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Edited by Brian Marcus, Karl Petersen and Tsachy Weissman

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Papers from the Banff International Research  
Station Workshop

Edited by

BRIAN MARCUS

*University of British Columbia, Vancouver*

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*University of North Carolina, Chapel Hill*

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*Stanford University, California*



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Frontmatter

[More information](#)


---

## Contents

---

	<b>Introduction</b>	<i>page</i> <b>1</b>
<b>1</b>	<b>Hidden Markov processes in the context of symbolic dynamics</b>	<b>5</b>
	Mike Boyle, University of Maryland, College Park Karl Petersen, University of North Carolina, Chapel Hill	
<b>2</b>	<b>On the preservation of Gibbsianness under symbol amalgamation</b>	<b>72</b>
	Jean-René Chazottes, École Polytechnique, Palaiseau Edgardo Ugalde, Universidad Autónoma de San Luis Potosí	
<b>3</b>	<b>A note on a complex Hilbert metric with application to domain of analyticity for entropy rate of hidden Markov processes</b>	<b>98</b>
	Guangyue Han, University of Hong Kong Brian Marcus, University of British Columbia Yuval Peres, University of California at Berkeley	
<b>4</b>	<b>Bounds on the entropy rate of binary hidden Markov processes</b>	<b>117</b>
	Erik Ordentlich, Hewlett-Packard Laboratories, Palo Alto Tsachy Weissman, Stanford University	
<b>5</b>	<b>Entropy rate for hidden Markov chains with rare transitions</b>	<b>172</b>
	Yuval Peres, Microsoft Research, Redmond Anthony Quas, University of Victoria, British Columbia	

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978-0-521-11113-3 - Entropy of Hidden Markov Processes and Connections to Dynamical Systems: Papers from the Banff International Research Station Workshop

Edited by Brian Marcus, Karl Petersen and Tsachy Weissman

Frontmatter

[More information](#)

---

vi

*Contents*

- |          |   |            |
|----------|---|------------|
| <b>6</b> | <b>The capacity of finite-state channels in the high-noise regime</b> | <b>179</b> |
|          | Henry D. Pfister, Texas A&M University                                |            |
| <b>7</b> | <b>Computing entropy rates for hidden Markov processes</b>            | <b>223</b> |
|          | Mark Pollicott, University of Warwick                                 |            |
| <b>8</b> | <b>Factors of Gibbs measures for full shifts</b>                      | <b>246</b> |
|          | Mark Pollicott, University of Warwick                                 |            |
|          | Thomas Kempton, University of Warwick                                 |            |
| <b>9</b> | <b>Thermodynamics of hidden Markov processes</b>                      | <b>258</b> |
|          | Evgeny Verbitskiy, Leiden University and<br>University of Groningen   |            |