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978-0-521-11782-1 - Variational Principles in Mathematical Physics, Geometry, and Economics:

Qualitative Analysis of Nonlinear Equations and Unilateral Problems

Alexandru Kristaly, Vicentiu D. Radulescu and Csaba Gyorgy Varga

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