643
Shanks' transformation	
for sequences	93
ship wave	790–791
sieve of Eratosthenes	
prime numbers	648
sigma function	<i>see</i> Weierstrass elliptic functions.
signal analysis	
spheroidal wave functions	706–707
simple closed contour	16
simple closed curve	11
simple discontinuity	4
simple zero	19
simply-connected domain	25
Sinc function	77
sine function	<i>see</i> trigonometric functions.
sine integrals	150
applications	
Gibbs phenomenon	154
physical	155
approximations	156
asymptotic expansions	153
exponentially-improved	154
auxiliary functions	<i>see</i> auxiliary functions for sine and cosine integrals.
Chebyshev-series expansions	156–157
computation	155
definition	150
expansion in spherical Bessel functions	153
generalized	188–189
graphics	151
hyperbolic analog	150
integral representations	152
integrals	154
Laplace transform	154
maxima and minima	155
notation	150
power series	151
relations to exponential integrals	151
sums	154

- | | | | |
|--|------------|--|------------|
| tables..... | 156 | addition theorems..... | 267 |
| value at infinity..... | 150 | analytic properties..... | 262 |
| zeros..... | 154 | applications | |
| asymptotic expansion..... | 154 | electromagnetic scattering..... | 276 |
| computation..... | 156 | Helmholtz equation..... | 276 |
| sine-Gordon equation | | wave equation..... | 276 |
| Jacobian elliptic functions..... | 565 | approximations..... | 281 |
| Painlevé transcendents..... | 739 | asymptotic approximations for large order..... | <i>see</i> |
| singularities | | uniform asymptotic expansions for large order | |
| movable..... | 724 | computation..... | 276–277 |
| singularity | | continued fractions..... | 266 |
| branch point..... | 20 | cross-products..... | 265 |
| essential..... | 19 | definitions..... | 262 |
| isolated..... | 19 | derivatives..... | 265 |
| isolated essential..... | 19 | zeros..... | 266, 280 |
| pole..... | 19 | differential equations..... | 262 |
| removable..... | 4, 19 | numerically satisfactory solutions..... | 262 |
| $6j$ symbols..... | 761 | singularities..... | 262 |
| addition theorem..... | 763 | standard solutions..... | 262 |
| applications..... | 765 | Dirac delta..... | 38 |
| approximations for large parameters..... | 764 | duplication formulas..... | 267 |
| computation..... | 765 | explicit formulas | |
| definition..... | 761 | modified functions..... | 264 |
| alternative..... | 763 | sums or differences of squares..... | 264 |
| generating functions..... | 763 | unmodified functions..... | 264 |
| graphical method..... | 765 | generating functions..... | 266 |
| notation..... | 758 | graphs..... | 262 |
| orthogonality..... | 763 | integral representations..... | 266 |
| recursion relations..... | 762 | integrals..... | 267 |
| Regge symmetries..... | 762 | computation..... | 278 |
| representation as | | interrelations..... | 262 |
| finite sum of algebraic quantities..... | 762 | limiting forms..... | 265 |
| finite sum of $3j$ symbols..... | 761 | modified..... | 262 |
| generalized hypergeometric functions..... | 761 | notation..... | 217 |
| special cases..... | 762 | of the first, second, and third kinds..... | 262 |
| sum rules..... | 763 | power series..... | 265 |
| summation convention..... | 760 | Rayleigh's formulas..... | 264 |
| sums..... | 763 | recurrence relations..... | 265 |
| symmetry..... | 762 | reflection formulas..... | 262 |
| tables..... | 765 | sums..... | 267 |
| zeros..... | 765 | addition theorems..... | 267 |
| $SL(2, \mathbb{Z})$ bilinear transformation..... | 579 | compendia..... | 267 |
| Sobolev polynomials..... | 477 | duplication formulas..... | 267 |
| soliton theory | | tables..... | 280 |
| classical orthogonal polynomials..... | 478 | uniform asymptotic expansions for large order..... | 266 |
| solitons | | Wronskians..... | 265 |
| Jacobian elliptic functions..... | 565 | zeros..... | 266 |
| Weierstrass elliptic functions..... | 582 | spherical Bessel transform..... | 278 |
| spatio-temporal dynamics | | computation..... | 278 |
| Heun functions..... | 718 | spherical coordinates..... | 8 |
| spectral problems | | spherical harmonics..... | 378 |
| Heun's equation..... | 720 | addition theorem..... | 379 |
| separation of variables..... | 720 | applications..... | 379 |
| spherical Bessel functions..... | 262 | basic properties..... | 378–379 |

- definitions 378
- Dirac delta 38
- distributional completeness 379
- Lamé polynomials 694
- relation to $3j$ symbols 760
- sums 379
- zonal 479
- spherical polar coordinates .. *see* spherical coordinates.
- spherical triangles
 - solution of 131
- spherical trigonometry
 - Jacobian elliptic functions 564
- sphero-conal coordinates 693
- spheroidal coordinates *see* oblate spheroidal coordinates and prolate spheroidal coordinates.
- spheroidal differential equation **698**
 - eigenvalues 698–699
 - asymptotic behavior 702–703
 - computation 707
 - continued-fraction equation 699
 - graphics 700
 - power-series expansion 699
 - tables 708
 - Liouville normal form 698
 - singularities 698
 - special cases 698
 - with complex parameter 700
- spheroidal harmonics
 - oblate 378
 - prolate 378
- spheroidal wave functions **698**
 - addition theorem 703
 - applications
 - signal analysis 706–707
 - wave equation 704–706
 - approximations 703
 - as confluent Heun functions 717
 - asymptotic behavior
 - as $x \rightarrow \pm 1$ 703
 - for large $|\gamma^2|$ 702–703
 - computation 707–708
 - convolutions 703
 - Coulomb 704
 - definitions 699, 700
 - differential equation 698
 - eigenvalues 698
 - elementary properties 699
 - expansions in series of Ferrers functions 702
 - asymptotic behavior of coefficients 702
 - tables of coefficients 708
 - expansions in series of spherical Bessel functions 703
 - Fourier transform 706
 - generalized 704
 - graphics 700–701
 - integral equations 703, 706
 - integrals 703
 - notation 698
 - oblate angular 699
 - of complex argument 700
 - of the first kind 699
 - of the second kind 700
 - orthogonality 699
 - other notations 698
 - power-series expansions 699
 - products 703
 - prolate angular 699
 - radial 703
 - scaled 706
 - tables 708
 - with complex parameters 700
 - zeros 699
- spline functions
 - Bernoulli monosplines 597
 - cardinal monosplines 597
 - cardinal splines 597
 - Euler splines 597
- splines
 - Bézier curves 100
 - definitions 100
- square-integrable function 6
- stability problems
 - Mathieu functions 679
- stable polynomials 23
 - Hurwitz criterion 23
- statistical analysis
 - multivariate
 - functions of matrix argument 773
- statistical applications
 - functions of matrix argument 773
- statistical mechanics
 - application to combinatorics 635
 - Heun functions 720
 - incomplete beta functions 189
 - Jacobian elliptic functions 564
 - modular functions 582
 - q -hypergeometric function 432
 - solvable models 146
 - theta functions 533
- statistical physics
 - Bernoulli and Euler polynomials 598
 - Painlevé transcendents 739
- Steed's algorithm
 - for continued fractions 96
- steepest-descent paths
 - numerical integration 83–84
- Stickelberger codes
 - Bernoulli numbers 598
- Stieltjes fraction (S -fraction) 95

- Stieltjes polynomials
- definition 718
 - orthogonality 719
 - products 719
 - zeros 718
 - electrostatic interpretation 719
- Stieltjes transform
- analyticity 30
 - asymptotic expansions 52–53
 - convergence 29
 - definition 29, 52
 - derivatives 30
 - generalized 53
 - inversion 30
 - representation as double Laplace transform 30
- Stieltjes–Wigert polynomials 471
- asymptotic approximations 474
- Stirling cycle numbers 631
- Stirling numbers (first and second kinds)
- asymptotic approximations 626
 - definitions 624
 - generalized 626
 - generating functions 624
 - identities 625
 - notations 618
 - recurrence relations 625
 - relations to Bernoulli numbers 596
 - special values 625
 - tables 624, 635
- Stirling’s formula 141
- Stirling’s series 141
- Stokes line 68
- Stokes multipliers 57
- Stokes phenomenon 67
- complementary error function 164
 - incomplete gamma functions 189
 - smoothing of 67
- Stokes sets 782–785
- cusps 783
 - definitions 782
 - umbilics 783
 - visualizations 784–785
- Stokes’ theorem for vector-valued functions 12
- string theory
- beta function 146
 - elliptic integrals 517
 - modular functions 582
 - Painlevé transcendents 739
 - Riemann theta functions 545
 - theta functions 533
- Struve functions ... *see* Struve functions and modified Struve functions.
- Struve functions and modified Struve functions ... **288**
- analytic continuation 291
 - applications
 - physical 298
 - approximations 300
 - argument $xe^{\pm 3\pi i/4}$ 294
 - asymptotic expansions
 - generalized 293
 - large argument 293
 - large order 293
 - remainder terms 293
 - computation 299
 - definitions 288
 - derivatives 292
 - with respect to order 292
 - differential equations 288
 - numerically satisfactory solutions 288
 - particular solutions 288
 - graphics 289–291
 - half-integer orders 291
 - incomplete 300
 - inequalities 291
 - integral representations
 - along real line 292
 - compendia 293
 - contour integrals 292
 - Mellin–Barnes type 293
 - integrals
 - compendia 294
 - definite 294
 - indefinite 293–294
 - Laplace transforms 294
 - products 294
 - tables 299
 - with respect to order 294
 - Kelvin-function analogs 294
 - notation 288
 - order 288
 - power series 288
 - principal values 288
 - recurrence relations 292
 - relations to Anger–Weber functions 297
 - series expansions
 - Bessel functions 292
 - Chebyshev 300
 - power series 288
 - sums 294
 - tables 299
 - zeros 292
- Struve’s equation ... *see* Struve functions and modified Struve functions, differential equations.
- Sturm–Liouville eigenvalue problems
- ordinary differential equations 89
- summability methods for integrals
- Abel 34
 - Cesàro 34

- Fourier integrals
 - conjugate Poisson integral 34
 - Fejér kernel 34
 - Poisson integral 34
 - Poisson kernel 34
- fractional derivatives 35
- fractional integrals 35
- summability methods for series
 - Abel 33
 - Borel 33
 - Cesàro 33
 - general 33
 - convergence 33
 - Fourier series
 - Abel means 33
 - Cesàro means 33
 - Fejér kernel 33
 - Poisson kernel 33
 - regular 33
 - Tauberian theorems 35
- summation by parts 63
- summation formulas
 - Boole 597
 - Euler–Maclaurin 597
- sums of powers
 - as Bernoulli or Euler polynomials 589
 - tables 598
- supersonic flow
 - Lamé polynomials 694
- support
 - of a function 35
- surface *see* parametrized surfaces.
- surface harmonics of the first kind 378
- surface-wave problems
 - Struve functions 298
- swallowtail bifurcation set
 - formula 781
 - picture 782
- swallowtail canonical integral **776**
 - asymptotic approximations 789–790
 - convergent series 787
 - differential equations 788
 - formulas for Stokes set 783
 - integral identities 787
 - picture of Stokes set 784
 - pictures of modulus 778
 - scaling laws 785
 - zeros 787
- swallowtail catastrophe **776**, 784
- symmetric elliptic integrals **497**
 - addition theorems 509–510
 - advantages of symmetry 497
 - applications
 - mathematical 514–516
 - physical 516–517
 - statistical 515
 - arithmetic-geometric mean 505
 - asymptotic approximations and expansions
 - 53, 510–511
 - Bartky’s transformation 504
 - change of parameter of R_J 504
 - circular cases 502–504
 - complete 486
 - computation 517–519
 - connection formulas 503
 - degree 498
 - derivatives 500
 - differential equations 501
 - duplication formulas 510
 - elliptic cases of $R_{-a}(\mathbf{b}; \mathbf{z})$ 498
 - first, second, and third kinds 486
 - Gauss transformations 497, 505
 - general lemniscatic case 502, 503
 - graphics 499–500
 - hyperbolic cases 502–504
 - inequalities
 - complete integrals 506–507
 - incomplete integrals 507
 - integral representations 506
 - integrals of 511
 - Landen transformations 497, 505
 - notation 486
 - permutation symmetry 497, 498
 - power-series expansions 501–502
 - reduction of general elliptic integrals 512–514
 - relations to other functions
 - Appell functions 509
 - Bulirsch’s elliptic integrals 508
 - hypergeometric function 509
 - Jacobian elliptic functions 508
 - Lauricella’s function 509
 - Legendre’s elliptic integrals 507, 508
 - theta functions 508
 - Weierstrass elliptic functions 509
 - special cases 502–503
 - tables 519
 - transformations replaced by symmetry
 - 497, 505, 508
- symmetries
 - of canonical integrals 777
- Szegő–Askey polynomials 475
- Szegő–Szász inequality
 - Jacobi polynomials 451
- tangent function *see* trigonometric functions.
- tangent numbers 596
 - tables 596
- Taylor series 18
 - asymptotic approximations for coefficients 65

- Taylor's theorem
 one variable 6, 18
 two variables 8
- tempered distributions 36, 52
 convergence 36
 Fourier transform 37
- term-by-term integration 18
- terminant function 68
 incomplete gamma functions 189
- tesseral harmonics 378
- test functions
 distributions 35
- Theorem of Ince
 Mathieu's equation 653, 657
- theta functions **524**
 addition formulas 531
 applications
 mathematical 533
 physical 533
 computation 534
 derivatives 529–530
 of ratios 531
 discrete analog 532
 double products 530
 duplication formula 531
 Fourier series 524
 fundamental parallelogram 524
 generalizations 532
 graphics
 complex variables 527–529
 real variables 525–527
 infinite products 529–530
 integrals 532
 Jacobi's identity 529
 Jacobi's inversion formula 532, 533
 Jacobi's original notation 524
 Jacobi's triple product 529
 Landen transformation 531
 Laplace transform with respect to lattice parameter
 532
 lattice parameter 524
 transformation of 531
 lattice points 524
 limit forms as $\Im\tau \rightarrow 0+$ 534
 McKean and Moll's 524
 Mellin transform with respect to lattice parameter
 532
 modular transformations 531
 multidimensional *see* Chapter 21.
 Neville's 524, 550
 nome 524
 rectangular case 524
 transformation of 531
 notation 524
- periodicity 524
- power series 530
- quasi-periodicity 524
- Ramanujan's 533
- Ramanujan's change of base 533
- rectangular case 524
- relations to other functions
 Dedekind's eta function 525
 elliptic integrals 532
 elliptic modular function 532
 Jacobi's epsilon function 562
 Jacobian elliptic functions 532, 550
 modular functions 579
 Riemann zeta function 532
 symmetric elliptic integrals 508
 Weierstrass elliptic functions 532, 574
- Riemann 538
- Riemann with characteristics 539
- sums of squares 530
- tables 534
- translation by half-periods 525
- values at $z = 0$ 529
- Watson's expansions 531
- Watson's identities 531
- with characteristics 533
- zeros 525
- Thomae transformation
 ${}_3F_2$ functions of matrix argument 772
- $3j, 6j, 9j$ symbols
 relation to generalized hypergeometric functions
 407, 418
- $3j$ symbols **758**
 angular momenta 758
 applications 765
 approximations for large parameters 764
 computation 765
 definition 758
 Gaunt coefficient 761
 Gaunt's integral 761
 generating functions 760
 graphical method 765
 notation 758
 orthogonality 760
 projective quantum numbers 758
 recursion relations 760
 Regge symmetries 759
- relations to other functions
 Legendre functions 760
 rotation matrices 761
 spherical harmonics 760
- representation as
 finite sum of algebraic quantities 758
 generalized hypergeometric functions 758
- special cases 759

INDEX

947

- summation convention 760
- sums 760
- symmetry 759
- tables 765
- triangle conditions 758
- zeros 765
- Toda equation
 - Hermite polynomials 478
- tomography
 - confluent hypergeometric functions 346
- tops
 - Jacobian elliptic, or hyperelliptic, integrals 566
- toroidal coordinates 371, 379
- toroidal functions **371**
 - applications 379
 - definitions 371
 - hypergeometric representations 371
 - integral representations 371
 - sums 372
 - Whipple's formula 372
- torus
 - complex 533
- transcendental equations
 - asymptotic solutions 43
- transcendental functions 724
- transition points 58, 63
- transport equilibrium
 - generalized exponential integral 190
- triangle conditions
 - $3j$ symbols 758
- triangle inequality 15
- triangles
 - solution of 130
- triangular matrices
 - confluent hypergeometric functions 345
- triconfluent Heun equation 718
- trigonometric functions **112**
 - addition formulas 117
 - analytic properties 112
 - applications
 - cubic equations 131
 - solution of triangles and spherical triangles ... 130
 - approximations 132
 - Chebyshev-series expansions 132
 - computation 131
 - conformal maps 113
 - continued fractions 121
 - definitions 112
 - derivatives 117
 - differential equations 117
 - elementary properties 115–116
 - graphics
 - complex argument 113–115
 - real argument 112
 - identities 117
 - inequalities 116
 - infinite products 118
 - integrals
 - definite 122
 - indefinite 122
 - inverse *see* inverse trigonometric functions.
 - Laurent series 116
 - limits 116
 - Maclaurin series 116
 - moduli 118
 - multiples of argument 118
 - notation 104
 - orthogonality 122
 - partial fractions 118
 - periodicity 112
 - poles 123
 - real and imaginary parts 118
 - relations to hyperbolic functions 123
 - special values 116
 - squares and products 117
 - sums 123
 - tables 132
 - zeros 112
- triple integrals 9
- truncated exponential series 180
- turning points 58, 63
 - fractional or multiple 61
- two-body relativistic scattering
 - Lamé polynomials 694
- ultraspherical polynomials **438**
 - *see also* classical orthogonal polynomials.
 - addition theorem 459
 - applications
 - zonal spherical harmonics 479
 - asymptotic approximations 452
 - case $\lambda = 0$ 437
 - computation 479
 - definition 439
 - derivatives 446
 - differential equation 445
 - expansions in series of 460, 461
 - Fourier transforms 456
 - generating functions 449
 - inequalities 450
 - integral representations 447, 448
 - for products 455
 - integrals 456
 - interrelations with other orthogonal polynomials
 - 444–445, 448
 - leading coefficients 439
 - limits to monomials 444
 - linearization formula 460
 - Mellin transform 458

- normalization 439
- notation 436, 437
- orthogonality property 439
- parameter constraint 439, 443
- recurrence relations 446
- relations to other functions
 - Ferrers functions 448
 - hypergeometric function 393, 442
- Rodrigues formula 442
- special values 444
- symmetry 444
- tables of coefficients 440
- upper bound 450
- weight function 439
- zeros 438, 454
- umbilics
 - normal forms 776
- umbral calculus
 - Bernoulli and Euler polynomials 590
- uniformization
 - algebraic equations via Jacobian elliptic functions 564
- unity
 - roots of 23
- vacuum magnetic fields
 - toroidal functions 379
- validated computing 72
- Van Vleck polynomials
 - definition 718
 - zeros 718
- Van Vleck's theorem for continued fractions 25
- Vandermonde 3
- variation of parameters
 - inhomogeneous differential equations 26
- variation of real or complex functions 6
 - bounded 6
 - total 6
- variational operator 44
- vector
 - equivalent 542
 - norms 74
- vector-valued functions 9–12
 - *see also* parametrized surfaces.
 - curl 10
 - del operator 10
 - divergence 10
 - divergence (or Gauss's) theorem 12
 - gradient 10
 - Green's theorem
 - three dimensions 12
 - two dimensions 11
 - line integral 11
 - path integral 11
 - reparametrization of integration paths
 - orientation-preserving 11
 - orientation-reversing 11
 - Stokes' theorem 12
- vectors 9
 - *see also* vector-valued functions.
 - angle 9
 - cross product 9
 - right-hand rule 10
 - dot product 9
 - Einstein summation convention 10
 - Levi-Civita symbol 10
 - magnitude 9
 - notations 9, 10
 - right-hand rule for cross products 10
 - scalar product *see* dot product.
 - unit 9
 - vector product *see* cross product.
- vibrational problems
 - Mathieu functions 678
- Voigt functions
 - applications 169
 - computation 169
 - definition 167
 - graphs 168
 - properties 168
 - relation to line broadening function 167
 - tables 169
- von Staudt–Clausen theorem
 - Bernoulli numbers 593
- Voronoi's congruence
 - Bernoulli numbers 593
- Waring's problem
 - number theory 645
- water waves
 - Kelvin's ship-wave pattern 790–791
 - Riemann theta functions 545
 - Struve functions 298
- Watson integrals
 - Appell functions 417
 - generalized hypergeometric functions 417
- Watson's ${}_3F_2$ sum
 - Andrews' terminating q -analog 427
 - Gasper–Rahman q -analog 426
- Watson's expansions
 - theta functions 531
- Watson's identities
 - theta functions 531
- Watson's lemma
 - asymptotic expansions of integrals 44, 46
- Watson's sum
 - generalized hypergeometric functions 406
- wave acoustics
 - generalized exponential integral 190
- wave equation

- *see also* water waves.
- Bessel functions and modified Bessel functions .. 276
- confluent hypergeometric functions..... 346
- ellipsoidal coordinates..... 693
- Mathieu functions..... 678
- oblate spheroidal coordinates..... 705–706
- paraboloidal coordinates..... 346
- prolate spheroidal coordinates..... 704–705
- separation constants..... 693
- spherical Bessel functions..... 276
- sphero-conal coordinates..... 693
- symmetric elliptic integrals..... 501
- wave functions
- paraboloidal..... 677
- waveguides..... 275
- Weber function..... *see* Anger–Weber functions.
- Weber parabolic cylinder functions
- *see* parabolic cylinder functions.
- Weber’s function
- *see* Bessel functions of the second kind.
- Weber–Schafheitlin discontinuous integrals
- Bessel functions..... 244
- Weierstrass M -test
- *see* M -test for uniform convergence.
- Weierstrass elliptic functions..... **570**
- addition theorems..... 577
- analytic properties..... 570
- applications
- mathematical..... 581
- physical..... 582–583
- asymptotic approximations..... 578
- computation..... 583
- definitions..... 570
- derivatives..... 571
- differential equations..... 571
- discriminant..... 571
- duplication formulas..... 578
- equianharmonic case..... 571–572, 574
- Fourier series..... 576
- graphics
- complex variables..... 573–574
- real variables..... 571–572
- homogeneity..... 578
- infinite products..... 577
- integral representations..... 578
- integrals..... 579
- lattice..... 570
- computation..... 583
- equianharmonic..... 574
- generators..... 570
- invariants..... 571
- lemniscatic..... 571–572, 574
- notation..... 570
- points..... 570
- pseudo-lemniscatic..... 574
- rectangular..... 574
- rhombic..... 574
- roots..... 571
- Laurent series..... 577
- lemniscatic case..... 571–572, 574
- n -tuple formulas..... 578
- notation..... 570
- periodicity..... 571
- poles..... 570
- power series..... 577
- principal value..... 577
- pseudo-lemniscatic case..... 574
- quarter periods..... 576
- quasi-periodicity..... 571
- relations to other functions
- elliptic integrals..... 576
- general elliptic functions..... 576
- Jacobian elliptic functions..... 575
- symmetric elliptic integrals..... 509
- theta functions..... 574
- rhombic case..... 574
- series of cosecants or cotangents..... 577
- tables..... 584
- zeros..... 570, 579
- Weierstrass \wp -function
- *see* Weierstrass elliptic functions.
- Weierstrass product..... 22
- Weierstrass sigma function
- *see* Weierstrass elliptic functions.
- Weierstrass zeta function
- *see* Weierstrass elliptic functions.
- weight functions
- cubature..... 84–85
- definition..... 79, 437
- Freud..... 475
- least squares approximations..... 99
- logarithmic..... 81–82
- minimax rational approximations..... 97
- quadrature..... 79–80
- weighted means..... 3
- Weniger’s transformation
- for sequences..... 94
- Whipple’s ${}_3F_2$ sum
- Gasper–Rahman q -analog..... 427
- Whipple’s formula
- associated Legendre functions..... 362
- toroidal functions..... 372
- Whipple’s sum
- generalized hypergeometric functions..... 406
- Whipple’s theorem
- Watson’s q -analog..... 429
- Whipple’s transformation
- generalized hypergeometric functions..... 407

- Whittaker functions **334**
 *see also* confluent hypergeometric functions.
 addition theorems 345
 analytic continuation 334
 analytical properties 334
 applications
 Coulomb functions 346
 groups of triangular matrices 345
 physical 346, 754
 uniform asymptotic solutions of differential equations 345
 asymptotic approximations for large parameters
 imaginary κ and/or μ 340
 large κ 341–342
 large μ 339–341
 uniform 339–342
 asymptotic expansions for large argument 339
 error bounds 339
 exponentially-improved 339
 computation 346
 connection formulas 335
 continued fractions 338
 definitions 334
 derivatives 336
 differential equation *see* Whittaker's equation.
 expansions in series of 344
 integral representations
 along the real line 337
 contour integrals 337
 Mellin–Barnes type 337
 integral transforms in terms of 344
 integrals 337
 compendia 344
 Fourier transforms 343
 Hankel transforms 343–344
 Laplace transforms 343
 Mellin transforms 343
 interrelations 335
 large argument 69
 limiting forms
 as $z \rightarrow 0$ 335
 as $z \rightarrow \infty$ 335
 multiplication theorems 345
 notation 322
 power series 334
 principal branches (or values) 334
 products 345
 recurrence relations 336
 relations to other functions
 Airy functions 338
 Coulomb functions 742, 748, 751
 elementary functions 338
 error functions 338
 incomplete gamma functions 338
 Kummer functions 334
 modified Bessel functions 338
 orthogonal polynomials 338
 parabolic cylinder functions 338
 series expansions 344–345
 addition theorems 345
 in Bessel functions or modified Bessel functions
 344
 multiplication theorems 345
 power 334
 Wronskians 335
 zeros
 asymptotic approximations 343
 distribution 342
 inequalities 343
 number of 343
 Whittaker's equation 334
 fundamental solutions 335
 numerically satisfactory solutions 335
 relation to Kummer's equation 334
 standard solutions 334
 Whittaker–Hill equation 676
 applications 678
 separation constants 678
 Wigner $3j$, $6j$, $9j$ symbols
 *see* $3j$ symbols, $6j$ symbols, and $9j$ symbols.
 Wilf–Zeilberger algorithm
 applied to generalized hypergeometric functions .. 407
 Wilkinson's polynomial 92
 Wilson class orthogonal polynomials 467–470
 asymptotic approximations 470
 definitions 467
 differences 469
 dualities 463
 generating functions 469
 interrelations with other orthogonal polynomials
 464, 468–469
 leading coefficients 468
 normalizations 467–468
 notation 436
 orthogonality properties 467
 relation to generalized hypergeometric functions
 468–469
 transformations of variable 467
 weight functions 467–468
 Wilson polynomials
 *see* Wilson class orthogonal polynomials.
 winding number
 of closed contour 16
 WKB or WKBJ approximation
 .. *see* Liouville–Green (or WKBJ) approximation.
 Wronskian
 differential equations 26
 Wynn's cross rule

- for Padé approximations 98
- Wynn's epsilon algorithm
 - for sequences 93
- zero potential
 - Coulomb functions 753, 754
- zeros of analytic functions
 - computation 90–92
 - conditioning 92
 - multiplicity 19, 90
 - simple 90
- zeros of Bessel functions (including derivatives)
 - analytic properties 235
 - approximations 281
 - asymptotic expansions for large order
 - uniform 237
 - asymptotic expansions for large zeros 236
 - error bounds 236
 - bounds 236
 - common 235
 - complex 235, 238
 - computation 277
 - distribution 235, 238–240
 - double 235
 - interlacing 235
 - monotonicity 236
 - notation 235
 - of cross-products 238
 - asymptotic expansions 238
 - purely imaginary 235, 236
 - relation to inverse phase functions 235
 - tables 132, 278
 - with respect to order (ν -zeros) 240
- zeros of cylinder functions (including derivatives)
 - 235–237
 - analytic properties 235
 - asymptotic expansions for large order
 - uniform 236
 - asymptotic expansions for large zeros 236
 - forward differences 235
 - interlacing 235
 - monotonicity 236
 - relation to inverse phase functions 235
- zeros of polynomials
 - *see also* stable polynomials.
 - computation 91–92
 - conditioning 92
 - degrees two, three, four 23
 - Descartes' rule of signs 22
 - discriminant 22
 - distribution 22
 - division algorithm 22
 - elementary properties 22
 - elementary symmetric functions 22
 - explicit formulas 91
 - Horner's scheme 22
 - extended 22
 - resolvent cubic 23
 - roots of constants 23
 - roots of unity 23
- zeta function *see* Hurwitz zeta function, Jacobi's zeta function, periodic zeta function, Riemann zeta function, *and* Weierstrass zeta function.
- zonal polynomials **769**
 - applications 773
 - beta integral 769
 - definition 769
 - Laplace integral 769
 - mean-value 769
 - normalization 769
 - notation 769
 - orthogonality 769
 - summation 769
 - tables 773
- zonal spherical harmonics
 - ultraspherical polynomials 479