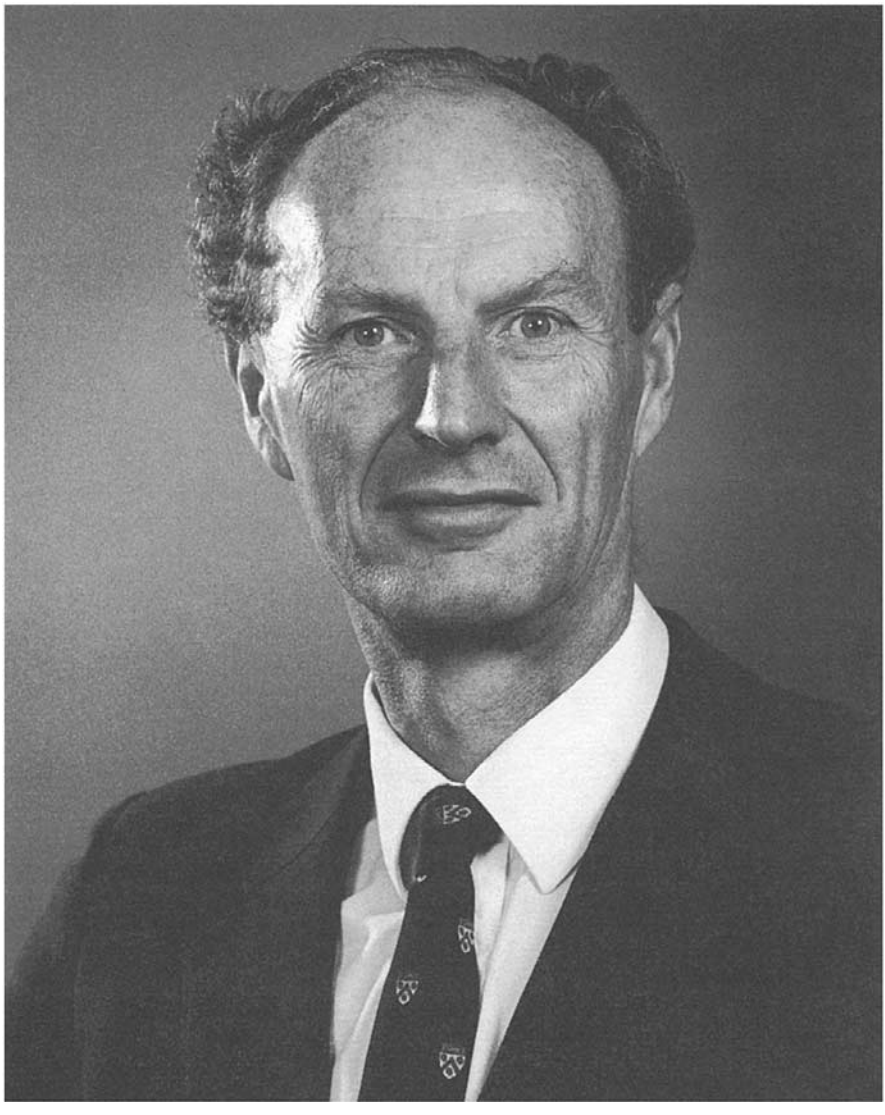
J. Frank Adams was one of the world's leading topologists. He solved a number of celebrated problems in algebraic topology, a subject in which he initiated many of the most active areas of research. He wrote a large number of papers during the period 1955–1988, and they are characterised by elegant writing and depth of thought. Few of them have been superseded by later work.

This selection brings together all his major research contributions. They are organised by subject matter rather than in strict chronological order. This first volume contains papers on: the cobar construction, the Adams spectral sequence, higher order cohomology operations, and the Hopf invariant one problem; applications of K-theory; generalised homology and cohomology theories. The second volume is mainly concerned with Adams' contributions to: characteristic classes and calculations in K-theory; modules over the Steenrod algebra and their Ext groups; finite H-spaces and compact Lie groups; maps between classifying spaces of compact groups.

THE SELECTED WORKS OF J. FRANK ADAMS



THE SELECTED WORKS OF J. FRANK ADAMS VOLUME I

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Introduction

Frank Adams was a great mathematician with a fine expository style. These two volumes contain the bulk of his numerous papers, grouped according to subject, and roughly chronologically within groups. We chose this organisation because of Adams' practice of returning periodically to certain subjects dear to his heart, re-examining them in the light of intervening research. We have added no editorial material since we prefer to let Adams' own introductions to his papers speak for themselves. We have also made no attempt to note errors since Adams was a scrupulously careful author. Several of his papers have published corrections; these we have included.

Adams' final bibliography contains 82 published items. These volumes contain 52 of them. The omitted items fall into five categories. There are five early papers (to 1957), an appendix to a paper of another author, and a published letter which we feel are probably not of lasting mathematical interest. There are four announcements of results which were published elsewhere. There are three biographical items. There are fifteen primarily expository articles, most of which were published in conference proceedings; we have chosen to include four of these, which we feel are of more lasting interest than the others.

The five remaining omitted items are books. The first of these, from 1964, is *Stable Homotopy Theory, Volume 3* in the Springer Lecture Notes in Mathematics. It contains material that has been superseded mathematically by results published elsewhere. Still, it has many delightful passages, and comparison of it with later work shows how rapidly algebraic topology in general, and Adams' work in particular, was progressing in those days. A second, *Algebraic Topology: a student's guide* consists primarily of selected reprints. The remaining three are highly recommended reading. Two of them, *Lectures on Lie Groups* and *Stable Homotopy and Generalized Homology* are being kept in print by the University of Chicago Press and remain among the best references on their subjects. The third, *Infinite Loop Spaces* is Study 90 of the Annals of Mathematics, published by Princeton University Press. It is vintage Adams, beautifully and humorously written, and it contains capsule summaries of various topics in algebraic topology other than the one described by the title.

The second volume ends with two expository articles which perhaps were not intended for publication. They well illustrate Adams' interest in the work of young algebraic topologists and his success in illuminating the most recent developments in his subject.

Biographical data

J. Frank Adams was born on 5 November 1930, in Woolwich, London, and died on 7 January 1989. He was married in 1953 and had one son and three daughters.

The main dates in his education were Bedford School until 1948, followed by military service in the Royal Engineers (1948–9) and attendance at Trinity College, Cambridge (1949–55), obtaining first class honours in Part II of the mathematical tripos (1951) and a distinction in Part III a year later. He was awarded his B.A. in 1952, his Ph.D. in 1955, his M.A. in 1956 and the higher degree of Sc.D. in 1982.

Following the completion of his Ph.D., he was appointed a Junior Lecturer at Oxford (1955–6). and he held a Research Fellowship at Trinity College, Cambridge, until 1958. During this time he also made his first visit to the University of Chicago (Summer 1957) and held a Commonwealth Fund Fellowship at the Institute of Advanced Study in Princeton (1957–8).

On returning to England he became an Assistant Lecturer at Cambridge, combining this with the Directorship of Studies in Mathematics at Trinity Hall. He was again a visiting member of the Institute at Princeton in the fall of 1961.

In 1962 Adams moved to the University of Manchester first as a Reader and then as Fielden Professor of Pure Mathematics (1964–71). In 1970 he returned to Cambridge as Lowndean Professor of Astronomy and Geometry, and he was also active as a Fellow of Trinity College.

His professional affiliations included the Association of University Teachers, the American Mathematical Society (since 1957) and the London Mathematical Society (since 1958). He was also elected a Fellow of the Royal Society in 1964 and a Foreign Associate of the National Academy of Sciences of the United States in 1985.

His frequent trips to the United States included ten lengthy visits to the University of Chicago, the last in 1985. Adams was in great demand as a speaker – his more important lectures included addresses to the International Congress of Mathematicians in Stockholm (1962) and in Moscow (1966), the Herman Weyl Lectures at the Institute in Princeton (1975) and an address to the American Mathematical Society as Bicentennial Exchange Lecturer (1976).

Over many years he contributed to the *Mathematical Reviews*, and in addition acted as referee for numerous other journals. He served as Editor, Member of the Editorial Board, or Editorial Advisor for three journals of the London Mathematical Society, the *Annals of Mathematics*, *Inventiones Mathematicae*, the *Journal of Pure and Applied Algebra*, and the *Mathematical Proceedings of the Cambridge Philosophical Society*. He also served on subcommittees to choose

speakers in algebraic topology for the International Congress (once as Chairman), on the committee to choose Fields Medallists, and on the Consultative (programme) committee of the ICM. He gave similar organisational help to other conferences.

As a Fellow of the Royal Society he served on the mathematical sectional committee (two terms, one as chairman) and on the Council. He was also a member of the Council of the London Mathematical Society, and of the mathematics committee of the Science and Engineering Research Council (two terms).

Adams' honours include the Junior Berwick Prize (1963), the Senior Whitehead Prize (1974) and the Sylvester Medal (1982). Besides the Sc.D. from Cambridge he was also made a Doctor (h.c.) by the Universität Heidelberg in 1986.

Acknowledgements

The following papers appear in these volumes with grateful thanks to the original publishers:

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