

Contents

<i>Preface</i>	<i>page</i> ix
<i>Basic definitions, notation and abbreviations</i>	xiv
1 Introduction	1
1.1 Nonlinear Markov chains	1
1.2 Examples: replicator dynamics, the Lotka–Volterra equations, epidemics, coagulation	6
1.3 Interacting-particle approximation for discrete mass-exchange processes	8
1.4 Nonlinear Lévy processes and semigroups	11
1.5 Multiple coagulation, fragmentation and collisions; extended Smoluchovski and Boltzmann models	13
1.6 Replicator dynamics of evolutionary game theory	24
1.7 Interacting Markov processes; mean field and k th-order interactions	28
1.8 Classical kinetic equations of statistical mechanics: Vlasov, Boltzmann, Landau	32
1.9 Moment measures, correlation functions and the propagation of chaos	34
1.10 Nonlinear Markov processes and semigroups; nonlinear martingale problems	39
Part I Tools from Markov process theory	41
2 Probability and analysis	43
2.1 Semigroups, propagators and generators	43
2.2 Feller processes and conditionally positive operators	54

vi	<i>Contents</i>	
	2.3	Jump-type Markov processes 64
	2.4	Connection with evolution equations 67
3	Probabilistic constructions	73
	3.1	Stochastic integrals and SDEs driven by nonlinear Lévy noise 73
	3.2	Nonlinear version of Ito's approach to SDEs 82
	3.3	Homogeneous driving noise 89
	3.4	An alternative approximation scheme 90
	3.5	Regularity of solutions 92
	3.6	Coupling of Lévy processes 96
4	Analytical constructions	102
	4.1	Comparing analytical and probabilistic tools 102
	4.2	Integral generators: one-barrier case 104
	4.3	Integral generators: two-barrier case 111
	4.4	Generators of order at most one: well-posedness 114
	4.5	Generators of order at most one: regularity 117
	4.6	The spaces $(C_\infty^l(\mathbf{R}^d))^*$ 120
	4.7	Further techniques: martingale problem, Sobolev spaces, heat kernels etc. 121
5	Unbounded coefficients	131
	5.1	A growth estimate for Feller processes 131
	5.2	Extending Feller processes 135
	5.3	Invariant domains 138
	Part II	Nonlinear Markov processes and semigroups 145
6	Integral generators	147
	6.1	Overview 147
	6.2	Bounded generators 149
	6.3	Additive bounds for rates: existence 154
	6.4	Additive bounds for rates: well-posedness 160
	6.5	A tool for proving uniqueness 165
	6.6	Multiplicative bounds for rates 169
	6.7	Another existence result 170
	6.8	Conditional positivity 173
7	Generators of Lévy–Khintchine type	175
	7.1	Nonlinear Lévy processes and semigroups 175
	7.2	Variable coefficients via fixed-point arguments 180

<i>Contents</i>		vii
7.3	Nonlinear SDE construction	184
7.4	Unbounded coefficients	186
8	Smoothness with respect to initial data	188
8.1	Motivation and plan; a warm-up result	188
8.2	Lévy–Khintchine-type generators	192
8.3	Jump-type models	201
8.4	Estimates for Smoluchovski’s equation	208
8.5	Propagation and production of moments for the Boltzmann equation	216
8.6	Estimates for the Boltzmann equation	219
Part III	Applications to interacting particles	223
9	The dynamic law of large numbers	225
9.1	Manipulations with generators	225
9.2	Interacting diffusions, stable-like and Vlasov processes	232
9.3	Pure jump models: probabilistic approach	236
9.4	Rates of convergence for Smoluchovski coagulation	245
9.5	Rates of convergence for Boltzmann collisions	250
10	The dynamic central limit theorem	252
10.1	Generators for fluctuation processes	252
10.2	Weak CLT with error rates: the Smoluchovski and Boltzmann models, mean field limits and evolutionary games	263
10.3	Summarizing the strategy followed	267
10.4	Infinite-dimensional Ornstein–Uhlenbeck processes	268
10.5	Full CLT for coagulation processes (a sketch)	270
11	Developments and comments	275
11.1	Measure-valued processes as stochastic dynamic LLNs for interacting particles; duality of one-dimensional processes	275
11.2	Discrete nonlinear Markov games and controlled processes; the modeling of deception	279
11.3	Nonlinear quantum dynamic semigroups and the nonlinear Schrödinger equation	282
11.4	Curvilinear Ornstein–Uhlenbeck processes (linear and nonlinear) and stochastic geodesic flows on manifolds	293
11.5	The structure of generators	300
11.6	Bibliographical comments	310

Appendices	319
A Distances on measures	319
B Topology on càdlàg paths	324
C Convergence of processes in Skorohod spaces	329
D Vector-valued ODEs	334
E Pseudo-differential operator notation	337
F Variational derivatives	338
G Geometry of collisions	343
H A combinatorial lemma	347
I Approximation of infinite-dimensional functions	349
J Bogolyubov chains, generating functionals and Fock-space calculus	352
K Infinite-dimensional Riccati equations	355
 <i>References</i>	 360
<i>Index</i>	373