Mathematical Foundations of Imaging, Tomography and Wavefield Inversion

Inverse problems are of interest and importance across many branches of physics, mathematics, engineering and medical imaging. In this text, the foundations of imaging and wavefield inversion are presented in a clear and systematic way. The necessary theory is gradually developed throughout the book, progressing from simple wave-equation-based models to vector wave models. By combining theory with numerous MATLAB-based examples, the author promotes a complete understanding of the material and establishes a basis for real-world applications.

Key topics of discussion include the derivation of solutions to source radiation and scattering problems using Green-function techniques and eigenfunction expansions; the propagation and scattering of waves in homogeneous and inhomogeneous backgrounds; and the concepts of field time reversal and field back propagation and the key role that they play in imaging and inverse scattering.

Bridging the gap between mathematics and physics, this multidisciplinary book will appeal to graduate students working in established areas of inverse scattering and to researchers developing new computational imaging modalities. Additional resources, including solutions to end-of-chapter problems and MATLAB codes for all the examples presented in the book, are available online at www.cambridge.org/9780521119740.

Anthony J. Devaney is Distinguished Professor of Engineering at Northeastern University, Boston and he has worked in the general area of inverse problems for more than 40 years. He did his Ph.D. at the Institute of Optics at the University of Rochester and his thesis was supervised by Professor Emil Wolf. Professor Devaney has experience in geophysics inverse problems and inverse problems related to radar, optical and acoustic imaging. His patent on diffraction tomography (covered in Chapter 8 of this book) was selected as one of the top 50 patents from Schlumberger Doll Research (the principal basic research center of the Schlumberger Corporation) over its first 50 years of existence.

Mathematical Foundations of Imaging, Tomography and Wavefield Inversion

ANTHONY J. DEVANEY
Northeastern University, Boston

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521119740

First published 2012

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data
Devaney, Anthony J.
Mathematical foundations of imaging, tomography and wavefield
inversion / Anthony J. Devaney, Northeastern University, Boston.
pages cm
ISBN 978-0-521-11974-0 (hardback)
1. Wave equation. 2. Inverse problems (Differential equations) I. Title.
QC174.26.W28D382 2012
515′.357–dc23
2012000073

ISBN 978-0-521-11974-0 Hardback

Additional resources for this publication at www.cambridge.org/9780521119740

Contents

Preface

I started this book roughly 20 years ago with the intention of producing a finished product within a year or so. But reality in the form of government research grants and "publish or perish" soon set in and so now, at long last, I have finally finished. The final product has of course changed significantly over these intervening years, both in content and in breadth. My original plan was to put together a six- or seven-chapter treatise on basic "Fourier-based" coherent imaging and diffraction tomography complete with Matlab codes implementing the imaging and inversion algorithms presented in the text. The current book certainly includes this material, but also includes a host of other material such as the chapter on time-reversal imaging and the four chapters on the propagation and scattering of waves in homogeneous and inhomogeneous backgrounds. More importantly, the "Fourier-based" inversion schemes originally used to develop much of coherent imaging and linearized inverse scattering (diffraction tomography) have been replaced by the much more powerful singular value decomposition (SVD). This approach allows virtually all of the linearized inverse problems associated with the wave and Helmholtz equation both in homogeneous and in inhomogeneous backgrounds to be treated in a uniform "turn the crank" manner.

My work on imaging and wavefield inversion began as a graduate student under Professor Emil Wolf at the University of Rochester. Originally I had intended to pursue my Ph.D. in quantum optics, but had my plans changed significantly by an off-hand remark by Professor Wolf during one of our meetings. We were discussing the classical theory of imaging by lenses, at which point he asked the question "what exactly is an image?" The answer to that seemingly simple question set us off on a road that included non-radiating sources, non-scattering scatterers, and other bizarre objects that the mathematician would recognize as being members of the null space of the mapping from object to "image." While the purely non-radiating sources and non-scattering scatterers are in the null space of the mapping from object to image, there are other strange objects that I have chosen to call "essentially" non-radiating sources (or scatterers). These objects are not in the null space but are very close to it, having the property that they only radiate (or scatter) evanescent waves outside of their support and are the cause for instability of inverse problems related to the wave and Helmholtz equations. I have tried to couple these physical interpretations of non-uniqueness and instability to the purely mathematical view of these properties throughout the book. Indeed, the melding of physics with mathematics is one of my major goals in this book.

The general areas of imaging and inverse scattering are multidisciplinary in that they require a strong foundation in physics, mathematics, and signal processing. I have tried to include the necessary background in all three areas, but assume that the reader is already proficient in complex-variable theory and linear algebra at the senior undergraduate/first-year graduate level and has at least a rudimentary familiarity with the wave and Helmholtz equations in a homogeneous medium such as free space. I have also tried to emphasize the underlying physics of the various topics covered in the book but, unfortunately, at the expense of mathematical rigor. This is especially true in the development of time-independent scattering theory in Chapters 6 and 9, which follow the purely formal approach used in non-relativistic quantum scattering (collision) theory.

The vast majority of the book treats scalar wave theory, with only the last chapter devoted to vector waves in the form of the electromagnetic (EM) field. The reasons for this are that all of the essential ingredients of coherent imaging and inverse scattering are already contained in the scalar theory and that the vector theory, at least for the EM field, can be reduced to three or fewer coupled scalar wave problems. Indeed, by using the so-called Whittaker or Debye representations presented in Chapter 11, EM inverse source and scattering problems for planar or spherical geometries can be reduced to two uncoupled scalar wave problems that are treated exactly in the manner presented in earlier chapters of the book. I have also, for the most part, restricted the treatment of the various inverse problems to *linearized* formulations of the corresponding forward problems. The exceptions to this are the inverse source problem which, by its nature, is a linear problem and one of the formulations of inverse scattering from conducting surfaces in Chapter 7.

The goal of this book is to present the mathematical (and physical) *foundations* of imaging and wavefield inversion rather than to push specific inversion schemes or algorithms or to present detailed results of the use of such algorithms on real data. To this end, I have concentrated on simple yet representative Matlab-based examples that are easily understood and directly related to the theoretical development presented in the book. The myriad details that attend any actual application of these algorithms to real data are not presented. Such details include the methods required to retrieve the phase of an optical field in an optical-imaging or inverse-scattering algorithm and the need to align, usually through the use of digital filters, the outputs from antenna or transducer arrays in ultrasound or EM inverse-scattering or time-reversal imaging experiments.

Finally, a word about the references cited in the book. Originally I intended to include as complete a list as possible of the majority of papers and books by workers in the general field of inverse scattering and wavefield inversion. I soon found the list growing beyond bound and was forced to limit the list to those references that I felt to be directly related to the material presented in the book. The book is mostly about *linearized* formulations of inverse scattering and, thus, I have left out an enormous number of references, especially within the mathematics community, to exact non-linear approaches to inverse scattering. I have also left out virtually all references to applications since the book is about the underlying *theory* of linearized inverse scattering and is not concerned with applications of this theory in various fields such as optics, acoustics, etc. I apologize to the many researchers who may feel slighted by not being included in the bibliography or not being suitably referenced.

I would like to thank my former professor and good friend and colleague over the past (can it be 40?) years Emil Wolf. Much of the material in the book can be traced back to my Ph.D. thesis and to joint papers by Emil and myself. I would also like to thank my colleague of many years' standing Dr. George Sherman and the dozens of current and

former students and colleagues who collaborated on the development and application of the material presented in the book. Special thanks go to my former friend and colleague Alan Witten, who died unexpectedly in 2005. Alan, who was professor of geophysics at the University of Oklahoma, used acoustic diffraction tomography to help find and unearth seismosaurus, the longest dinosaur yet discovered (see *NY Times* "New X-Ray Technique Helps Dinosaur Hunters," Science Section, Dec. 12, 1989), and whose work was, at least partially, the motivation for the opening scenes in the original *Jurassic Park* movie. I would also like to thank Dr. Arje Nachman of the AFOSR and Dr. Richard Albanese, director of the mathematical products division at the Brooks Air Force Base in San Antonio, for financial and inspirational support over the past 20 years. Finally, I must thank Simon Capelin and the wonderful staff at Cambridge University Press. Simon first met me about the book in 1990 in my company office in downtown Boston to discuss the project that I promised would be finished in less than a year.