CAMBRIDGE

Cambridge University Press
978-0-521-11367-0 - Zeta Functions of Graphs: A Stroll through the Garden
Audrey Terras
Index
More information

Index

adjacency matrix, 10
algebraic variety, 27
Alon conjecture, 17
analytic density, 196
Artin–Ihara L-function properties, 154
Artinized
  adjacency matrix, 156
  edge adjacency matrix, 149, 165
  path matrix, 179
  start, terminal, and J matrices, 170

backtrack, 11
Bass proof of Artin–Ihara three-term determinant formula, 89, 173
boundary of a set of vertices, 71
bounds
  on poles of Ihara zeta, 94–95
bouquet of loops, 14

Catalan number, 69–70
centralizer of element of group, 15
character
  of cyclic group, 63–64
  of representation, 146
character table for S_3, 148
Chebotarev density theorem, 194–199
closed path, 11
complexity of a graph, 78
conjectured inequality between ρ_X, R_X, and $\overline{d_X}$, 54
conjugacy class in group, 14
conjugate intermediate graph, 122
  equivalent to covering isomorphic graph, 122
conjugate prime, 131
conjugate subgroup, 122
constructing graph covers using the Frobenius automorphism, 142, 190, 191
covering isomorphic graphs, 118
covering isomorphism, 118
covering map π, 20, 106
covering of graphs, 20, 106
cycle, 11
cycle code, 213
cycle graph, 10

decomposition group, 137
Dedekind conjecture, 115
degree
  of representation, 144
  of vertex, 10
δ, 201
Δ_X, 21
determinant formula, three term,
  for Artin–Ihara zeta, 156
  for Ihara zeta, 17
determinant formula, two term,
  cohomological, for zeta of algebraic variety, 27
  for edge zeta, 84
  for Ihara zeta, 28, 86
  for path Artin L-function, 179
  for path zeta, 100
  for Artin–Ihara L-function, 149
  with edge adjacency matrix W_1, 28
dichotomy of quantum chaos, 39
direct sum of representations, 145
Dirichlet theorem on primes in progressions, 7
divisibility of zetas of covers, 110

edge adjacency matrix W_1, 28
  irreducible, 93
edge Artin L-function properties, 165
edge Artin matrix, 149, 165
edge matrix W, 83
edge norm, 83

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-11367-0 - Zeta Functions of Graphs: A Stroll through the Garden
Audrey Terras
Index
More information

elementary reduction operations on fundamental group, 99
equal intermediate graphs, 118
equidistributed sequence, 39
equivalence class of closed paths, 12
equivalent representations, 145
error-correcting code, 211
Euclid's fifth postulate, 23
Euler characteristic, 16
Euler product, 3
expander family, 71
expansion ratio, 71
experiment
  Mathematica on random graphs, 57
  Matlab
    on abelian cover, 225
    on random cover, 224
    on random graph, 88, 221
  Newland using Mathematica on random regular graphs, 40–41
  ramification, 115, 116
explicit formulas, 4, 216–217
extended Riemann hypothesis (ERH), 7

factorization
  of edge zeta of a normal cover, 165
  of Ihara zeta of a normal cover, 155
  of path zeta of a normal cover, 184
facts about edge adjacency matrix, 91
$f(D, Y/X)$, residual degree of D, 128
fission, 86
Fourier inversion, 144
Fourier transform, 144
fractional linear transformation, 22
free group, 13
freely homotopic cycles, 15
Frobenius
  automorphism of graph, properties, 137
  automorphism of a path with respect to a Galois cover, 135
  character formula, 146
  morphism of algebraic variety, 27
  normalized automorphism of a path with respect to a Galois cover, 133
  reciprocity law, 147
functional equation
  regular graph, 50
  Riemann zeta, 3
fundamental domain of modular group, 23
fundamental group, 13, 99
fundamental theorem of graph Galois theory, 118

$\widehat{G}$, set of all irreducible unitary representations of group G, 145
Galois group action on sheets of cover, 110
Galois group $G(Y/X)$, 109
Gauss sum, 64
Gaussian orthogonal ensemble (GOE), 34
Gaussian unitary ensemble (GUE), 4, 35
$g(D, Y/X)$, number of primes above C in X if $D|C$, 128
generalized Riemann hypothesis (GRH), 54
geodesic in upper half plane, 22, 24
graph
  bipartite, 47
  bouquet of loops, 14
  Cayley, 50
  complete, 10
  cycle, 10
  dumbbell, 86
  Euclidean, 65
  Harold and Audrey, 192
  K_4, tetrahedron, 15
  $K_4 - e$, 15
  Lubotzky, Phillips, and Sarnak, 67
  normal for a cycle code, 213
  Paley, 64
  regular, 10
  simple, 10
  Tanner, 212
graph automorphism, 109
graph prime counting function, 21
group
  free, 13
  fundamental, 13
group representation, 144

hypotheses, the usual, 10

induced representation, 145
induction property
  of Artin–Ihara L-functions, 154
  of edge Artin L-functions, 166, 175
  of path Artin L-functions, 183
inequality for ρ, p, q, R, 54, 96
inflated graph, 45, 206
intermediate covering, 117
irreducible matrix, 90
irreducible representation, 144
isospectral non-isomorphic graphs, 188
  Buser's example, 187
  Stark and Terras example, 192

K_4, 16
Kloosterman sum, 66

L-function
  Artin–Ihara, 148
  Artin of number fields, 134
  Dirichlet, 6, 7
  edge Artin, 164
  path Artin, 178

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-11367-0 - Zeta Functions of Graphs: A Stroll through the Garden
Audrey Terras
Index
More information

labeling edges
  left out of spanning tree, 98
  of a graph, 11
Landau's theorem, 13
Laplace operator for Poincaré arc length, 22
lemma
  expander mixing, 71
  $R_Y = R_X$, 195
length $\nu(C)$ of path C, 12
level spacings picture, 31, 33, 35, 41, 220, 225, 226
lift, 109

Möbius function, 76
Möbius inversion formula, 76
Markov transition matrix, 61
matrix identities with S and T, 88
  with representations, 170
modular group, 23

naive Ramanujan inequality, 53
neighborhood of a vertex, 106
Newland's experiments, 40, 41
N_m, number of closed paths of length m without backtracking and tails, 29
node, 200
non-Euclidean triangle, 23
normal or Galois covering, 109
number of primes D above C, 128

orthogonality relations, 147

parity check matrix, 211
path matrix Z, 99
path norm, 100
permutation matrix, 90
$\pi(n)$, prime counting function, 21
Plancherel measure, 39
Poincaré arc length, 22
Poincaré upper half plane, 22
prime ideal theorem, 7
prime in a graph, 12
prime number theorem
  graph theory version, 21, 75
  ordinary integer version, 4
prime or primitive path in graph, 12
primitive conjugacy class, 15
probability vector, 61
properties
  of Artin–Ihara L-functions, 154
  of edge Artin L-functions, 165
  of path Artin L-functions, 179

quantum graph, 192

R, radius of circle of convergence of Ihara zeta, 13

Ramanujan
  graph, 17
  irregular graph, 53
ramification, 8, 24, 115
rank of free group, 13
Rayleigh quotient, 94
reduced word in fundamental group, 99
k-regular graph, 10
relation between δ and Δ, 201
representation
  direct sum, 145
  equivalent, 145
  induced, 145
  of cyclic group, 144
  right regular, 145
  trivial, 145
  unitary, 144
residual degree $f(D, Y/X)$, 128
ρ_X, 52
ρ'_X, 52
Riemann hypothesis (RH)
  graph theory, 53
  regular graph, 48
  Riemann zeta, 3
  weak graph theory, 54

S_3 cover of tetrahedron minus an edge, 124
Seifert–Van Kampen theorem, 13
shape of a drum, 186, 192
sheets of a covering, 107
Siegel poles, 201
Siegel zero
  Dedekind zeta, 5
spanning tree, 13, 107
specialization, 84
  edge zeta
    to Hashimoto edge zeta, 84
    to Ihara zeta, 84
    to weighted zeta, 84
  for induction property of path Artin L-functions, 181
  of path zeta to edge zeta, 100
spectrum adjacency matrix, regular graph properties, 47
splitting of primes
  in coverings, 128
  in cube cover K_4, 130, 131
  in non-normal cubic cover of the tetrahedron minus an edge, 132
  in non-normal cubic extension of $\mathbb{Q}$, 129
  in quadratic extension of $\mathbb{Q}$, 8
start matrix, 87
subshift of finite type, 29
symmetric generating set S for Cayley graph $X(G, S)$, 50

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-11367-0 - Zeta Functions of Graphs: A Stroll through the Garden
Audrey Terras
Index
More information

tail, 11
tensor product, 170
terminal matrix, 87
tessellation of upper half plane, 24
tetrahedron, 16
theorem
  Alon–Boppana, 68
  Bowen–Lanford, 30
  constructing graph covers using the Frobenius automorphism, 142
  Fan Chung, on graph diameters, 73
  fundamental theorem of graph Galois theory, 118
  graph theory prime number theorem, 75
  Hoory inequality, 53
  Ihara generalized by Bass, Hashimoto, etc., 17
  Koetter, Li, Vontobel, and Walker, 214
  Kotani and Sunada, 52
    proof of, 95
  McKay, 40
  Perron–Frobenius, 94
  random walker gets lost, 62
  right regular representation is mother of all representations, 145
transitivity property of residual degrees, 131–132
tree, 13
trivial representation, 145
types of fractional linear transformation, 24

unique lifts to covers, 108
universal covering tree, 15
unramified covering graph, 20

weighted length of a path in a graph, 45
Weil conjectures, 27
Weil's proof of the Riemann hypothesis for zetas of curves, 66, 67
Wigner semicircle distribution, 33
Wigner surmise, 34, 218, 220

$X(G, S)$, Cayley graph, 50
X-specialized edge matrix, 165

zeta function
  algebraic variety, 27
  Artin–Mazur, 28
  Bartholdi, 84
  Dedekind, 5–6
  edge, 83
  Epstein, 9
  function field, 5
  Hashimoto edge, 84
  Ihara, 12, 16
  path, 100
  Riemann, 3
  Ruelle, 28
  Selberg, 25
  weighted graph, 45

© in this web service Cambridge University Press www.cambridge.org