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# RADIO WAVES IN THE IONOSPHERE

THE MATHEMATICAL THEORY OF THE  
REFLECTION OF RADIO WAVES FROM  
STRATIFIED IONISED LAYERS

BY

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*To the memory of*  
DOUGLAS R. HARTREE

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## PREFACE

This book is based on a course of lectures given annually in Cambridge since 1956, and repeated in 1957 at the U.S. National Bureau of Standards Laboratories, Boulder, Colorado, U.S.A. Its object is to set out the mathematical basis of the theory of the propagation of radio waves in a horizontally stratified ionosphere. It is hoped that the book will serve both as a text-book, for those comparatively new to the subject, and as a reference book for more experienced readers. Some of the more advanced topics are printed in smaller type and could be omitted on a first reading. Throughout the book the stress is on the understanding of the mathematical methods rather than on their immediate practical use, since the radio engineer who really understands the mathematics is much better equipped to solve practical problems than one who does not.

The reader is assumed to be familiar with calculus, the theory of complex variables, vectors including the operators div, curl, grad, and electromagnetic theory as far as Maxwell's equations. Matrices are used in a few places, but the reader unfamiliar with them need not be deterred from studying the rest of the book.

For standard mathematical techniques there are references throughout the book to well-known mathematical treatises. One of the most useful of these is *Methods of Mathematical Physics* by Sir Harold and Lady Jeffreys which is a mine of valuable information.

It is inevitable that some important topics are omitted. Throughout the book it is assumed that the ionosphere is horizontally stratified, but this is only approximately true, since horizontal variations and irregularities play an important part in radio wave-propagation. The extensive recent work in this field is not covered here. Nor is there any discussion of reflection or scattering of radio waves from cylindrical structures such as meteor trails. The statistical mechanics of an ionised medium, and phenomena such as wave-interaction, which depend on electron temperature, are also excluded. In the theory of the propagation of radio waves to great distances, the space between the earth and the ionosphere is often treated as a wave-guide and the propagation constants of the 'wave guide modes' are found. This is sometimes called the 'mode theory' of radio wave-propagation. It is a large topic, which is beyond the scope of this book and really needs one to itself.

I am indebted first to Sir Edward Appleton whose work is the basis of

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## P R E F A C E

most of the theory of radio waves in the ionosphere. I have made very extensive use both of his published papers and of notes taken at his lectures and colloquia at Cambridge in the years 1935–39. At this time, too, Professor H. G. Booker and Mr E. Cunningham lectured on radio waves at Cambridge. I attended both courses, and still make frequent use of the lecture notes. On returning to Cambridge after the war I received a great deal of help and encouragement from the late Professor D. R. Hartree.

There are many diagrams in this book which are the result of computations made on EDSAC, the automatic digital computer in the University Mathematical Laboratory, Cambridge. I am greatly indebted to the Director, Dr M. V. Wilkes, and his staff for permission to use the EDSAC, and for able instruction in its use. To Dr Wilkes I am further indebted for his share in my initiation into the radio-wave field before the war.

The writing of this book was started at Boulder, Colorado, U.S.A., in 1957, while I was on sabbatical leave from Cambridge. I should like to thank the Director and staff of the Boulder Laboratories of the U.S. National Bureau of Standards for making it possible for me to work in Boulder and for their encouragement and valuable discussions of many aspects of the work.

Numerous other colleagues have given valuable advice and help. It is impossible to list them all, but I should mention particularly Dr D. W. Barron, Dr B. L. Briggs, Dr P. C. Clemmow, Dr J. O. Thomas, Mr G. Millington, Dr M. L. V. Pitteway, Dr H. Pöeverlein, Dr D. Shinn, and Dr K. Weekes.

But above all I am indebted to Mr J. A. Ratcliffe. I decided to write this book as a result of his suggestion, and some of the problems discussed in it were propounded by him. He, too, read the typescript and made valuable suggestions for improving it. Mr Ratcliffe's recent book on *The Magneto-ionic Theory* gives an excellent insight into the physical principles underlying radio wave-propagation, and should be studied by all readers of the present book.

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