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978-0-521-12431-7 - New Directions in Hopf Algebras
Edited by Susan Montgomery and Hans-Jurgen Schneider
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Hopf algebras have important connections to quantum theory, Lie algebras, knot and braid theory, operator algebras, and other areas of physics and mathematics. They have been intensely studied in the last decade; in particular, the solution of a number of conjectures of Kaplansky from the 1970s has led to progress on the classification of semisimple Hopf algebras and on the structure of pointed Hopf algebras. There has been much progress also on actions and coactions of Hopf algebras and on Hopf Galois extensions. Many new methods have been used for these results: modular and braided categories, representation theory, algebraic geometry, and Lie methods such as Cartan matrices.

The contributors to this volume of expository papers were participants in the Hopf Algebras Workshop held at MSRI as part of the 1999–2000 Year on Noncommutative Algebra. Together the papers give a clear picture of the current trends in this active field, with a focus on what is likely to be important in future research.

Among the topics covered are results toward the classification of finite-dimensional Hopf algebras (semisimple and non-semisimple), as well as what is known about the extension theory of Hopf algebras. Some papers consider Hopf versions of classical topics, such as the Brauer group, while others are closer to recent work in quantum groups. The book also explores the connections and applications of Hopf algebras to other fields.

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Preface

This collection of expository papers highlights progress and new directions in Hopf algebras.

Most of the contributors were participants in the Hopf Algebras Workshop held at MSRI in late 1999, although the papers are not necessarily tied to lectures given at the workshop. The workshop was very timely, as much progress has been made recently within Hopf algebras itself (for example, some long-standing conjectures of Kaplansky have been solved) as well as in studying Hopf algebras that have arisen in other areas, such as mathematical physics and topology.

The first two papers discuss progress on classifying certain classes of Hopf algebras.

In the paper by Andruskiewitsch and Schneider, pointed Hopf algebras are studied in terms of their infinitesimal braiding. Important examples of pointed Hopf algebras are group algebras and the quantum groups coming from Lie theory, that is, $U_q(\mathfrak{g})$, $\mathfrak{g}$ a semisimple Lie algebra, introduced by Drinfel'd and Jimbo, and Lusztig's finite-dimensional Frobenius kernels $\mathfrak{u}(\mathfrak{g})$ where q is a root of unity. The classification of all finite-dimensional pointed Hopf algebras with abelian group of group-like elements in characteristic zero seems to be in reach today. In this classification, Hopf algebras that are closely related to the Frobenius kernels play the main role.

In the second paper, Gelaki gives a survey on what is known on finite-dimensional triangular Hopf algebras. In a sense these Hopf algebras are close to group algebras. Over the complex numbers, triangular semisimple, and more generally triangular Hopf algebras such that the tensor product of simple representations is semisimple, are Drinfeld twists of group algebras of finite groups or supergroups. Thus these Hopf algebras are completely classified by means of group-theretical data. A main ingredient in these results is Deligne's theorem on Tannakian categories.

Thus twisting is an important method of describing new Hopf algebras that in general are neither commutative nor cocommutative. Another important idea is to study extensions. The papers by Masuoka and Schauenburg explain the modern theory of extensions of Hopf algebras and (co)quasi Hopf algebras.

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Here, recent versions and generalizations of an old exact sequence of G. I. Kac for Hopf extensions of group algebras from 1963 are crucial. Masuoka defined this sequence for universal enveloping algebras of Lie algebras. He showed how this sequnce can be used to compute many concrete examples of extensions in the group and Lie algebra case.

In Schauenburg's paper, a very general formulation and a completely new interpretation of Kac's sequence are given. The basic tools are reconstruction theorems, which allow one to construct Hopf algebras or (co)quasi Hopf algebras from various monoidal categories.

Letzter generalizes the classical representation theory of symmetric pairs, consisting of a semisimple complex Lie algebra and its fixed elements under an involution, to the quantum case. Since $U_q(\mathfrak{g})$ does not contain enough sub Hopf algebras, the theory has to deal with left coideal subalgebras instead of sub Hopf algebras. For general Hopf algebras, left and right coideal subalgebras are the natural objects to describe quotients and invariants.

Takeuchi's "short course on quantum matrices" starts with an elegant treatment of the 2×2 quantum matrices and their interpretation in the theory of knot invariants. Among many other aspects of quantum matrices for GL_n and SL_n, Takeuchi's theory of q-representations of quantum groups is discussed.

Radford's paper is a survey on his recent work with L. Kauffmann on invariants of knots and links. Motivated by the applications of quasitriangular and coquasitriangular Hopf algebras to topology, axioms for abstract quantum algebras and coalgebras are given. These new algebras and coalgebras are used in a very direct way to obtain topological invariants.

In another direction, the paper by Nikshych and Vainerman also goes beyond Hopf algebras in the strict sense. They study quantum groupoids, also called weak Hopf algebras. The groupoid algebra of a groupoid is a quantum groupoid, and a Hopf algebra if the groupoid is a group. One motivation comes from operator algebras in connection with depth 2 von Neumann subfactors. The theory is developed from scratch, and applications to topology and operator algebras are given.

The paper by van Oystaeyen and Zhang is an exposition of recent work on the Brauer group of a Hopf algebra. This new invariant of a Hopf algebra H is defined by introducing the notion of H-Azumaya algebras in the braided category of Yetter-Drinfeld modules over H. Thus Hopf algebra theory is related to the classical theory of the Brauer group of a commutative ring and of the Brauer-Long group of a graded algebra.

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