PROBIT ANALYSIS

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A STATISTICAL TREATMENT OF THE SIGMOID RESPONSE CURVE

BY

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FOREWORD

In our work at Rothamsted on insecticides, their action and relative potency, we have been so dependent and have made so many calls on the skill and patience of the Statistical Department and in particular, in recent years, of Mr Finney, that it is a pleasure to learn that the statistical methods and techniques which he has placed so willingly at our disposal for the solution of our various problems are now to be expressed in a more permanent form and to be given to a wider field of workers. If this book receives its due, the investigators of toxicological problems throughout the world will find it a standby.

To many of us, engaged on the practical issues of devising the experimental material and methods to be employed in our laboratory tests, there will be much in these pages that has to be taken upon trust. They are meant to act primarily as an aid in computation, but there is a profound thread of reasoning running through them all, giving a coherence to the several chapters. One cannot but feel, therefore, that the more mathematical readers will find the book suggestive and stimulating.

Twenty-five or more years ago, when I entered the field of research from which Mr Finney takes so many examples for detailed computational study, the very whisper of the need for statistical analysis, falling upon the ears of the biological expert, was enough to bring down a storm of denial upon one's head. Although there may be some small residue of such a reaction still in existence, it now only persists in obscure nooks and crannies of the world of biological research. Much of this change is due to the school of statisticians founded by R. A. Fisher at Rothamsted, of which Mr Finney has been a distinguished member. They always showed a willingness to enter into one's experimental difficulties and an understanding of the limitations imposed by time and space upon the amount of work it was possible to do. The measure of the thought given to these matters can be gauged by the fact that some of the most important tables, given by Mr Finney in the following pages, were in process of computation in actual anticipation of problems which we later carried to him. This brings up the now rather hackneyed, but none-the-less important, matter of the prior consultation with a competent statistician before designing experiments. Repeatedly is such a course justified by events, in that the plan can not only be often simplified, but so designed as to yield more information with little, if any, more labour.

Another issue, raised by such text-books as this, concerns the extent to which a training in statistics should enter into the curriculum of the biological and biochemical student. I have found, relatively late in life, how hard a task it is to pick up, during busy years, the requisite amount of basic statistical knowledge to follow the arguments set out in recent text-books written for one's own benefit. I therefore feel that more attention might be given to such a matter by the academic powers-that-be. The engineer may not need to know all there is to be scientifically known about the composition and manufacture of his tools and materials, but at least he should know enough to use them rightly. So too the quantitative biological investigator.

Mr Finney's text-book has a very long and dignified ancestry ranging back to early Egyptian and Mesopotamian times, when texts of mathematics were used for the training of scribes, the professional computers of those distant days. But this is a forward-looking book and is primarily meant for experimentalists. Enlarged editions may well follow this, the first, as the subject grows and new problems arise; and that there are many just round the corner anyone who has discussed these matters with the author is very well aware. Having had the benefit of his personal advice and help throughout the last six years I cannot but wish this book Godspeed on its helpful mission to others.

F. TATTERSFIELD

PREFACE

FROM the theory of probability, originally investigated in order to explain nothing more important than the results of games of chance, has developed the science of applied statistics. Over one hundred years ago Laplace wrote that

> . . . la théorie des probabilités n'est au fond, que le bon sens réduit au calcul: elle fait apprécier avec exactitude, ce que les esprits justes sentent par une sorte d'instinct, sans qu'ils puissent souvent s'en rendre compte,

and these words might equally well be written of statistics to-day. In many fields of scientific research, and especially in the biological sciences, numerical studies are complicated by the inherent variability of the material under investigation, and conclusions must be based on averages derived from series of observations. The estimation of these averages and the assessment of their reliability are statistical operations, in the performance of which the experimenter inevitably employs a statistical technique even though he himself may not always recognize this fact. The operations may be simple or complex, depending upon the circumstances; if, however, they cease to be 'le bon sens réduit au calcul', they can no longer be expected to contribute to the understanding of the problem under investigation.

The recent rapid advances in the application of rigorous statistical methods to biological data began with the publication, in 1925, of R. A. Fisher's *Statistical Methods for Research Workers*. Not only did Fisher develop exact methods for the analysis of data from small samples to replace the older approximations from large-sample theory, but he also introduced new and powerful techniques for making the most efficient use of experimental results. Of equal importance to the growth of the present-day philosophy of experimentation was Fisher's suggestion that the statistician should be consulted during the planning of an experiment and not only when statistical analysis of the results is required, as his advice on experimental design may greatly increase the value of the results eventually obtained. Since Fisher's book first appeared, the principles of experimental design and the methods of statistical analysis have been extended so rapidly as to make it increasingly difficult for any but the professional statistician to be familiar with the variety of methods needed in biological problems.

Many books since Fisher's have been written with the aim of surveying a wide field of biological statistics, but these can give only an outline of some important topics. There is to-day a need for books in which the specialized statistical methods appropriate to certain branches of science will be discussed in sufficient detail to enable biologists to appreciate them and apply them to their own problems.

One subject requiring fuller discussion than can reasonably be expected in any general text-book of statistics is the method of probit analysis, for the development of which J. H. Gaddum and C. I. Bliss are largely responsible. This method is widely used for the analysis of data from toxicity tests for the assay of insecticides and fungicides, and also of data from other types of assay dependent upon a quantal response. In this book I have tried to give a systematic account of the theory and practice of probit analysis, including as much as possible of the most recent extensions and refinements, in such a form that it may be understood by biologists, chemists, and others who have some knowledge of elementary statistical procedure; at the same time, I have endeavoured to satisfy the mathematical statistician by showing the theoretical background of the method. The less mathematically minded reader will no doubt be content to omit, or at most to read cursorily, Appendix II and other sections concerned with the mathematical basis of the technique. Full understanding and appreciation of statistical methods can be gained only by experience in their use, but careful study of the numerical examples should enable many who were previously unfamiliar with probit analysis to apply it satisfactorily to their own data.

This book has been written as a result of several years of close collaboration with members of the Insecticides Department

at Rothamsted Experimental Station, especially with Dr F. Tattersfield, Dr C. Potter, and, until he left Rothamsted, Dr J. T. Martin. I wish to express my gratitude to them for discussing with me a wide variety of their problems, for advising me on the experimental aspects of their results, and for the generosity with which they have permitted me to use their data both in this book and in earlier publications. I am also very grateful to my colleagues in the Statistical Department at Rothamsted for much helpful discussion, and particularly to Dr F. Yates for his detailed and constructive criticism in the preparation of my book. Others to whom my thanks are due include Miss G. M. Ellinger for assistance in German translation, Dr C. G. Butler for permission to use the numerical data of Ex. 33, Dr A. E. Dimond and Dr J. G. Horsfall for giving me very full information on the results discussed in § 41 and for permission to use their data, Professor G. H. Thomson for assistance in tracing the history of the probit method, the Editors of the *Annals of Applied Biology* for permission to reproduce the first half of Table II, Professor R. A. Fisher, Dr F. Yates, and Messrs Oliver and Boyd, Ltd. for permission to reproduce Tables I, VI and VII from their book *Statistical Tables for Biological, Agricultural and Medical Research*, and my father, Robt. G. S. Finney, for very considerable help in the correction of proofs.

D. J. FINNEY

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p. 43.

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