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978-0-521-11418-9 - The Theory of Finite Linear Spaces: Combinatorics of Points and Lines

Lynn Margaret Batten and Albrecht Beutelspacher

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THE THEORY OF FINITE LINEAR SPACES

This is the first comprehensive text to cover finite linear spaces. It contains all the important results that have been published up to the present day, and is designed to be used not only as a resource for researchers in this and related areas, but also as a graduate level text. In eight chapters the authors introduce and review fundamental results, and go on to cover the major areas of interest in linear spaces. A combinatorial approach is used for the greater part of the book, but in the final chapter recent advances in group theory relating to finite linear spaces are presented. At the end of each chapter there is a set of exercises which are designed to test comprehension of the material, and there is also a section of research problems. It will be an invaluable book for researchers in geometry and combinatorics as well as an excellent text for graduate students.

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In memory of
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Preface

The aim of this monograph is to give a comprehensive and up-to-date presentation of the theory of finite linear spaces. For the most part, we take a combinatorial approach to the subject, but in the final chapter group theory is introduced.

The text is designed as a research resource for those working in the area of finite linear spaces, while the structure of the book also encourages its use as a graduate level text. At the end of each chapter, there is a section of exercises designed to test and extend a student's knowledge of the material in that chapter. There is also a research problem section containing current open problems which can be tackled with the aim of producing a thesis or a journal publication.

In the first chapter, constructions of affine and projective spaces are reviewed, and the fundamental results on finite linear spaces are given. Chapters 2 through 6 cover the work done on the major problem areas in linear spaces taking the 'planar' view: classification of linear spaces with given parameters, embeddability of linear spaces in 'suitably small' projective planes. In Chapter 7 we consider problems of embedding higher dimensional linear spaces in projective spaces. Finally, in Chapter 8, assumptions are introduced on the collineation groups of linear spaces, and the recent results on characterization are presented.

There are several people we wish to thank for their assistance, encouragement and patience while this book was being written. Jean Doyen, Université Libre de Bruxelles, provided us with the appendix of small linear spaces which appears at the end of the book. We also wish to thank him, and the numerous other people whose work is included in our text and reference sections, for the contributions they have made in the area, and for their support for the assimilation of their results in the present

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