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978-0-521-11862-0 - Neural Network Learning: Theoretical Foundations

Martin Anthony and Peter L. Bartlett

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