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REIKO SAKAMOTO

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HYPERBOLIC  
BOUNDARY VALUE  
PROBLEMS

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## PREFACE

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The book is designed to introduce the reader to the general theory of the existence and uniqueness of solutions of hyperbolic initial value problems in partial differential equations of higher orders. The subject has been my main research theme for more than a decade.

A few years ago when I decided to write this book at the suggestion of Professor Hiroshi Fujita I thought that the project was very tempting in the sense that I could retrace the old course of inquiries along which I had struggled for ten years, and I should be able to put various ideas in perspective.

However, after setting down some ideas, I soon realised that this was an audacious task; I must confess that I often went astray while writing the book.

Even now that the book has eventually taken shape I fear that some readers may blame me for my obsession with logical consistency at the expense of the wider view of the theory which I originally had in mind.

To such a reader, I can only reply by pointing out that my presentation is merely one of many alternatives for expounding the subject matter; drawing a beautiful global map of the present state of the art is beyond my capabilities.

The book is divided into three chapters.

Chapter 1 is a collection of facts later used in the main body of the text, which are basic to the theory of second-order partial differential equations.

Chapter 2 discusses hyperbolic boundary value problems with constant coefficients by the method of Fourier–Laplace transforms.

Chapter 3 deals with hyperbolic boundary value problems with variable coefficients by the method of energy inequalities.

I have presented the above-mentioned subjects while presupposing a minimal background. The reader is assumed to have some prior acquaintance with the elementary theories of a complex variable and Lebesgue

integrals, and when extra tools are needed, brief explanations are provided to make the text as self-contained as possible.

I wish to thank my teachers, Professor Shigeru Mizohata and Professor Masaya Yamaguchi for their kind guidance throughout my career as a researcher. My academic indebtedness to them does not end there. Several ideas concerning hyperbolic initial value problems, which have resulted from their vast research, underpin the arguments that I have presented.

Special thanks go to Professor Hiroshi Fujita, who gave me the chance to write this book, and to Mr Hideo Arai, editorial staff of Iwanami Shoten, who showed remarkable patience and gave willing assistance in spite of the slow progress of my writing.

Finally, I must thank my husband, Kazuichi, without whose constant encouragement this book would never have come into being.

*February 1978*

Reiko Sakamoto

*PREFACE TO THE ENGLISH EDITION*

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I am pleased to see that this English edition has appeared through the enduring labour of Katsumi Miyahara and the skilful arrangement by the editorial staff of Cambridge University Press. As a result, I believe that the book may appeal to a wider audience outside my country.

As the translation progressed, I took the opportunity of incorporating various corrections and revisions in the text. I am indebted to Dr Tatsuo Nishitani and other Japanese readers of the original edition for their valuable suggestions.

All this has led to a significant improvement of the original text. I am sincerely grateful to everyone concerned.

*October 1980*

Reiko Sakamoto