AN INTRODUCTION
TO
ANALYTICAL GEOMETRY

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by

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PREFACE

This is an introduction to the use of coordinates and analytical methods in geometry. It is expected that the reader will usually have taken a previous course of elementary calculus and that during that course and from his study of graphs he will have gained some knowledge of rectangular cartesian coordinates.

The early chapters contain numerous exercises suitable for a beginner, so that a previous course of coordinate geometry is not necessary. The book is intended to be easy throughout. For the better students some harder questions are included in the illustrative examples and in the exercises.

The aim has been to introduce a large variety of methods and ideas. The use of parameters, envelope coordinates, and duality is emphasised, and vectors are used when this seems desirable.

The importance of the parabola, ellipse, and hyperbola must be recognised, although it has been exaggerated in the past. The analytical processes that are introduced can profitably be illustrated by applications to other curves as well as the conics, and many of the curves have an interest of their own. It has been found that the necessary knowledge of the conics is not easily acquired from an analytical course alone; therefore in this book some of the important properties are dealt with by pure geometry. Enough about the conics for the beginner is contained in Chapter 6, and it is recommended that chapters 13, 14, 15 should not be taken before any of the previous chapters; these three chapters contain sufficient detail for those who require a treatment of analytical conics on traditional lines.

An attempt is made in Chapter 8 to justify the use of complex coordinates and points at infinity. It is felt that they certainly ought not to be used without some justification, and further that the important idea of an abstract geometry is one which should be presented to ordinary students.

There is nowadays no definite line to be drawn between pure and analytical geometry. Much of the bookwork of what is usually called elementary projective geometry is included in Volume I or Volume II. But the book is intended to be mainly analytical and it will be necessary for the student to supplement his reading by working sets of examples from some text-books on pure geometry on such subjects as cross-ratio, homography, involution, and inversion. Even then he should always feel at liberty to apply analytical methods when they are the most convenient. He needs to learn to choose for himself the most suitable method for particular geometrical problems.

The distinction between metrical and projective geometry has been kept in mind in writing the book, with a view to making it a good introduction to the work which will be done later at the university. It is a distinction which the teacher himself will do well to emphasise in the schools.

My thanks are due to Mr J. C. Manisty for the help he has given me in the production of the book.

A. R.

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CONVENTIONS AND ABBREVIATIONS

The following conventions or abbreviations are used in this book:

0·1. (a, b) means the point whose coordinates are a, b.
(a, b, c) means the point whose coordinates are a, b, c.
P_n means (x_n, y_n) or (x_n, y_n, z_n) according to the context.

0·2. "Line" means "straight line", and lines are supposed to be unlimited except when it is otherwise stated.
$[a, b]$ means the line whose coordinates are a, b.
$[a, b, c]$ means the line whose coordinates are a, b, c.
p_n means $[X_n, Y_n]$ or $[X_n, Y_n, Z_n]$ according to the context.

0·3. If the angle ω between the cartesian axes is relevant, it is assumed to be $\frac{1}{2}\pi$ unless the contrary is stated. When it is to be specially noted that the axes are oblique, the symbol $\{\omega\}$ or $\{\omega = \alpha\}$ is used.

0·41. The word "respectively" is omitted unless the omission is likely to lead to misunderstanding.

0·42. The words "whose equation is" are often omitted in such expressions as "the line whose equation is $ax+by+c = 0$".

0·43. The word "wo" means "with respect to".

0·5. If in an equation $x_1 : x_2 = a_1 : a_2$ it happens that $a_1 = 0$, the equation is taken to mean that $x_1 = 0$; if $a_2 = 0$, it is taken to mean that $x_2 = 0$; if $a_1 = a_2 = 0$, the equation has no meaning.

If $x_1 : x_2 : x_3 = a_1 : a_2 : a_3$ and, for example, $a_3 = 0$, the equation is taken to mean $x_1 : x_2 = a_1 : a_2$ and $x_3 = 0$;

if $a_2 = a_3 = 0$, it is taken to mean that $x_2 = x_3 = 0$;

if $a_1 = a_2 = a_3 = 0$, the equation has no meaning.

Similar conventions are made for equations of the same type with n variables.

0·6. A knowledge of the notation and elementary properties of determinants is assumed.

The determinant $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ which arises in connexion with the quadratic forms

$$ax^2+2hxy+by^2+2gx+2fy+c, \quad ax^2+2hxy+by^2+2gxz+2fyz+cz^2$$

is denoted by δ. Also it is assumed that if

$$A = bc - f^2, \quad B = ca - g^2, \quad C = ab - h^2,$$
$$F = gh - af, \quad G = hf - bg, \quad H = fg - ch,$$

then
$$\delta = Aa + Hh + Gg = Hh + Bb + Ff = Gg + Ff + Cc,$$
$$0 = Ah + Hb + Gf = Ha + Bh + Fg = Ga + Fh + Cg$$
$$0 = Ag + Hf + Gc = Hg + Bf + Fc = Gh + Fb + Cf,$$

and
$$\begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix} = \delta^2 = \Delta.$$

Also
$$BC - F^2 = a\delta, \quad CA - G^2 = b\delta, \quad AB - H^2 = c\delta,$$
$$GH - AF = f\delta, \quad HF - BG = g\delta, \quad FG - CH = h\delta.$$

0·7. In mathematics the phrase "in general" is often used to qualify a statement. It means that the statement is true unless the "constants" involved satisfy some special condition.

For example:

(1) In general $ax + b = 0$ is true for just one value of x.

(2) In general two lines in a plane have just one common point.

In (1) for the special value $a = 0$, $ax + b = 0$ for no value of x unless also $b = 0$ when it is true for all values of x. In (2) if the lines are distinct and parallel, they have no common point; if they are coincident, they have an unlimited number of common points.

The "constants" may be explicit algebraic constants as in (1) or they may be implicit. In (2) the "constants" might be the coefficients in the equations of the lines or they might be implicit in geometrical conditions determining the lines.

0·8. The following abbreviations are used for references in the text:

P.M. *Pure Mathematics*, Hardy (Cambridge).
A.A. *Advanced Algebra*, 3 vols., Durell and Robson (Bell).
A.T. *Advanced Trigonometry*, Durell and Robson (Bell).
E.C. *Elementary Calculus*, 2 vols., Durell and Robson (Bell).
M.G. *Modern Geometry*, Durell (Macmillan).
P.G. *Projective Geometry*, Durell (Macmillan).
H.M. *A Short Account of the History of Mathematics*, W. W. R. Ball (Macmillan).