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# Partial Differential Equations and Fluid Mechanics

*Edited by*

JAMES C. ROBINSON & JOSÉ L. RODRIGO

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To Tania and Elizabeth

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## Preface

This volume is the result of a workshop, “Partial Differential Equations and Fluid Mechanics”, which took place in the Mathematics Institute at the University of Warwick, May 21st–23rd, 2007.

Several of the speakers agreed to write review papers related to their contributions to the workshop, while others have written more traditional research papers. All the papers have been carefully edited in the interests of clarity and consistency, and the research papers have been externally refereed. We are very grateful to the referees for their work. We believe that this volume therefore provides an accessible summary of a wide range of active research topics, along with some exciting new results, and we hope that it will prove a useful resource for both graduate students new to the area and to more established researchers.

We would like to express their gratitude to the following sponsors of the workshop: the London Mathematical Society, the Royal Society, via a University Research Fellowship awarded to James Robinson, the North American Fund and Research Development Fund schemes of Warwick University, and the Warwick Mathematics Department (via MIR@W). JCR is currently supported by the EPSRC, grant EP/G007470/1.

Finally it is a pleasure to thank Yvonne Collins and Hazel Higgins from the Warwick Mathematics Research Centre for their assistance during the organization of the workshop.

Warwick,  
December 2008

*James C. Robinson*  
*José L. Rodrigo*

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