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# OCEAN ACOUSTIC TOMOGRAPHY

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*To our partners in the original  
ocean acoustic tomography group*

David Behringer  
Theodore Birdsall  
Michael Brown  
Bruce Cornuelle  
Robert Heinmiller  
Robert Knox  
Kurt Metzger  
John Spiesberger  
Robert Spindel  
Doug Webb

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## PREFACE

Over drinks in the Cosmos Club in 1979, Athelstan Spilhaus, who had perfected the bathythermograph for measuring temperature profiles to predict the ranges at which submarines could be detected acoustically, held forth that it should be done the other way around: the measured sonar transmission should serve to determine the ocean temperature field. Unknown to Spilhaus, we were in Washington to persuade the Office of Naval Research and the National Science Foundation to fund an experiment to do just that.

In seismology, the inversion of travel times to map the interior of the Earth has been the time-honored procedure, since the Earth is not readily accessible to direct intrusive measurements. In medicine, intrusive methods are viewed with some reluctance (at least on the part of the patient), and this has led to the development of computed tomographic inverse methods using X-rays. In contrast, the oceans are accessible to direct intrusive measurements; the limits are set by the availability of costly platforms for adequate sampling. Unlike the seismological and medical applications, ocean time variability is an essential component, and the requirements for sampling in space and time are severe. With only a few research vessels plying the world's oceans, it is not surprising that the first century of oceanography had a strong climatological flavor.

It came as a great shock in the 1960s that the oceans, like the atmosphere, had an active *weather* at all depths. The storms within the sea are called eddies. Typical spatial scales are 100 km; time scales are 100 days. Ocean eddies are far more compact and long-lived than their atmospheric counterparts. The intensity of the eddies is such that they contain the predominant fraction of kinetic energy in midocean regions. There is a great difference between an ocean with currents of  $10 \pm 1$  cm/s and an ocean with currents of  $1 \pm 10$  cm/s: With the appreciation of this intensive *mesoscale* field (Russian-speaking scientists refer to it as the *synoptic* scale), it became evident how inadequate the existing observational sampling strategy was, a strategy that had permitted 99% of the kinetic energy

to slip through the grid. The proposal for *Ocean Acoustic Tomography* was a direct consequence of that realization.<sup>1</sup>

The Office of Naval Research of the United States Navy supported our initial research to develop acoustic techniques for ocean monitoring and has strongly supported our work ever since, without attempting to influence the direction of the research; we are particularly indebted to Gordon Hamilton and Hugo Bezdeck for their early encouragement. The National Science Foundation also began supporting us in 1981, and we have continued to enjoy support from both agencies. The work was an informal collaborative effort by scientists from several institutions. This book is dedicated to our partners in the first three-dimensional test of ocean acoustic tomography: D. Behringer, T. Birdsall, M. Brown, B. Cornuelle, R. Heinmiller, R. Knox, K. Metzger, J. Spiesberger, R. Spindel, and D. Webb. In addition, we have worked with many other scientists over the years, all of whom have made significant contributions to the field: S. Flatté, J. Guoliang, B. Howe, J. Lynch, P. Malanotte-Rizzoli, J. Mercer, J. Miller, J. Romm, F. Zachariasen, and B. Zetler. The following have read the manuscript and made many helpful suggestions: B. Cornuelle, J. Colosi, B. Dushaw, M. Dzieciuch, B. Howe, D. Menemenlis, and U. Send. Last, but by no means least, recognition is due the engineers, programmers, and technicians who were responsible for developing the instrumentation and performing the experiments required to test the concepts of *Ocean Acoustic Tomography*.

Elaine Blackmore and Breck Betts have worked with us for many years in preparing this volume; we are deeply indebted to them. K. Rolt has greatly contributed to the final preparation of the manuscript.

We have worked together on acoustic tomography by telemail and with shared enthusiasm, without a professional coordinator. At one stage a reviewer termed our organizational structure a disaster, but gave the proposal his reluctant support when we pointed to forty published papers. It had been intended that the tomography group would disband after a few years, but we are still working together. It is only fitting that this book is dedicated to our partners.

The reader will find a multitude of errors. Please inform Walter Munk, Scripps Institution of Oceanography, UCSD, 9500 Gilman Drive, La Jolla, CA 92093-0225. FAX 619/534-6251 or [wmunk@igpp.ucsd.edu](mailto:wmunk@igpp.ucsd.edu)

<sup>1</sup> Application of tomography has subsequently broadened to include shorter- and longer-scale ocean processes.

## NOTATION

Only the symbols that appear throughout the book are here identified; a notation that is used in one section only is defined locally. Several symbols have different meanings in different chapters. This duplication is impossible to avoid in a subject covering oceanography, acoustics, and inverse methods. When a choice had to be made between an established convention and some degree of ambiguity, our decision was with convention. The notation  $f$ ,  $\mathbf{f}$ ,  $\mathbf{F}$ ,  $\mathbf{F}^T$  refers to scalar, vector, matrix, and matrix transpose representations, respectively, of any quantity  $f$ .

$\mathbf{r} = (x, y, z)$	coordinates, $z$ is upward from the sea surface
$z_B, z_A, z_S$	pertaining to bottom, sound axis, surface
$t, \tau^\pm$	clock time, travel time in direction $\pm x$
$R^\pm, T^\pm, A^\pm$	range, travel time, action for upper/lower ray loop
$r, t, a = n(R, T, A)$	total range, travel time, action for $n$ double loops
$\delta R, \delta T, \delta A$	fractional ray loops
$R = R^+ + R^-$ , etc.	range of double loop
$\theta, \theta = \arctan(m/k)$	inclination of ray, of modal wavenumber
$\Gamma, \Gamma(-)$	ray path, unperturbed ray path
RR, RSR, RBR	refracted refracted, refracted surface-reflected, refracted bottom-reflected
SLR, BLR	surface/bottom-limited ray
$\Delta\tau, \delta\tau$	perturbation in $\tau$ , error in $\tau$
$D\tau = \tau_{n+1} - \tau_n$	interval between ray arrivals
$s, d = \frac{1}{2}(\Delta\tau^+ \pm \Delta\tau^-)$	sum and difference in travel-time perturbations
$C, S = 1/C$	sound-speed, sound-slowness

<p>xiv</p> <p><math>\tilde{C} = C(\tilde{z}^{\pm})</math></p> <p><math>\Delta C, \Delta S</math></p> <p><math>\widehat{\Delta C}, \widehat{\Delta S}</math></p> <p><math>\widehat{\Delta C}(t_0, -)</math></p> <p><math>c_p = \omega/k, c_g = d\omega/dk</math></p> <p><math>s_p = k/\omega, s_g = dk/d\omega</math></p> <p><math>\sigma^2 = (S_0^2 - S^2)/S_0^2</math></p> <p><math>\phi^2 = \sigma^2/(\gamma_a h)</math></p> <p><math>i(m, n)</math></p> <p><math>j</math></p> <p><math>u, v</math></p> <p><math>T, T_p, Sa</math></p> <p><math>N</math></p> <p><math>\gamma_a = 0.0113 \text{ km}^{-1}</math></p> <p><math>k, \ell, m</math></p> <p><math>f, \omega = 2\pi f</math></p> <p><math>a, b, c</math></p> <p>RI, RD, RA, LA</p> <p><b>E</b></p> <p><b>P, P<sub>n</sub></b></p> <p><b>x, y</b></p> <p>BT, XBT, AXBT</p> <p>CTD</p> <p><math>\langle \rangle, \text{rms}</math></p> <p><math>\equiv, \approx</math></p> <p><math>O(\dots)</math></p> <p><math>i</math></p>	<p>NOTATION</p> <p>sound-speed at upper/lower turning depth, etc.</p> <p>perturbations of sound-speed, sound-slowness</p> <p>estimated perturbations</p> <p>estimate prior to measurements at <math>t_0</math></p> <p>phase and group speeds</p> <p>phase and group slowness</p> <p>dimensionless sound-slowness</p> <p>normalized <math>\sigma^2</math></p> <p>acoustic mode number, ray number</p> <p>dynamic mode number, layer number</p> <p>components of particle velocity</p> <p>temperature, potential temperature, salinity</p> <p>buoyancy frequency</p> <p>adiabatic gradient <math>-C^{-1} dC/dz</math></p> <p>modal wavenumbers</p> <p>frequency in cyclical and circular units</p> <p>coefficient in canonical profile (2.5.8)</p> <p>range-independent, range-dependent, range-averaged, loop-averaged</p> <p>observation or design matrix</p> <p>solution uncertainty, solution variance</p> <p>statevector, observation vector</p> <p>bathythermograph, expendable BT, airborne XBT</p> <p>instrument measuring conductivity, temperature, and depth</p> <p>averaging operator, root-mean-square</p> <p>equals by definition, approximately equals</p> <p>order of magnitude ...</p> <p><math>\sqrt{-1}</math></p>
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