

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

525

INDEX OF DEFINITIONS OF THE MORE IMPORTANT SYMBOLS

<i>a</i>	half thickness of parabolic profile (also used with other meanings)	<i>page</i> 153
a	gradient of $-n^2$ or $-q^2$ or X in linear profile	134, 284
B	magnetic induction in wave	15
\mathfrak{B}	magnetic induction of earth's magnetic field	26
\mathfrak{B}	magnitude of \mathfrak{B}	27
b	displacement of origin of height z in Epstein theory	375
C	$\cos \theta_I$, cosine of angle of incidence	85, 121
\mathcal{C}	cylinder function; any solution of Bessel's equation	
c	velocity of electromagnetic waves in free space (also used with other meanings)	18
D	electric displacement in wave (also used with other meanings)	15
D_x, D_y, D_z	components of D	17
D	horizontal range denominator in one form of Appleton–Hartree formula (also used with other meanings)	180 200
E	electric intensity in wave	13
E	magnitude of E	21
E_x, E_y, E_z	components of E	17, 146, 176
E_{\parallel}	component of E parallel to plane of incidence	118
E_L	longitudinal component of E	120
e	charge on the electron (also used for the exponential)	14
$F(q)$	left side of Booker quartic F is also used to denote other functions	122
$\mathfrak{F}, \mathfrak{F}_o, \mathfrak{F}_x$	field variables in Försterling's equations	397
f	frequency (also used with other meanings)	12
$f_H, f_H^{(e)}, f_H^{(i)}$	gyro-frequencies	27, 32
f_N	plasma frequency	25
f_p	penetration frequency	153
H	magnetic intensity in wave	13
H_x, H_y, H_z	components of H	16
\mathcal{H}	$Z_0 \mathbf{H}$, alternative measure of magnetic intensity	20

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

526

INDEX OF SYMBOLS

$\mathcal{H}_x, \mathcal{H}_y, \mathcal{H}_z$	components of \mathcal{H}	page 20
$h, h(f)$	phase height	150
h_0	height of base of ionosphere	150
$h', h'(f)$	equivalent height of reflection	149
\mathcal{I}	imaginary part of	
i	$\sqrt{-1}$	
\mathbf{J}	current density in wave	14
k	$\omega/c = 2\pi/\lambda = 2\pi f/c$ propagation constant in free space	20
k_p	value of k at the penetration frequency	365
l	x -direction cosine of \mathbf{Y} , opposite to earth's magnetic field (also used with other meanings)	27
\mathbf{M}	susceptibility matrix	29
$M_{i,j}$ ($i, j = x, y, z$)	elements of \mathbf{M}	29
\mathcal{M}	ray refractive index	255
m	y -direction cosine of \mathbf{Y} , opposite to earth's magnetic field (also used to denote an integer)	27
m, m_e	mass of electron	24, 32
m_i	mass of ion	32
N	number of electrons per unit volume	4
N_e, N_i	number of electrons and ions, respectively, per unit volume	32
n	z -direction cosine of \mathbf{Y} , opposite to earth's magnetic field (also used to denote an integer, and with other meanings)	27
n	(complex) refractive index	18, 38
n'	(complex) group refractive index	170
\mathbf{P}	electric polarisation	14
P_x, P_y, P_z	components of \mathbf{P}	27
P	phase path (usually for oblique incidence)	173
P'	equivalent path (usually for oblique incidence)	173
Q	effective value of q	344
q	solution of Booker quartic equation (also used with other meanings)	121, 175
\mathcal{R}	real part of	
R	reflection coefficient	ch. 7
R_0, R_1, R_2, R_3	values of R in specified conditions	ch. 7
\mathbf{R}	reflection coefficient matrix	90
$\parallel R_{\parallel}, \parallel R_{\perp}, \perp R_{\parallel}, \perp R_{\perp}$	elements of \mathbf{R}	89
r	used to denote an integer, and with other meanings	
S	$\sin \theta_I$, sine of angle of incidence	85, 121

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

INDEX OF SYMBOLS

527

s	distance along ray path	<i>page</i> 279
	kz ; height measured in units of $\lambda/2\pi$	433
	(also used with other meanings)	
\mathbf{T}	4×4 matrix	389
T_{ij} ($i, j = 1, 2, 3, 4$)	elements of \mathbf{T}	390
$T_{\parallel}, T_{\perp}, T_{\parallel, \perp}, T_{\perp, \parallel}$	transmission coefficients	87, 88
t	time	
T and t	also used with other meanings	
U	$1 - iZ$	26
	group velocity	148
U_z	upward component of group velocity	148
V	wave velocity	38
	(also used with other meanings)	
V_R	ray velocity	255
X	$Ne^2/(\epsilon_0 m \omega^2)$	25
X_e, X_i	X for electrons and ions respectively	31
X_0, X_1	values of X at specified levels	
X	$X_e + X_i$	81
x	Cartesian coordinate	16
\mathbf{Y}	$e\mathfrak{B}/(m\omega)$	27
Y	magnitude of \mathbf{Y}	27
\mathbf{Y}_i	\mathbf{Y} for ions	32
Y_e, Y_i	Y for electrons and ions respectively	31
Y_L	nY , longitudinal component of \mathbf{Y}	49
Y_T	transverse component of \mathbf{Y}	49
y	Cartesian coordinate	16
Z	ν/ω	26
Z_e, Z_i	Z for electrons and ions respectively	80
Z_c	critical value of Z	49
Z_0	(μ_0/ϵ_0) , characteristic impedance of free space	19
z	Cartesian coordinate	17
z_0	level of reflection; (complex) value of z which makes $n = 0$,	136
	or $q = 0$	142
z_P	(complex) value of height z at coupling point	417
α	coefficient of q^4 in Booker quartic	122
	gradient of f_N^2 in linear profile	150
	coefficient of z in exponent, for exponential profile	151
	angle between wave normal and ray	253
	(also used with other meanings)	

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

528

INDEX OF SYMBOLS

β	coefficient of q^3 in Booker quartic (also used with other meanings)	<i>page</i> 122
γ	coefficient of q^2 in Booker quartic arc $\tan \rho_o$ (also used with other meanings)	122 51
δ	coefficient of q in Booker quartic	122
ϵ	coefficient of q^0 in Booker quartic (also used for arbitrarily small quantity, and with other meanings)	122
ϵ_0	electric permittivity of free space	11
$\epsilon_1, \epsilon_2, \epsilon_3$	parameters in Epstein theory	376
ζ	scaled value of height z (the method of scaling depends on the problem) (also used with other meanings)	
Θ	angle between earth's magnetic field and vertical angle between earth's magnetic field and wave normal when wave normal is vertical	119 50
ϑ	angle between earth's magnetic field and wave normal when wave normal is oblique	245
θ	angle between wave normal and vertical, within ionosphere	121
θ_I	angle of incidence; value of θ below ionosphere	97
θ_R, θ_T	angles of reflection and transmission at sharp boundary	97
λ	wavelength in free space	20
λ_p	λ at penetration frequency	365
μ	real part of refractive index	41
μ_o, μ_x	μ for ordinary and extraordinary waves respectively	199
μ'	group refractive index	148
μ'_o, μ'_x	μ' for ordinary and extraordinary waves respectively	201
μ_0	magnetic permittivity of free space	11
ν	collision frequency for electrons	6, 25
ξ	scaled value of height z (similar to ζ)	
Π	Poynting vector	22
$\bar{\Pi}$	average Poynting vector	23
$\bar{\Pi}_x, \bar{\Pi}_y, \bar{\Pi}_z$	components of $\bar{\Pi}$	44
ρ	wave polarisation (used with subscripts to denote polarisation of specified waves)	47
σ	scaling factor in Epstein profiles wave polarisation referred to axes at 45° to magnetic meridian (also used with other meanings)	375 434

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

INDEX OF SYMBOLS

529

ϕ	usually denotes (complex) phase of a wave, or polar co-ordinate angle	
χ	minus imaginary part of refractive index n (also used with other meanings)	page 41
χ_o, χ_x	χ for ordinary and extraordinary waves respectively	199
Ψ	factor of coupling parameter	413
ψ	coupling parameter (also used with other meanings)	397
ω	$2\pi f$; angular frequency	12
ω_N	angular plasma-frequency	25
ω_H	angular gyro-frequency	27
ω_c	critical value of ν	49

NOTES ADDED IN SECOND PRINTING

Note 1. The equivalent height of reflection for a parabolic profile of electron density is given by the expressions (17.54) and (17.56). In the derivation it was assumed that the two asymptotic solutions (17.32) can be taken as the upgoing and downgoing waves. But these solutions should be multiplied by an asymptotic series which was intentionally omitted from (17.32). For the process to be valid it is necessary that the ratio of the second to the first term in the series shall be small at the top and bottom of the layer, which leads to the condition $|\frac{1}{2}n(n-1)/\zeta_a^2| \ll 1$. This is violated when $|D|$ (equation (17.43)) is large, and then the full wave solutions (17.54) (17.56) may not agree with the ray theory solutions (10.33) and (10.37). The discrepancy increases as the difference Δf (17.47), between the penetration frequency and the wave frequency is increased. The author is indebted to Dr. Kenneth Davies for pointing out this possibility.

Note 2. In §23.4 the assumption that the extraordinary wave has polarisation $1/\rho_A$ is valid only when the plane of propagation is the magnetic meridian plane, that is for propagation from magnetic north to south or south to north. The theory for other directions of propagation has been given by Budden, K. G. and Jull, G. W., 1964, *Can. Journ. Phys.*, **42**, 1113.

The author is greatly indebted to numerous colleagues who, in correspondence, reviews or personal discussion, have pointed out errors in the first printing.

SUBJECT AND NAME INDEX

Page numbers in bold type refer to definitions or descriptions.

- Abel, 162, 165
 Abel's integral, **165**
 abnormal component (of reflected wave), **489**
 absorption (of energy in radio wave), 172, 195, 279, 472
 absorption (of sun's radiation), 4
 accumulation of energy, 480
 admittance matrix, **493**, 496
 adjoint matrix, **390**
 aerials, 91, 177, 257, **502**, 506
 Ai, *see* Airy integral function
 Airy, Sir G. B., 290, 313, 512
 Airy integral function, 169, 184, **283** (ch. 15), 287, **290** (fig. 15.5), 291, 303, 309, 311, 316, 320, 409, 410, 440
 Al'pert, Ya. L., 278, 512
 ambiguities in refractive index, 261, 264, 266
 ambiguity (of terms 'ordinary' and 'extraordinary'), 69, **73**, 114, 454
 Ampère's circuital theorem, 16, 96
 amplitude (and imaginary part of phase), 331
 amplitude (complex), **12**
 amplitude-phase diagram, 309, 316
 amplitude, planes of constant, **43**
 analogues of progressive waves, 137
 analytic continuation, 372, 373
 angle of incidence (θ_i), **85**, 97, 225, 259, 283
 angular frequency, **12**
 angular gyro-frequency, **27**
 angular plasma frequency, **25**
 angular spectrum of plane waves, 92, 175
 anisotropic ionosphere, 26, 29, 35 (§3.11), 47 (ch. 5), 59 (ch. 6), 88 (§§7.4–7.7), 112 (§§8.12–8.20), 143, 148, 199 (ch. 12), 225 (ch. 13), 271 (ch. 14), 385 (ch. 18), 412 (ch. 19), 450 (§§20.6–20.8), 458 (ch. 21), 482 (ch. 22)
 anisotropic medium, 39, 42
 antennas, *see* aerials
 anti-Stokes lines, **293**, 304, 307, 311, 350, 355, 361, 441, **452**, 455, 510
 antisymmetric (matrix), 399
 antisymmetric (phase function), 147
 aperture, diffraction by, 168, 309
 apparent bearing, 250
 apparent loss of energy, 479
 Appleton, Sir E. V., xxiii, 8, 25, 26, 27, 94, 173, 191, 194, 209, 512
 Appleton–Hartree formula, **52**, 57, **59** (ch. 6), 112, 122, 123, 144, 200, 233, 246, 256, 272, 274, 275, 397, 402, 417, 418, 438, 440, 450, 460, 472
 approximations, 5, 119, *see also* asymptotic; W.K.B.
 arbitrary constants (Stokes phenomenon), 292, 294
 argument of complex number, 12
 argument of reflection coefficient, 86, 102 (figs. 8.2–8.8)
 asymptotic approximation, 295, **297**, **310**, 313, 320, 355, 360; *see also* W.K.B.
 asymptotic expansions, **310**, 372
 atmospherics, 29, 257, 258
 attachment coefficient, 4
 attenuation, 173, **250**, **280**, 506, 507; *see also* absorption
 audible frequencies, 57, 257
 audio frequency amplifier, 257
 auxiliary sub-routine, 484
 average (over small irregularities), 31
 average (velocity or displacement of electron), 25
 average energy flow, **23**, 44, 247
 average magnetic force on electron, 28
 axes, change of, 50
 axis ratio, **51**, 214
 backscatter, **428**, 500
 Bailey, V. A., 491, 512
 Banerjee, B. K., 29, 512
 band width, 147, 500
 Barber, N. F., 96, 127, 512
 Barnes, E. W., 373, 512
 Barron, D. W., xxiv, 127, 435, 513
 base of ionosphere, 150, 207, 319, 322, 334, 336
 basic equations, 11 (ch. 2)
 bearing error, 250
 Becker, W., 201, 513
 Berkner, L. V., 479, 513
 Bessel functions, 291, 355, 369, 461, 475
 Beynon, W. J. G., 191, 194, 512, 513
 Bi, *see* Airy integral function
 binomial theorem, 132, 143
 bivariate interpolation, 292

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

INDEX

531

- Blair, J. C., 220, 513
 Boardman, E. M., 257, 515
 Booker, H. G., xxiv, 77, 92, 121, 191, 230, 233, 238, 245, 250, 251, 388, 418, 438, 479, 491, 513
 Booker quartic (equation), 117, **120**, **122**, **123**, 140, 144, 225, **226** ff., **233** ff., 246, 248 ff., 271, 274, 387, 389, 400 ff., 408, 410, 438, 439, 497, 506
 boundary conditions, 69, **96**, 99, 106, 111, 115, 118, 321, 335, 422, 427, 428
 boundary condition at infinity, 320, 324, 458
 Bracewell, R. N., 119, 463, 513
 Bradbury, N. E., 8, 513
 Brainerd, J. G., 369, 513, 517
 branch cut, 296, 372, 439
 branch point, 94, 296, 327, 372; *see also* reflection branch point
 Breit, G., 186, 514
 Breit and Tuve's theorem, **186**, 188, 194, 281
 Bremmer, H., 96, 228, 438, 514
 Brewster angle, 94, **101**, 105
 Briggs, B. H., xxiv, 10, 382, 514
 broadening (of pulse), 166, 170, 204
 Brown, J. N., 220, 513
 Browne, J., 504, 514
 Budden, K. G., 31, 96, 104, 119, 120, 127, 402, 408, 411, 435, 438, 459, 472, 485, 488, 491, 509, 513, 514
 Builder, G., 209, 512
 Bullen, K. E., 3, 515
 Burton, E. T., 257, 515
- calculus of variations, 272, 279
 Caminer, D. T., 217
 canonical equations for a ray, 272, **276**, 279, 281, 282
 cardinal points (of compass), 250
 Carson, J. R., 502, 515
 Cartesian coordinates, **16**, 20, 43, **85**, 279, 282, 385
 cathode ray oscillograph, 146, 166, 200, 424
 Cauchy relations, 297
 caustic curve, 184, 191, 313, 318
 cavity definitions of **E**, **D**, 13, 15, 16
 Chapman layer, 3, **5**, 8, 134, 155, 354, 436
 Chapman, S., 3, 515
 characteristic equation, 387, 389
 characteristic impedance of free space, **19**, 26, 492
 characteristic roots, 389, 399
 characteristic waves, 200, 252, 255, **388**, 402, 432; *see also* ordinary, extraordinary
- charge on electron, 14, 26
 Chatterjee, B., 233, 515
 check of numerical methods, 465, 472
 choice of dependent variable, 484
 choice of solution, 320, 324, 330, 355, 360, 369, 461, 470, 474, 477, 486, 487
 choice of starting solution, 483, **487**, 492, 495, 511
 cigar-shaped surface, 262
 circuit elements (electric), 502, 503
 circuit relations, **313**, 353, **369**, 372, 375, 378
 circular polarization, **48**, 60, 69, 116, 200, 214, 460, 461, 506
 circularly polarised components (resolution into), 90, 116
 Clemmow, P. C., xxiv, 92, 252, 396, 398, 402, 408, 411, 485, 513, 514, 515
 col, 298
 collision frequency, 7, **26**, 80, 85, 139, 173, 250, 280, 432
 collision frequency, critical, **50**, **67**, 71, 413
 collisions, 2, **25**, 40, 64, **66**, 69, 70, 113, 142, 195, 224, 248, 265, 280, 326
 collisions and equivalent path, 197
 collisions and equivalent height, 171, 212
 collisions and group refractive index, 170 ff., 204 ff.
 column matrix, 90, 389, 398, 489
 compensating singularities, 291
 complete reciprocity, 506, 508
 complex amplitude, 12
 complex angle, 42, 43, 87, 92, 98, 100
 complex angle of incidence, 87, 98
 complex Brewster angle, 101
 complex conjugate, 22, 30, 37, 231, 388, 393, 507
 complex direction of electric field, 45
 complex Fresnel integral, 308
 complex group refractive index, 170, 204
 complex height, 40, 41, 59, **326**, 413, 438
 complex numbers and harmonic time variation, **12**, 22
 complex phase, **130**, 142, 145, 331
 complex point of reflection, 331
 complex Poynting vector, **23**, 54, 131, 247, 388, 393
 complex refractive index, 38, **41**, 43, 66, 98, 101, 133, 135, 170, 171, 199, 280, 482
 complex values of *X* and *Z*, 40; *see also* complex height; complex *z*-plane
 complex variables, 297
 complex *z*-plane, 136, 171, 212, 285, **326**, 329, 423, 438, 473
 component frequency, 147
 component plane wave, 92, 175, 225, 229, 249

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

532

components of the reflection and transmission coefficient, **88 ff.**, 486, 489
 composition of ionosphere, 1
 Computation Laboratory, Cambridge, Mass., 291, 515
 computing, 205, 482
 condenser, 20
 conduction current, 15, 21
 conductivity of ionosphere, 104
 cone of angles of incidence, 265
 cone of radiation, 146
 confluent hypergeometric functions, 464, 476
 conjugate complex roots of quartic, 231, 233; *see also* complex conjugate
 connection formula, **313**, 353, 356, 374, 453
 constancy of energy flow, 131
 constant collision frequency, 172, 327, 354, 358, 367, 460
 constant electron density, 383, 464
 constitutive relations, 14, **24** (ch. 3), 29, 39, 40, 48, 53, 56, 226, 396, 408, 484
 continuous waves, 499
 contour integral, 136, 142, 171, 174, 212, 288, 297, 302, 309, 330, 372, 423, 437
 contour map, 298
 convex lens, 113
 conversion of ordinary to extraordinary wave, 265
 convolution theorem, 168
 coordinate system, 16, 96, 128, 252
 Copson, E. T., 372, 374, 515
 Cornu spiral, 308, 310, 313, 317
 Coulomb law of force, 11
 coupled equations, 385 (ch. 18), **394**, 398, 401, 412 (ch. 19), 473, 485, 500
 coupled equations, matrix form, 398
 coupling, 63, 144, **385** (ch. 18), 396, 401, 404, **412** (ch. 19), 418, 430, 432, 440, 450
 coupling between upgoing and downgoing waves, 137
 coupling branch points, 440, 442, **450**, 452, 455
 coupling coefficient, 401, 409, 421
 coupling echo, 405, 412, **428**, 500
 coupling level, 144, 387, **412**, 425, 473
 coupling parameter, ψ , **397**, 403, **412**, 417, 427, 432, 436, 473, 485, 487
 coupling point, 413, **417**, 418, 452, 454
 coupling region, 421, 422, 427, 428, 432, 433, 453, 454, 455, 485, 500; *see also* coupling level
 coupling terms, **394**, 399, 401, 409, 410, 434
 Courant, R., 36, 515
 Crary, J. H., 258, 518

INDEX

critical angle, 94, **101**, 105
 critical case (Booker quartic), 238, 264, 268
 critical collision frequency, **50**, 67, 71, **413**
 critical condition (Appleton–Hartree formula), 67, 68
 critical coupling, **413**, 415, 429, 431, 455, 485
 critical coupling frequency, **413**, 430
 critical height, 413
 critical value of Z , **49**, 450
 Crombie, D. D., 96, 127, 512
 Crompton, R. W., 7, 515
 crystalline dielectrics, 36
 crystal optics, 36, 252
 cubic equation, 234, 245, 303
 cubic lattice, 30
 cubic law (of phase near caustic), 315
 cumulative coupling, 412, **433**
 cumulative phase change, 131
 Cunningham, E., xxiv
 curl, 17
 current density, 14, 21
 curvature of earth, 1, 192, 509
 curvature of $h'(f)$ curve, 168
 curvature of $N(z)$ profile, 5, 8, 153, 155, 157, **333**, 367
 curvature of wave front, 91, 175
 cusp, 233, 262
 cut, 296, 372, 439
 cylindrical waves, 314
 Dällenbach, W., 502, 515
 damping of electron motion, **25**, 27, 34, 480
 Darwin, Sir C. G., 30, 515
 Davids, N., 37, 485, 515, 516
 De Groot, W., 163, 516
 derivatives of electron density, 485
 determinant, 338, 342, 387
 determinant form of curl, 17
 diagonal matrix, 36, 398, 497
 dielectric, 14, 33
 dielectric constant, 53, 104, 502, 503
 differential equation for **A**, **494**, 498
 differential equation for limiting polarization, 434
 differential equation for **R**, 127, 491, 501
 differential equation for 'vertical' polarization, **343 ff.**, 349, 510
 differential equation of fourth order, 67, **387**, 469
 differential equation of Riccati type, 132, 394, 435, 492
 differential equations, **128**, 129, **140**, 143, 285, 327, 353, **385**, 458
 differential equations for ray path, 229, 271, **276 ff.**

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

INDEX

533

- differential equations for vertical incidence, 391
- diffraction, 31, 168, 308
- digital computer, 201, 205, 213, 278, 452, 482, 484, 487, 511
- Dingle, R. B., 310, 516
- dipole moment per unit volume, 30
- dipole term (in radiation), 92
- direction cosines of ray, 273
- direction cosines of \mathbf{Y} , 27
- direction cosines of wave normal, 121, 126, 225, 245, 272
- direction finding, 250
- direction of energy flow, 44, 124, 247, 479
- direction of ray, 253, 273
- direction of wave normal, 121, 126, 225, 232, 245, 272
- discontinuity of gradient, 156, 334, 362, 368
- discontinuity of the constants, 292
- discrete strata, 138, 225, 228, 385
- displacement current, 14, 16, 21
- displacement, electric, 15, 20, 24, 386, 397
- distortion of pulses, 168, 170, 204
- diurnal and seasonal variations, 8
- divergent series, 301, 310
- divergence theorem, 22
- D*-layer, 9
- dominant term, 293, 311, 320, 361, 442, 510
- double linear profile, 340 ff.
- double root, 144, 230 ff., 235, 237, 401; *see also* equal roots
- double saddle point, 298
- doubly refracting medium, 2, 98, 225; *see also* anisotropic
- downgoing wave, 52, 124, 231, 393, 406, 442, 489
- D*-region, 9
- drift of electrons, 34
- dynamics, 276
- earth's curvature, 1, 192, 509
- earth's magnetic field, 2, 26, 47 (ch. 5), 112, 116, 143, 199 (ch. 12), 225 (ch. 13), 458 (ch. 21), 466; *see also* anisotropic
- earth's magnetic field horizontal, 115, 123, 395
- earth's magnetic field vertical, 116, 406, 408, 459, 464
- earth's surface, 92, 94, 101
- east-west propagation, 123, 233, 236 ff., 248, 498
- echos, 200
- Eckart, C., 370, 516
- Eckersley, T. L., 257, 331, 410, 437, 438, 516
- EDSAC, xxiv, 201, 213, 452, 511
- effective refractive index, 344, 481
- effective value of q , 344, 467
- Eigen values of matrix, 36, 389
- Eigen vector of matrix, 390, 399
- Eikonal function, 132, 272, 274 ff.
- elastic waves, 3
- E*-layer, 8, 9, 157, 165, 363
- electric displacement, 15, 20, 24, 386, 397
- electric intensity, 13, 24
- electric permittivity of free space, 11
- electric polarisation \mathbf{P} , 14, 24, 29, 30, 386
- electric vector horizontal, 123
- electric vector in plane of incidence, 101 ff., 343 ff., 510
- electromagnetic induction, 16, 38
- electromagnetic units, 12
- electron (number) density, 3, 25, 85, 160 ff., 215 ff.
- electron velocities, 33
- electron waves, 2, 149
- electrons, 209
- electrons, free undamped, 24, 101
- electrons prevented from moving, 466
- electrons, rate of production, 4
- electrostatic units, 12
- ellipse in plane of propagation, 44, 215
- elliptical polarisation, 19, 39, 45, 48, 88, 129, 214, 430, 463, 486, 504
- Ellis, G. R., 426, 516
- energy absorbed, 35, 468, 479
- energy flow, 12, 21, 44, 46, 54, 111, 131, 328, 393, 402, 481
- energy stored, 20, 33, 135
- England, ionosphere over, 120, 218
- envelope of pulse, 166
- envelope of refractive index surface, 266, 270
- Epstein, P. S., 370, 467, 516
- Epstein profile, 378, 446
- Epstein theory, 369 ff., 446, 463
- equal roots of Booker quartic, 230 ff., 401, 409, 418, 439; *see also* double root
- equation of motion of electron, 14, 24, 26, 34
- equation of ray, 177, 178 ff., 276; *see also* ray path
- equation of ray surface, 273
- equation of refractive index surface, 272
- equator (magnetic), 123, 127, 392
- equivalent frequency at vertical incidence, 187, 194
- equivalent height, 94, 149 ff., 167, 171, 172, 205 ff., 212, 215, 223, 365, 382, 424, 428, 499
- equivalent height, exponential profile, 151, 207
- equivalent height, linear profile, 150, 206
- equivalent height, parabolic profile, 152, 209, 365

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

534

equivalent path, 173, **185** ff., 197, 279
E-region, 78
 error (in Abel method), 165
 error elimination, 488
 error integral, 302, 308
 essential singularities, 371
 Euler's equations, 279
 Euler's theorem, 274
 evanescent wave, **41**, 43, 55, 108, 135, 321, 328, 337, 402, 454, 473, 479
 exponential profile, 5, **151**, 172, 180, 207, **354** ff., 379, **460** ff., 488
 extraordinary, 50, **63**, 70 ff., 81, 115, 160, 199, 202, 205 ff., 214, 215, 223, 232, 237 ff., 247, 262 ff., 387, 393, 395, 403, 406, 422 ff., 432 ff., 450 ff., 500, 506
 Fabry–Perot interferometer, 109
 factorial function, 358, 372, 382
 failure of W.K.B. solutions, 133, 136, 141, 322 ff., 329, 410
 Faraday's law, 16, 38, 96
 Farmer, F. T., 173, 512, 516

 Feinstein, J., 348, 516
 Fejer, J. A., 9, 516
 Fermat's principle, 271, **279**, 314
 fictitious homogeneous medium, 53, 495
 finitely conducting earth, 92, 94
 first-order coupled equations, **398** ff., 402, 406, 433
 fixed electron density, 383, 464
F-layer, 8, 157, 165, 220, 258, 363, 364, 424
*F*₁-layer, *F*₂-layer, 8, 160
 flow of energy, 12, **21**, 44, 46, 54, 111, 131, 328, 393, 402, 481
 flux, *see* flow
 force on electron, 24, 26, 28
 formation of layers, 3
 Forsgren, S. K. H., 265, 516
 Försterling, K., 346, 348, 395, 403, 408, 412, 516
 Försterling's coupled equations, 394, **396** ff., 416, 418, 426 ff., 473, 485, 487
 four characteristic waves, 144, 228, **387**, 394, 395, 396
 Fourier analysis, 166, 347
 Fourier integral, 147, 166
 Fourier transforms, 168
 four parts of *h'(f)* curve, 210
 four reflection coefficients, 90
 four steps for finding reflection coefficient, 353
 fourth-order differential equation, 2, 67, **387**, 464, 469, 483
 fourth reflection condition, 472, 481
 free electrons, 24, 101
 free space, 19, 54, 413, 433

INDEX

free space below ionosphere, 320, 413, 432, **433**, 465
F-region, 60, 78
 frequency, 12, 40, 108, 147
 Fresnel diffraction, 168, 309, 314
 Fresnel integral, 308
 Fresnel reflection coefficient matrix, 497
 Fresnel reflection coefficients, **98** ff., 115, 119, 340, 380, 449, 497
 frictionless slope, 162
 Friis, H. T., 502, 522
 full wave theory, 2, 144, 149, 155, 214, 319, 325, 334, 342, **353** (ch. 17), 364, 444, **458** (ch. 21), 474, 485, 508
 functions of mathematical physics, 370, 464
 Furry, W. H., 296, 517
 Furutsu, K., 248, 517

 Gans, R., 129, 517
 Gardner, F. F., 9, 517
 Gauss formula, 213, 482
 Gaussian units, 12, 20
 Gauss's theorem, 16
 general coordinate system, 271, 272, 279
 generalisation of Snell's law, 276, 277
 generalisation of W.K.B. solution, 401
 generalised wave admittance matrix, 491
 geometrical optics 145; *see also* ray theory
 Gerson, N. C., 517
 Gibbons, J. J., 204, 205, 409, 426, 485, 517
 Gibson, G. A., 274, 301, 517
 Gill, S., 484, 511, 517
 Goubau, G., 84, 206, 505, 517
 Goubau's reciprocity theorem, 505, 508
 gradient of collision frequency, 413
 gradient of electron density, 207, 413
 Gray, H. J., 369, 517
 great heights, 324, 353, 355, 461, 470, 486, 487, 495
 greater than critical, **429**, 456
 ground wave, 1, 94
 group refractive index, **148**, 153, 160, 170, 187, **200** ff., 204, 205, 212, 217, 220, 223, 254, 280, 482
 group retardation, 157, 158, 212, 365
 group velocity, **147**, 155, 158, 170, 187, 200, 254, 279, 480
 group velocity surface, **254**, 257
 guided waves, 94, 101, 257
 Guillemin, E. A., 502, 517
 gyrator, 503
 gyro-frequency, **27**, 32, 69, 160, 206 ff., 223, 247, 256, 462

 half thickness (semi-thickness), 5, **153**, 157, 191, 194, 211, 214
 Hamilton's canonical equations, 276

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

INDEX

355

- Hankel functions, 291, 355, 461, 475
 Harang, L., 424, 517
 harmonics, 347, 481
 harmonic waves, 12, 21
 Hartree, D. R., xxiv, 58, 336, 340, 342, 459, 482, 517
 Haselgrove, J., 218, 271, 278–81, 518, 523
 Heading, J., 182, 356, 396, 398, 408, 438, 442, 445, 459, 464, 469, 471, 472, 477, 515, 518
 Heading and Whipple's method, 459, 464
 Heading's rule, 442
 heat, 21, 35, 351, 468, 480
 heavy ions, 1, 31, 56, 78 ff., 258, 270
 height, *see* equivalent height, phase height, true height, Cartesian co-ordinates
 height as complex variable, 40, 41, 59, 326, 413, 438
 Helliwell, R. A., 258, 518
 helical path, 27
 Hermitian matrix, 30, 36, 390
 Hermitian orthogonal, 37
 Hertzian dipole, 92, 504
 Hertz vector, 92
 high frequencies, 60, 221
 Hilbert, D., 36, 515
 Hilbert space, 36, 37
 Hines, C. O., 248, 258, 270, 491, 518
 homogeneous functions, 274
 homogeneous medium, 6, 17, 19, 38 (ch. 4), 47 (ch. 5), 108, 131, 214, 256, 495
 horizontal dipole aerials, 508
 horizontal direction of transmission path, 120, 250
 horizontally stratified, 10, 85, 128, 277
 horizontal polarisation, 99, 105, 140, 285, 319 (ch. 16), 336, 340, 376, 440, 493
 horizontal range, 180, 181, 182, 192, 195, 281
 horizontal ray, 230 ff., 260
 horizontal variations, 10, 191
 Houston, R. A., 111, 518
 Huxley, L. G. H., 7, 26, 502, 515, 518
 Huyghens's principle, 313, 318
 hyperbolic secant, *see* sech
 hypergeometric equation, 370, 375, 377
 hypergeometric function, 370, 372 ff.; *see also* confluent hypergeometric functions
 hypergeometric series, 371, 374
 image of transmitter, 93
 imaginary part of q , 124, 196, 251
 imaginary part of refractive index, 40 ff., 66 ff., 173, 280
 imaginary refractive index, 41, 43, 55, 102, 108
 impedance, 18, 388, 491
 impulse signal, 146, 166, 258
 inclination of ray to vertical, 246, 258 ff.
 independence of characteristic waves, 395, 398, 416, 418, 432
 independent second-order equations, 129, 140, 460, 471
 independent solutions, 491
 India, 425
 indicial equation, 346, 370, 371
 induction, electric, *see* electric displacement
 induction, electromagnetic, 16, 38
 infinite equivalent height, 157, 159, 160, 163, 206
 infinite fields, vertical polarisation, 347
 infinite group refractive index, 201, 202
 infinite refractive index, 59, 60, 61, 62, 64, 79, 82, 203, 206, 447, 468, 472 ff., 479
 infinite root of Booker quartic, 123, 233, 285, 410
 infinity and zero of refractive index, 476
 inhomogeneous plane waves, 42 ff., 45, 87, 92, 98, 101, 106
 initial conditions, 434; *see also* starting solutions
 integral equation, 160 ff., 215 ff.
 integrals of Barnes type, 373, 470
 integration, 213, 300, 307, 483
 intermediate inclination of earth's field, 61, 65
 intrinsic impedance, 390
 inverse square law, 16
 inversion of integral equation, 161 ff., 215 ff., 482
 inversion of matrix, 29, 490
 ions, 1, 31, 56, 78 ff., 258, 270
 irregularities of electron density, 10, 31, 191, 426, 428, 436
 isolated infinity of refractive index, 474
 isolated zero of n or q , 283, 285, 334
 isotropic medium, 17, 38 (ch. 4), 53, 98 ff., 252, 471 (chs. 16, 17), 491 ff., 501, 506
 Jackson, J. E., 165, 522
 Jeffreys, Sir H., 3, 313, 518
 Jeffreys, Sir H. and Lady B. S., xxiii, 92, 129, 287, 290, 297, 310, 312, 370, 372, 519
 Johnson, W. C., 436, 520
 Jones, R. E., 519
 Keitel, G. H., 484, 519
 Kelso, J. M., 405, 426, 428, 485, 519
 Kimura, I., 258, 278, 519
 kinetic energy of electrons, 33, 34
 King, G. A. M., 221, 519
 kinks in refractive index curves, 72 ff.

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

536

Landmark, B., 431, 436, 519
 Lange-Hesse, G., 37, 519
 Langer, R. E., 313, 334, 519
 Lassen, H., 516
 lateral deviation, 94, 178, 229, 231, 233, 238, 246 ff., 261, 479
 latitude variations of N , 247
 ledge, 158, 212
 left-handed circular polarisation, 48, 91, 460, 461
 Leo Computers Ltd., 217
 Lepechinsky, D., 431, 519
 less than critical, 429, 456
 level line, 297, 307, 308
 level of coupling, 144, 387, 412, 425, 473
 level of reflection, 137, 143, 144, 149, 153, 176, 187, 196, 206, 215, 230 ff., 237, 260, 261, 265, 320, 333, 339, 385, 387, 409, 488
 Lewis, R. P. W., 519
 Lied, F., 431, 519
 lightning flash, 29, 258
 limiting points of spiral, 309, 317
 limiting polarisation, 200, 412, 414, 416, 432 ff., 507
 limiting region, 433 ff.
 Lindquist, R., 405, 519
 linear and homogeneous equations, 387, 487
 linear electric circuit, 502
 linear profile, 5, 134, 150, 172, 179, 188, 206, 213, 283 (ch. 15), 319 (ch. 16), 336, 340, 440, 474
 linear polarisation, 18, 28, 39, 44, 48, 57, 70, 77, 90, 112, 129, 137, 200, 214, 462, 466, 489
 linear time base, 146, 166
 lines of force (earth's magnetic), 257
 lines of steepest descent, 297 ff.
 logarithmic term in E_x , 347
 logarithm of frequency, 221
 longitudinal component of electric field, 18, 38, 53, 67, 70, 77, 78, 117, 120, 125, 215; *see also* vertical component of electric field
 longitudinal component of \mathbf{Y} , 49, 117, 126
 longitudinal curves (Booker quartic), 245
 longitudinal propagation, 54, 60, 64, 69, 79, 80, 245, 264, 266, 460 ff.
 long whistler, 258
 Lorentz, H. A., 2, 30, 60, 519
 Lorentz polarisation term, 30
 loss-free medium, 42, 43, 111, 131, 390, 392
 lower triangular matrix, 216
 lowest ionosphere, 1, 9, 80, 354, 358, 412, 432 ff., 455, 465

INDEX

low frequencies, 64, 145, 333, 353 (ch. 17), 354, 458 (ch. 21), 462; *see also* very low frequencies
 Maeda, K., 258, 278, 519
 magnetic energy, 21
 magnetic equator, 123, 127, 233, 392, 406, 408
 magnetic field of earth, 2, 26, 47 (ch. 5), 112, 116, 143, 199 (ch. 12), 225 (ch. 13), 458 (ch. 21), 466; *see also* anisotropic magnetic field of wave, 28, 44, 131, 134, 320, 328
 magnetic induction, 15
 magnetic induction of earth's field, 26
 magnetic intensity, 11, 12, 13, 20
 magnetic meridian, 47, 51, 119, 215, 231, 246, 260 ff., 277, 406, 499, 508
 magnetic permeability, 15, 53, 99, 127, 502, 503
 magnetic permittivity, 11
 magnetic pole, 247, 392
 magnetic rotation, 58, 481 (Ex.)
 magnetoionic medium, 19, 38, 54, 472, 481, 502; *see also* anisotropic magnetoionic splitting, 199
 magnetoionic theory, 25, 26, 47 (ch. 5), 59 (ch. 6), 121, 214, 392, 396, 432, 507
 Manning, L. A., 161, 519
 Marconi's Wireless Telegraph Co., Ltd., 224
 Marcou, R. J., 278, 519
 Martyn, D. F., 188, 197, 520
 Martyn's theorem for absorption, 188, 195 ff.
 Martyn's theorem for equivalent path, 186 ff., 194, 281
 mass of electron, 24, 32
 Mathieu's equation, 369
 matrix for inverting integral equation, 216 ff.
 matrix form of differential equations, 389, 398, 494, 498
 matrix form of Maxwell's equations, 226
 matrix, reflection coefficient, 90 ff., 470, 489 ff., 496, 500, 505 ff.
 matrix, susceptibility, 27, 29, 32, 36, 227, 386, 391, 459
 maximum in $h'(f)$ curve, 159
 maximum of electron density, 5, 6, 8, 9, 153, 157, 163, 353, 358, 381, 443; *see also* penetration frequency
 maximum usable frequency, 190 ff., 281
 Maxwell's equations, 13, 16, 17, 20, 22, 31, 38, 40, 42, 46, 48, 52, 94, 125, 127, 128, 133, 137, 140, 141, 143, 226, 274, 275, 320, 385, 408, 419, 484

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

INDEX

537

- mechanical model, 163
 Meek, J. H., 424, 520
 Mervin, R., 369, 517
 metal sheet, 93
 Miller, J. C. P., 287, 291, 310, 312, 339, 520
 Millington, G., xxiv, 224, 233, 247, 249, 250, 491, 516, 520
 Milne, W. E., 484, 520
 m.k.s. units, 11, 491
 modified Hankel functions, 291
 modulus of reflection coefficient, 86, 87, 102 ff., 136, 173, 196, 197, 321, 329, 342, 351, 357, 363, 369, 380 ff., 437, 445, 461 ff., 478, 507
 monotonic profile, 161, 215
 Morgan, M. G., 436, 520
 movements of ionosphere, 10, 436
 M.U.F. *see* maximum usable frequency
 Mullaly, R. F., 252, 515
 multiple reflections, 109, 192
 Musgrave, M. J. P., 3, 520
- Namba, S., 96, 524
 National Bureau of Standards, xxiv, 195, 520
 Nearhoof, H. J., 405, 426, 485, 519
 Nertney, R. J., 405, 409, 426, 485, 516, 519, 520
 Newbern Smith, 194, 195, 281, 522
 Newstead, G., 424, 520
 Newton's laws of motion, 24
 Nicolet, M., 7, 520
 non-linear differential equation(s), 132, 142, 492, 494, 498, 501
 non-singular matrix, 490, 494
 normal component, **489**
 normal form of differential equation, **343**, 349, 376, 427
 normal incidence, 89, 108, 112, 115; *see also* vertical incidence
 normal modes, xxiii, 438
 normal to ray surface, 255, 273
 normal to refractive index surface, 253, 254, 255, 273
 northern hemisphere, 52, 67, 69, 116, 119, 247
 north-south propagation, 124 ff., 238 ff., 246, 250, 260 ff., 471, 508
 nose whistlers, 258
 numerical methods, 127, 215, 278, 369, 482 (ch. 22)
- oblique incidence, 86, 101 ff., 116, 120 ff., 138 ff., 175 (ch. 11), 225 (ch. 13), 283, 285, 319, 354, 406, 408, 464 ff.
- one magnetoionic component (reciprocity), 506
 one magnetoionic component, **R** for, 500
 optics, 109, 111, 168, 252, 308, 313 ff.
 ordinary, 50, **63**, 70 ff., 81, 115, 199, 202, 205 ff., 214, 215, 218 ff., 232, 237 ff., 247, 259 ff., 268 ff., 281, 387, 392, 395, 403, 406, 422 ff., 432 ff., 450 ff., 476, 500, 506
 ordinary point of differential equation, 286, 419, 447
 orthogonal, 36, 37
 oscillating function, 112, 138, 288
 oscillatory reflection coefficient, 500
 Ott, H., 94, 520
- parabolic cylinder function, $D_n(\zeta)$, 311, 359, 361, 367, 443
 parabolic profile, 5, 8, 152, 157, 182, 188, 191, 194, 209 ff., 214, 281, 285, 358 ff., 381, 382, 464
 parabolic ray path, 180
 parallel-plate condenser, 20
 parallel-sided slab, 108 ff.
 Parkinson, R. W., 37, 485, 516, 520
 partial fields, 394
 partial penetration and reflection, 113, 138, 145, 225, 285, 342, 353, **363**, 369, 382, 500
 partial standing wave, **19**, 39
 particle, motion of, 162
 particle velocity, 149
 partitioning of matrix, 490, 494
 passive electric circuit, 502
 passive medium, **41**, 66
 path of wave packet, 178, 196, 204, **229**, 248, 250; *see also* ray, ray path
 Pawsey, J. L., 9, 517
 Pedersen, P. O., 101, 521
 Pedersen ray, 183, 190
 Pekeris, C. L., 521
 pencil of radiation, 177
 penetration, 157, 160, 363, 445
 penetration frequency, 8, 113, 145, 149, **153** ff., 163 ff., 172, 182, 190, 191, **208**, 214, 220, 247, 342, 364, 382, 424, 444
 permeability, 15, 53, 99, 127, 502, 503
 permittivity of free space, 11
 Perry, L. B., 104, 119, 523
 Pfister, W., 278, 425, 464, 519, 521
 phase, **12**, 111, 130, 504; *see also* complex phase
 phase advance on reflection, 326
 phase change on reflection, 102 ff., 108, 380
 phase difference between electric and magnetic fields, 42
 phase height, **150** ff., 167, 171 ff., 205, 212, 365

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

538

phase integral, 136, 142, 150; *see also* phase memory
 phase integral formula for coupling, 410, 423 ff., 431, 452 ff.
 phase integral formula for reflection, 136, 171, 213, 329 ff., 331, 334, 336, 348, 350, 357, 368, 409, 416, 430, 442, 446
 phase integral method, 136, 437 (ch. 20), 452; *see also* phase integral formula
 phase memory, 130, 144, 147, 199, 228, 275, 392
 phase path, 173, 197, 507
 phase, planes of constant, 43, 46, 326
 phase velocity, 255, 466
 physical optics, 109, 111, 168, 308, 313 ff.
 Piggott, W. R., 165, 521
 Pitteway, M. L. V., xxiv, 235, 521
 plane polarised, *see* linear polarisation
 planes of constant amplitude, 43, 46
 planes of constant phase, 43, 46, 326
 planes of incidence, 85, 88, 114, 231, 260
 plane waves, 1, 17, 19, 92, 225, 274
 plasma frequency, 25, 40, 113, 147, 152, 158, 160, 179, 180, 186, 205, 212, 216, 221, 223
 plasma oscillations, 45
 plate-like cavity, 15
 Poeverlein, H., xxiv, 258, 262, 521
 Poeverlein's construction, 258 ff.
 Poincaré, H., 310, 311, 355, 361, 521
 point of inflection (q curves), 244
 point source, 91 ff., 175
 polar coordinates, 92, 252, 278, 282
 polar diagram, 281
 polarisation, electric, 14, 24 ff., 32, 36, 227, 386
 polarisation ellipse, 19, 45, 49, 51 ff., 215, 463
 polarisation equation, 48 ff., 57, 78, 214, 403
 polarisation term, 30
 polarisation, wave, 14, 19, 45, 47 (ch. 5), 59 (ch. 6), 114 ff., 124 ff., 199, 214, 230, 248, 388 ff., 396, 424, 430, 432 ff., 456, 458, 462, 487, 500, 504 ff.; *see also* limiting polarisation
 pole of refractive index, 476; *see also* infinite refractive index, infinite root
 pole of ψ , 417
 Pope, J. H., 258, 518
 potential field (electron waves in), 2, 149
 potential function, 299
 Poynting's theorem, 21
 Poynting vector, 13, 22, 42, 44, 46, 54, 131, 135, 247, 248, 388, 393, 402
 precursor, 169
 prediction of M.U.F., 191, 194, 281
 predominant frequency, 147, 149, 150, 166 ff., 169

INDEX

predominant plane wave, 93
 predominant values, 177, 178, 229, 246, 253, 256
 pressure of radiation, 28
 principal axes, 35 ff.
 principal axis components, 37, 485
 principal terms, 394, 397, 399, 409, 410
 principal value of integral, 166
 probability, 25
 progressive (plane) waves, 17 ff., 38, 41, 44, 45, 48, 67, 108, 129 ff., 137, 392, 402
 projected path, 249, 251
 projected thickness, 338, 342
 propagation constant, k , 20, 177, 438
 propagation to great distances, xxiii, 104, 191 ff., 438
 pulsating energy flow, 42, 135
 pulses, 146, 150, 155, 163, 166 ff., 177, 199, 200, 205, 424, 499, 506
 pulse shape, 147, 166 ff.
 purely imaginary refractive index, 41, 43, 55, 102, 108
 q , 121 ff., 141, 144, 175 ff., 225 ff., 319 ff. (chs. 16, 17), 387, 398 ff., 438 ff.
 quadratic equation for polarisation, 48 ff., 57, 78, 214, 403, 417
 quadratic equation for q^2 , 123, 233, 236 ff., 248, 402, 407
 quadrupole terms, 92
 quantum mechanics, 36
 quartic equation, 117, 120, 122, 123, 140, 144, 225, 226 ff., 233 ff., 246, 248 ff., 271, 274, 387, 389, 400 ff., 408, 410, 438, 439, 497, 506
 quasi-Brewster angle, 103
 quasi-longitudinal, 76, 117 ff., 429
 quasi-transverse, 77, 429
 radar, 146
 radiation links (and reciprocity), 502, 503
 radiation pressure, 28
 radio sounding, 146, 160
 random distribution of electrons, 30
 random velocities of electrons, 24, 25
 range, 180, 181, 182, 192, 195, 281
 Rao, R., 204, 205, 517
 Ratcliffe, J. A., xxiv, 10, 26, 59, 77, 163, 173, 191, 512, 513, 516, 521
 rate of production of electrons, 4
 rationalised units, 11
 Rawer, K., 8, 375, 381, 382, 383, 521
 ray, 124, 145, 177, 178, 179, 190, 197, 253, 255, 258 ff.
 ray path, 179 ff., 190, 230 ff., 245, 247, 255, 258 ff., 271 (ch. 14)
 ray path horizontal, 230

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

INDEX

539

- ray path in magnetic meridian plane, 238 ff., 260 ff.
- ray refractive index, **255**, 272, 273, 279
- ray space, 273
- ray surface, **255**, 256, 261, 263, 265, 271, 273, 274, 278
- ray theory, 2, **144**, **146** (ch. 10), **175** (ch. 11), **199** (ch. 12), **225** (ch. 13), 271 (ch. 14), 364, 383, 437
- ray tracing, 94, 177, 258 ff., **271** (ch. 14), 278
- ray velocity, **255**, 271, 273
- receiving aerial, 503
- reciprocal property of surfaces, 255, 256, 274
- reciprocity, 127, 230, 252, 502 (ch. 23)
- reciprocity in phase, 504
- reciprocity theorem, 499, **502**
- reciprocity with full wave solutions, 508
- recombination coefficient, 4, 5
- rectangular pulse, 168
- reference level for reflection and transmission coefficients, **85** ff., 93, 467, 470, 471
- reflection at sharp boundary, 96 (ch. 8), 340, 380, 449, 497, 509
- reflection at vertical (or normal) incidence, 85, 108, 112, 115, 146 (ch. 10), 199 (ch. 12), 459 ff., 471
- reflection (branch) point, 144, 327, 330, 401, 418, **438** ff., 443, 450
- reflection coefficient(s), **85** (ch. 7), 98 ff., 136, 141, 173, 196, 197, 283, 319 (ch. 16), 353 (ch. 17), 437, 444 ff., 449, 458, 461 ff., 467 ff., 478
- reflection coefficient matrix, **90** ff., 470, 489 ff., 496, 500, 505 ff.
- reflection level, 137, 143, 144, 149, 153, 176, 187, 196, 206, 215, **230** ff., 237, 260, 261, 265, 320, 333, 339, 385, 387, 409, 488
- reflection of pulse, 149, 150, 166 ff., 205
- reflection process, 129, 134, 139, 231, 283, 396, 404, 418
- refractive index space, **272**
- refractive index surface, **252**, 254 ff., 271 ff., 278
- refractive index (wave), 3, 18, 28, **38** (ch. 4), 52, 56, **59** (ch. 6), 97 (ch. 8), 147, 148, 172, 199 ff., 391 ff., 397, 414 ff., 430, 434, 438, 450 ff., 460, 466, 472 ff., 491; *see also* group refractive index
- region I, 465, 467, 471
- region I(a), 465, 466
- region II, 465, 468 ff.
- regular singularities, 370
- rejects, **504**, 506
- relativistic effects neglected, 27
- removal of electrons, 4
- residuals, 218, **220**, 223
- retardation, group, 157, 158, 212, 365
- retarding force on electron, 2, 25, 26
- reversal of sense (wave polarisation), 116
- reversibility of ray path, **230**, 252, 506
- reversion of series, **301**, 308
- Ricatti-type equation, 132, 394, 435, 492
- Riemann surface, **438** ff., 450
- right-handed circular polarisation, 48, 90, 460, 461
- right-handed system, 19, 50, 504
- Robbins, A., 218, 523
- roots of Booker quartic, **121** ff., 144, **225** ff., 387, 398 ff.
- rotation in complex z -plane, 327
- rotation, magnetic, 58, 481 (Ex.)
- rotation of axes, 36, 50, 434
- Runge-Kutta process, 483, 511
- Rydbeck, O. E. H., 96, 168, 169, 363, 365, 366, 409, 424, 426, 445, 446, 522
- saddle point, **298** ff.
- scale height, 4, 7, 8, 134, 154, 436
- scattering, 58, 191, 426, 428
- Schelkunoff, S. A., 491, 501, 502, 522
- Scott, J. C. W., 248, 522
- Seaton, S. L., 191, 513, 517
- sech² profile, 154, 156, 380 ff.
- second-order coupled equations, 391, 397, **408**, 411, 426, 469, 485
- Seddon, J. C., 165, 522
- seismology, 3
- semi-thickness (half thickness), 5, **153**, 157, 191, 194, 211, 214
- separation into upgoing and downgoing, 137, 486, 489 ff.
- separation of differential equations, 18, 129, 140, 460, 465
- series solution of differential equation, 287, 339, 346, 356, 370 ff., 419
- shape of pulse, 147, **166** ff.
- sharp gradient of refractive index, 134, 473
- sharply bounded anisotropic medium, 68, 114 ff., 497
- sharply bounded homogeneous medium, 86, 96 (ch. 5), 134, 340, 380, 449
- Shinn, D. H., xxiv, 200, 201, 206, 207, 224, 522
- Shire, E. S., 12, 28, 522
- short whistler, 258
- side-band frequencies, 169
- sign convention for reflection coefficients, **88**, 470, 490, 505, 507
- sign convention for square roots, 403
- sign of electronic charge, 25, 27
- silvered glass, 127 (Ex.)

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

540

INDEX

- simultaneous differential equations, 483 ff.
singular matrix, 94 (Ex.), 500
singularities of refractive index, 447, 450, 451, 476
singular point, singularity, 291, **299**, 344
singularities of ψ , 417
sink of energy, 480
sinusoidal layer, 368
size of step, 484
skip distance, **183**, 190, 281, 318
slab model of ionosphere, 108 ff.
slab of ionised medium, 45
slit, diffraction by, 168
Slough, 218
slowly varying function, 94, 176
slowly varying medium, 128 ff., **133**, 144, 228, 252, 256, 259, 271, 275, 322, 329, 358, 413, 417, 420, 479, 506
small irregularities, 31
Smith, N., 194, 195, 281, 522
Smith, R. L., 258, 518
smoothing out, 13, 31
Snell's law, 97 ff., 114, 116, 119, 121, 139, 179, 226, 259, 277, 487
Sommerfeld, A., 502, 522, 523
sound waves, 3
source, dimensions of, 2, 91, 175, 177, 504
southern hemisphere, 247
south-north propagation, 124 ff., 238 ff., 246, 250, 260 ff., 471, 508
space charge, 16
space charge waves, 24
spectrum function $F(f)$, 147, 166 ff.
specular reflection, 96 (ch. 8), 509; *see also* reflection at sharp boundary
Spencer, M., 10, 514
spherical polar coordinates, 92, 252, 278, 282
spherical waves, 1, 91 ff., 146, 175 ff., 509
spiral, 309, 316, 317
spitze, 236, **260** ff., 266 ff.
splitting, **199**, 232, 253, 424
square law increase (profile), **366** ff., 464
square root in Appleton-Hartree formula, 67, 201, 417, 450
square root, sign convention, 403
standing wave, **19**
Stanley, J. P., 463, 523, 524
starting solutions, **486**, 487, 495, 511; *see also* initial conditions
statistical mechanics, 25
stationary phase, 92, 93, 148, 149, 170, 171, 176, 178, 185, 204, 253, **307** ff.
stationary time, 271, 279, 314
steepest descents, **297** ff., **300** ff., 470
steep gradient of electron density, 134, 425, 473
step-by-step process for integration, **483**, 486
step-size, 484
Stirling's formula, 358, 365, 368, 382, 449
Stokes, Sir G. G., 293, 523
Stokes constant, **295**, 306, 313, 325, 348 ff., 356, 362, 437, 446, 510
Stokes diagram, **294**, 307, 312, 356, 361, 441, 442, 453 ff.
Stokes (differential) equation, **286** ff., 302, 312, 319 (ch. 16), 371, 409, 420, 447, 450
Stokes lines, **293** ff., 296, 305, 307, 311, 312, 350, 355, 359 ff., 440, 450 ff., 510
Stokes phenomenon, 283 (ch. 15), **292**, 302, 310, 372, 440, 452, 511
stored energy, 20, **33**, 135
Storey, L. R. O., 252, 255, 257, 523
Straker, T. W., 513
strata, 138, 225, 228, 385
Stratton, J. A., 92, 170, 204, 365, 523
strong coupling, 409; *see also* critical coupling, cumulative coupling
structure of the ionosphere, 7 ff.
subdominant term, **293**, 311, 320, 324, 339, 350, 355, 442, 454, 456, 510
successive approximations, 137, 395, 409, **421**, 426, 485
Suchy, K., 275, 523
summation convention, 389, 399
surface integral, 22
surfaces of revolution, 252 ff.
susceptibility matrix, 27, **29**, 32, 36, 227, 386, 391, 459
Sutton, D. J., 7, 515
symmetrical ionosphere, 340; *see also* parabolic profile, sech^2 profile, sinusoidal layer
symmetric (amplitude function), 147
symmetric matrix, 36, 498, 499
symmetric tensor, 502
symmetry properties of equations, 499
table for $z(f_N)$ from $h'(f)$, 218
tables of Airy-integral functions, 291
tail (of pulse), 169
Taylor, M., 523
Taylor series, 167, 298, 301, 308, 349, 417, 419
temperate latitudes, 425
thermal motions, 24
thickness of ionosphere, layer, 363, 382, 445
Thomas, J. O., xxiv, 218, 523
three equal roots, 238
tilt angle, **51**, 52
time average of energy flow, **23**, 44, 247

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

INDEX

541

- time of travel of pulse or wave packet, 149, 167, 170 ff., 205; *see also* equivalent height, equivalent path
- time of travel of wave crest or wave front, 279; *see also* phase height, phase path
- Titheridge, J. E., 224, 523
- top of trajectory, 180, 230 ff.
- Toshniwal, G. R., 424, 425, 523
- total current density, **15**, 16
- total reflection, 102, 107, 380
- trains of whistlers, 258
- transition through critical coupling, 429, 455
- transmission coefficient, **85** (ch. 7), 98 ff., 342, 362 ff., 369, 377 ff., 381, 467, 478
- transmitter, 28, 146, 167, 175 ff., 229
- transmitting aerial, 91, 175 ff., 229, 253, 502 ff.
- transpose of matrix, 399, 499
- transverse components of **Y**, **49**, 126
- transverse curves (Booker quartic), 245
- transverse field components, 17, 18, 38, 39, 47, 77, 504
- transverse propagation, 54, **61**, **65**, **70**, 79, 81; *see also* east-west propagation
- triangulated path, **186**, 187
- true bearing, 250
- true height, **150**, 152, 154, 157, 205
- truncation errors, 484
- Tuve, M. A., 186, 514
- two separate parabolic layers, 157, 210 ff.
- two transmitted waves, 116 ff.
- Ulwick, J. C., 278, 519
- unitary matrix, 37, 91
- unitary transformation, 36, 91
- units, 11
- University Mathematical Laboratory, Cambridge, xxiv, 201, 213, 511
- upgoing wave, 122, 124, 231, 393, 405, 406, 442, 486 ff., 492, 495
- upward velocity of pulse, 148, 200; *see also* group velocity
- validity of W.K.B. solutions, **133**, 141, 143, **410**
- variation of parameters, **426**
- variations of path, 271, **279**
- varying collision frequency, 383, 457 (Ex.), 464
- vector diagram, 308, 309, 316
- vector, refractive index as, 121, 272, **276**
- vertical component of electric field, 347, 466; *see also* longitudinal component of electric field
- vertical component of group velocity, 148, 200, 480
- vertical dipole, 508
- vertical incidence, 85, 114, 123, 128 ff., 146 (ch. 10), 299 (ch. 12), 233, 246, 365, 384, 395 ff., 402 ff., 412 (ch. 19), 438, 446, 459 ff., 473 ff., 485, 495; *see also* normal incidence
- vertical magnetic field, 116, 406 ff., 411 (Ex.), 459 ff., 464 ff.
- vertical motions of electrons, 347, 466
- vertical polarisation, 99, 101 ff., **140**, 142, 286, **343** ff., 510
- vertical tangents to q curves, 238; *see also* equal roots
- very low frequencies, 9, 57, 103, 117, 119, 145, 220, 256, 353 (ch. 17), 389, 458 (ch. 21), 463, 487; *see also* low frequencies
- Vice, R. W., 9, 516
- Wait, J. R., 104, 119, 523
- Walkinshaw, W., 438, 513
- Watson, G. N., 161, 291, 346, 355, 361, 369, 370, 372, 374, 438, 475, 476, 478, 524
- Watson's lemma, 310
- Watts, J. M., 220, 513
- wave admittance, 485, 486, 488, **491** ff.
- wave admittance matrix, 493 ff.
- wave crest, 150, 255, 276, 279
- wave front, 2, 91, **255**, 271; *see also* ray surface
- wave guide, xxiii, 87, 104, 149, 438
- wave impedance, **18**, 388, 491
- wave interaction, xxiii, 7, 24
- wavelength in free space, 20, 484
- wavelength in medium, 150, 484, 487
- wave mechanics, 2
- wave normal, 17, 28, 35, 41, 45, **97**, 121, 124, 231, 232, 252, 255, 272
- wave packet, 146, 148, 166, 173, 175, **177** ff., 182, 185 ff., 196, 199, 204, 214, 225, 229 ff., 246 ff., 253, 259, 271, 279, 388, 479, 506
- wave polarisation, 14, 19, 45, **47** (ch. 5), 59 (ch. 6), 114 ff., 124 ff., 199, 214, 230, 248, 388 ff., 396, 424, 430, 432 ff., 456, 458, 462, 487, 500, 504 ff.; *see also* limiting polarisation
- wave refractive index, **148**, 200; *see also* refractive index
- wave surface, 253
- wave velocity, 148, **255**
- Waynick, A. H., 405, 426, 485, 519
- Weekes, K., xxiv, 7, 513
- Weber's equation, 359, 367
- Westfold, K. C., 37, 524
- west-east propagation, 123, 233, 236 ff., 248, 498
- Weygandt, C. N., 369, 513
- Weyl, H., 92, 524

Cambridge University Press

978-0-521-11439-4 - Radio Waves in the Ionosphere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers

K. G. Budden

Index

[More information](#)

542

Whale, H. A., 201, 206, 522, 524
 Whipple, R. T. P., 182, 459, 464, 469, 518
 whistlers, 30, 32, 256 ff., 269, 278
 Whittaker, Sir E. T., 161, 166, 346, 361, 369, 370, 372, 374, 476, 478, 524
 Wilkes, M. V., 366, 464, 469, 524
 winds, 10
 W.K.B. method, 128 (ch. 9), 131 ff., 292, 323, 350
 W.K.B. solution, derivation, 128 (ch. 9), 131, 137, 405
 W.K.B. solution with earth's field, 143, 199, 228, 385, 389, 392 ff., 401, 405, 406, 412, 427, 445, 452, 465, 468, 470, 474, 485, 487, 495
 W.K.B. solution, without earth's field, 128 (ch. 9), 133, 141, 143, 283, 292, 303, 321 ff., 329 ff., 335, 339, 355, 362

INDEX

Woodward, P. M. and Woodward, A. M., 292, 524
 Wronskian, 313, 342, 426
 Wüster, H. O., 346, 348, 516
 Yabroff, I. W., 127, 524
 Yokoyama, E., 96, 524
 zenith angle of sun, 4, 5
 Zenneck, E. H. J., 101, 524
 zero of q , 143, 231, 234, 237, 283 ff., 320, 322 ff., 327 ff., 357
 zero of refractive index, 40, 59, 64, 78, 81, 137, 149, 438, 443, 450, 473
 zero-order approximation, 421, 427
 zero refractive index, 45
 zeros of Airy integral function, 185
 Z-trace, 209, 404, 412, 416, 424 ff., 430