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An Introduction to the Theory of Graph Spectra

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Preface

This book has been written primarily as an introductory text for graduate students interested in algebraic graph theory and related areas. It is also intended to be of use to mathematicians working in graph theory and combinatorics, to chemists who are interested in quantum chemistry, and in part to physicists, computer scientists and electrical engineers using the theory of graph spectra in their work. The book is almost entirely self-contained; only a little familiarity with graph theory and linear algebra is assumed.

In addition to more recent developments, the book includes an up-to-date treatment of most of the topics covered in *Spectra of Graphs* by D. Cvetković, M. Doob and H. Sachs [CvDSa], where spectral graph theory was characterized as follows:

> The theory of graph spectra can, in a way, be considered as an attempt to utilize linear algebra including, in particular, the well-developed theory of matrices, for the purposes of graph theory and its applications. However, that does not mean that the theory of graph spectra can be reduced to the theory of matrices; on the contrary, it has its own characteristic features and specific ways of reasoning fully justifying it to be treated as a theory in its own right.

Spectra of Graphs has been out of print for some years; it first appeared in 1980, with a second edition in 1982 and a Russian edition in 1984. The third English edition appeared in 1995, with new material presented in two Appendices and an additional Bibliography of over 300 items. The original edition summarized almost all results related to the theory of graph spectra published before 1978, with a bibliography of 564 items. A review of results in spectral graph theory which appeared mostly between 1978 and 1984 can be found in *Recent Results in the Theory of Graph Spectra* by D. Cvetković, M. Doob, I. Gutman and A. Torgašev [CvDGT]. This second monograph, published in 1988, contains over 700 further references, reflecting the rapid growth of interest in graph spectra. Today we are witnessing an explosion of the literature on the topic: there exist several thousand papers in mathematics, chemistry, physics, computer science and other scientific areas that develop or use some parts of the theory of graph spectra. Consequently a truly comprehensive text with a complete bibliography is no longer practicable, and we have concentrated on what we see as the central concepts and the most useful techniques.

The monograph [CvDSa] has been used for many years both as an introductory text book and as a reference book. Since it is no longer available, we decided to write a new book which would nowadays be more suitable for both purposes. In this sense, the book is a replacement for [CvDSa]; but it is not a substitute because *Spectra of Graphs* will continue to serve as a reference for more advanced topics not covered here. The content has been influenced by our previous books from the same publisher, namely *Eigenspaces of Graphs* [CvRS2] and *Spectral Generalizations of Line Graphs: on Graphs with Least Eigenvalue* -2 [CvRS7]. Nevertheless, very few sections of the present text are taken from these more specialized sources. For further reading we recommend not only the books mentioned above but also [BroCN], [Big2], [Chu2] and [GoRo].

The spectra considered here are those of the adjacency matrix, the Laplacian, the normalized Laplacian, the signless Laplacian and the Seidel matrix of a finite simple graph. In Chapters 2–6, the emphasis is on the adjacency matrix. In Chapter 1, we introduce the various matrices associated with a graph, together with the notation and terminology used throughout the book. We include proofs of the necessary results in matrix theory usually omitted from a first course on linear algebra, but we assume familiarity with the fundamental concepts of graph theory, and with basic results such as the orthogonal diagonalizability of symmetric matrices with real entries. Chapter 2 is concerned with the effects of constructing new graphs from old, and graph angles are used in place of walk generating functions to provide streamlined proofs of some classical results. Chapter 3 deals with the relations between the spectrum and structure of a graph, while Chapter 4 discusses the extent to which the spectrum can characterize a graph. Chapter 5 explores the relation between structure and just one eigenvalue, a relation made precise by the relatively recent notion of a star complement. Chapter 6 is concerned with spectral techniques used to prove graph-theoretical results which themselves make no reference to eigenvalues. Chapter 7 is devoted to the Laplacian, the normalized Laplacian and the signless Laplacian; here the emphasis is on the Laplacian because the normalized Laplacian is the subject of the monograph *Spectral Graph Theory* by F. R. K. Chung [Chu2], while the theory of the signless

Laplacian is still in its infancy. In Chapter 8 we discuss sundry topics that did not fit readily into earlier sections of the book, and in Chapter 9 we provide a small selection of applications, mostly outwith mathematics.

The tables in the Appendix provide lists of the various spectra, characteristic polynomials and angles of all connected graphs with up to 5 vertices, together with relevant data for connected graphs with 6 vertices, trees with up to 9 vertices, and cubic graphs with up to 12 vertices. We are indebted to M. Lepović for creating the graph catalogues for Tables A1, A3, A4 and A5, and for computing the data. We are grateful to D. Stevanović for the graph diagrams that appear with these tables: they were produced using *Graphviz* (open source graph visualization software developed by AT&T, www.graphviz.org/), in particular, the programs 'circo' (Tables A1,A3,A5) and 'neato' (Table A4). Table A2 is taken from *Eigenspaces of Graphs*.

Chapters 2, 4 and 9 were drafted by D. Cvetković, Chapters 1, 5 and 6 by P. Rowlinson, and Chapters 3, 7 and 8 by S. Simić. However, each of the authors added contributions to all of the chapters, which were then re-written in an effort to refine the text and unify the material. Hence all three authors are collectively responsible for the book. We have endeavoured to find a style that is concise enough to enable the extensive material to be treated in a book of limited size, yet intuitive enough to make the book readily accessible to the intended readership. The choice of consistent notation was a challenge because of conflicts in the 'standard' notation for several of the topics covered; accordingly we hope that readers will understand if their preferred notation has not been used. The proofs of some straightforward results in the text are relegated to the exercises. These appear at the end of the relevant chapter, along with notes which serve as a guide to a bibliography of over 500 selected items.

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