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ZETA FUNCTIONS OF GRAPHS

Graph theory meets number theory in this stimulating book. Ihara zeta functions of finite graphs are reciprocals of polynomials, sometimes in several variables. Analogies abound with number-theoretic functions such as Riemann or Dedekind zeta functions. For example, there is a Riemann hypothesis (which may be false) and a prime number theorem for graphs. Explicit constructions of graph coverings use Galois theory to generalize Cayley and Schreier graphs. Then non-isomorphic simple graphs with the same zeta function are produced, showing that you cannot "hear" the shape of a graph.

The spectra of matrices such as the adjacency and edge adjacency matrices of a graph are essential to the plot of this book, which makes connections with quantum chaos and random matrix theory and also with expander and Ramanujan graphs, of interest in computer science. Pitched at beginning graduate students, the book will also appeal to researchers. Many well-chosen illustrations and exercises, both theoretical and computer-based, are included throughout.

Audrey Terras is Professor Emerita of Mathematics at the University of California, San Diego.

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Cambridge University Press
978-0-521-11367-0 - Zeta Functions of Graphs: A Stroll through the Garden
Audrey Terras
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Zeta Functions of Graphs

A Stroll through the Garden

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521113670

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First published 2011

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication Data

Terras, Audrey.
Zeta functions of graphs : a stroll through the garden / Audrey Terras.
p. cm. – (Cambridge studies in advanced mathematics ; 128)
ISBN 978-0-521-11367-0 (Hardback)
1. Graph theory. 2. Functions, Zeta. I. Title. II. Series.
QA166.T47 2010
511′.5–dc22
2010024611

ISBN 978-0-521-11367-0 Hardback

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Preface

The goal of this book is to guide the reader in a stroll through the garden of zeta functions of graphs. The subject arose in the late part of the twentieth century and was modelled on the zetas found in other gardens.

Number theory involves many zetas, starting with Riemann's – a necessary ingredient in the study of the distribution of prime numbers. Other zetas of interest to number theorists include the Dedekind zeta function of an algebraic number field and its analog for function fields. Many Riemann hypotheses have been formulated and a few proved. The statistics of the complex zeros of zeta have been connected with the statistics of the eigenvalues of random

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Hermitian matrices (the Gaussian unitary ensemble (GUE) distribution of quantum chaos). Artin L-functions are also a kind of zeta associated with a representation of a Galois group of number or function fields. We will find graph analogs of all of these.

Differential geometry has its own zeta, the Selberg zeta function, which is used to study the distribution of the lengths of prime geodesics in compact or arithmetic Riemann surfaces. There is a third zeta function, known as the Ruelle zeta function, which is associated with dynamical systems. We will look at these zetas briefly in Part I. The graph theory zetas are related to these zetas too.

In Part I we give a brief glimpse of four sorts of zeta function, to motivate the rest of the book. In fact, much of Part I is not necessary for the rest of the book. Feel free to skip all but Chapter 2 on the Ihara zeta function.

Prerequisites for reading this book include linear algebra and group theory. What do groups have to say about graphs which appear to have no symmetry? The answer comes with an understanding of the fundamental group whose elements are closed paths through a vertex. This group is intrinsic to our subject. We will find that the theory Galois developed at a young age has its applications here. Our zetas are reciprocals of polynomials, sometimes in several variables. We will obtain determinant formulas for these zetas. And Galois theory will lead to factorizations of the zetas of normal covering graphs, just as it leads to factorizations of Dedekind zeta functions of Galois extensions of number fields.

Most of this book arises from joint work with Harold Stark. Thanks are due to the many people who listened to my lectures on this book and helped with the research, Matthew Horton, Derek Newland, Tom Petrillo, Adriano Garsia, Angela Hicks, Paul Horn, and Yeon Kyung Kim at the University of California, San Diego. I would also like to thank the people who encouraged me by attending my Ulam Seminar at the University of Colorado, Boulder, especially Lynne Walling, David Grant, Su-ion Ih, Vinod Radhakrishnan, Erika Frugoni, Jonathan Kish, and Mike Daniel.

In addition, I want to thank the Newton Institute in Cambridge, England for its hospitality while I worked on this manuscript, which was prepared using Scientific Workplace.

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