UNDERSTANDING FLUID FLOW

African Institute of Mathematics Library Series

The African Institute of Mathematical Sciences (AIMS), founded in 2003 in Muizenberg, South Africa, provides a one-year postgraduate course in mathematical sciences for students throughout the continent of Africa. The **AIMS LIBRARY SERIES** is a series of short innovative texts, suitable for self-study, on the mathematical sciences and their applications in the broadest sense.

AIMS Library Series

UNDERSTANDING FLUID FLOW

GRAE WORSTER
Institute of Theoretical Geophysics
Department of Applied Mathematics and Theoretical Physics
University of Cambridge

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

The Water Club, Beach Road, Granger Bay, Cape Town 8005, South Africa

www.cambridge.org
Information on this title: www.cambridge.org/ 9780521132893

First published 2009

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-13289-3 Paperback

Cambridge University Press
978-0-521-13289-3 - Understanding Fluid Flow: AIMS Library Series
Grae Worster
Frontmatter
More information

Contents

Cambridge University Press
978-0-521-13289-3 - Understanding Fluid Flow: AIMS Library Series
Grae Worster
Frontmatter
More information

Preface

This book is an account of lectures that I gave at AIMS (the African Institute of Mathematical Sciences) in 2008, which were developed from two earlier courses I had given there in 2004 and 2006. I did not set out to write a textbook but rather to present a diary, summarising and reviewing what I had tried to convey to the students in the hour or two immediately preceding writing. This reflects the fact that lectures at AIMS are dynamic, interactive sessions and I had prepared little by way of formal lecture notes in advance. To be sure, there was some didactic delivery of ideas, but that lecturing activity was interspersed with problem-solving activities and many, many questions from students so that the educational process was much more of a conversation than is typical of most university courses I have given. Having now put this material in writing, I hope that it might interest and inspire students and teachers alike but will not constrain or encourage future lecturers at AIMS or elsewhere to teach rigidly from the book.

Where, in the course of writing, I discovered some small point that I had omitted to mention or some additional clarification or illustration that I felt would be helpful, I allowed myself to include it. Other than that, I have made no attempt to expand upon what I taught or to provide a comprehensive introduction to fluid dynamics. A student interested in pursuing the ideas further should consult any of the many excellent texts that exist in this subject, a few of which are described in the Bibliography.

In my view, fluid dynamics as a discipline sits squarely between physics and mathematics. It is important to understand the physical interactions involved well enough to codify them within mathematical

Cambridge University Press
978-0-521-13289-3 - Understanding Fluid Flow: AIMS Library Series
Grae Worster
Frontmatter
More information

equations, and to have sufficient mathematical insight to know whether the equations are likely to be manageable. The equations must then be solved, often by making approximations with enough physical insight to know whether the important interactions have been retained, and with enough mathematical rigour to have confidence in the resulting predictions. And finally, the mathematical solution must be interpreted in a way that sheds light on the physical problem being studied. I tried to convey that balance in the course, with the result that this is neither a physics text nor a mathematics text but a text aimed at understanding fluid flow from fundamental physical ideas explained and developed by mathematical studies.

I anticipate that readers will have a basic understanding of Newtonian dynamics, and be familiar with ordinary differential equations and vector calculus to the level usually taught in introductory undergraduate courses.

The first two times that I lectured at AIMS, I shared the three-week course with another lecturer: Daya Reddy from the University of Cape Town in 2004 and Keith Moffatt from Cambridge University in 2006. I nevertheless wanted my half course to be complete in itself and to convey something meaningful about fluid dynamics as an important topic of current research to students who had never met the subject before. I felt it important that the students learn about real, viscous fluids, to have a taste of analytical, numerical and experimental approaches to understanding and quantifying the flow of fluids, and to experience the satisfaction and some of the limitations of making mathematical predictions of experimental flows.

To these ends, I took as a case study the spreading of a pool of syrup poured onto a horizontal surface. This is a relatively straightforward and robust experiment that can be done in class by inexperienced students; it introduces a fundamental balance between pressure gradients and viscous stresses; and the equations describing the flow can be solved analytically and also numerically using simple methods. The major assignments given to the students to be completed outside of the classroom combined all of these aspects. I have reproduced the assignments here but have deliberately not provided any form of model answers. There is a tremendous amount of teaching that can be achieved through such extended and open-ended projects and I do not wish to circumscribe those opportunities.

The first assignment involves analysing data from experiments. I encourage students to try setting up and running these for themselves, from which they will gain first-hand experience of some fluid flows. Failing that, measurements can be made from recorded experiments available at http://www.cambridge.org/worster.

I have included within the text the data taken by students at AIMS, which could also be used to complete the assignment, though allowance must then be made for human error, an appreciation of which will perhaps be missed by those students not performing the experiments themselves.

The lectures and activities associated with the case study took up the first week of the course and form the first three chapters of this book. A more conventional approach to teaching fluid dynamics begins at Chapter 4 with a statement of the Navier–Stokes equations and discussions of the physical interactions they embody. I designed much of the central part of the course around the flow associated with a stagnation point on a rigid flat plate. This flow illustrates the role of dynamic pressure in enforcing mass conservation in an incompressible fluid, and also introduces key ideas about viscous boundary layers and the inviscid approximation away from boundaries.

In the third week of the course I aimed to introduce a number of significant phenomena of which I feel every fluid dynamicist should be aware: the dynamics of vorticity; d'Alembert's paradox and its resolution in terms of separation of viscous boundary layers; lift on an aerofoil; water waves and instability.

There are a number of short exercises scattered throughout the book. They are important and should be read as an integral part of the text even if not attempted until later. Many of them were given to students to solve in class. I have generally not included solutions. However, in some cases I wanted students to discover important ideas for themselves but, since these ideas were key foundational material, I also lectured on them subsequently. In such cases, I have provided some discussion of the solution here, but I encourage any students reading this book to try the exercises for themselves first.

In preparing these lectures I drew significantly on my own experiences as a student and the inspiration given to me by the many excellent lecturers from whom I was privileged to learn aspects of fluid dynamics: John Hinch in Part IB; George Batchelor, Michael McIntyre and

Cambridge University Press
978-0-521-13289-3 - Understanding Fluid Flow: AIMS Library Series
Grae Worster
Frontmatter
More information

Michael Proctor in Part II; and Adrian Gill, Douglas Gough, Herbert Huppert, Tim Pedley and Ron Smith in Part III of the Cambridge Mathematics Tripos. I have also drawn many of the examples and exercises from those used in the Tripos. I thank all my colleagues who have contributed to and moulded the excellent stock of pedagogical questions designed and refined there over the years.

I am extremely grateful to AIMS students Khumbo Kumwenda, Doreen Mbabazi and Mercy Njima who prepared some of the figures and tables for inclusion in an early draft of the notes, and to Mark Hallworth who prepared all the figures for publication. Mark was also invaluable in setting up the experiments and preparing the videos of them, assisted by Andrew Lawrie. The text was much improved following valuable input from Colm Caulfield, Herbert Huppert, Richard Katz, Keith Moffatt, Jerome Neufeld and John Wettlaufer, to all of whom I am indebted. In particular, Colm lectured this course at AIMS in 2009 and gave me invaluable feedback based on his experience. I could not have undertaken this project without the able assistance of Doris Allen who shouldered the considerable burden of setting everything in LaTeX. And finally, I am grateful to my wife Jacqui and daughters Susannah, Lizzie, Katherine and Caroline, who allowed me time in South Africa without them.