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Introduction

1.1 The wonders of the heavens

The planets have been a subject of wonder to man from earliest recorded times. Their very name, the Wandering Ones, recalls the fact that their apparent positions in the sky change continually, in contrast to the fixed stars. Greek astronomers, Ptolemy particularly, had shown how the motions of the planets, the Sun and the Moon could be accounted for if they were all supposed to move around a stationary Earth, and in mediaeval times an elaborate cosmology was created, at its most allegorical, evocative and poetic in the *Paradiso* of Dante. The men of the Renaissance overthrew these ideas but provided fresh cause for wonder in their place. Placed in motion around the Sun by Copernicus, their paths observed with care by Kepler, the planets led Newton to his ideas of universal gravitation. Galileo, his telescope to his eye, showed that they had discs of definite size and that Jupiter had moons, the Medicean satellites, which formed a system like the planets themselves.

The discoveries of the seventeenth century settled notions of the planets for three centuries, but within that framework a most extraordinary flowering of the intellect attended the working out of the ideas of Newton. Closer and closer observation showed ever more intricate departures of the paths of the planets from the simple ellipses of Kepler, and each was accounted for by ever subtler applications of mechanics as the consequence of the gravitational pull of each planet upon its fellows. For some while the system of planets was tacitly or explicitly supposed to be closed, until William Herschel, in almost his first excursion into astronomy from his profession of music, saw from Bath an unknown planet – Uranus. Much later Adams predicted, a great achievement of dynamical theory, the existence of a further planet, and Leverrier found it. Now we know of yet a further planet, Pluto, and of the belt of asteroids between Mars and Jupiter. The major dynamical features of the motions

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of the planets are now well understood, as are many minor features, among them the effects of the general-relativistic description of gravitation.

No doubt there are still discoveries to be made in orbital dynamics; they will come from the exploitation of modern developments in theory and, in particular, the execution of lengthy algebraic calculations by computer, and they will come also from the ever more precise measurements of the motions of the Moon and the planets and space probes that go close to them, by means of radio, radar and lasers. But these will probably be refinements; the heroic age of dynamical studies is almost closed, and other fields are now more productive. Early in this century, definite ideas about the internal constitution of the Earth began to develop, while estimates of the densities of the planets were available from dynamical investigations. It was realized that Mercury, Venus and Mars, as well as the Moon, must be in a general way similar to the Earth while the major planets, with much lower densities, were essentially different. In the 1930s the physics of the solid state and the quantum mechanics of materials at high pressures, rudimentary though they were by present day standards, were nonetheless adequate guides to thought about the nature of the planets, and the conjunction of dynamical studies on the one hand with the physics of condensed matter on the other is the theme to be developed in this book.

Space research has certainly contributed to the dynamical study of the planets, but rather in the way of refinement, for the observations of natural objects had led to considerable knowledge of the masses, densities, gravitational fields and moments of inertia of the Moon and a number of the planets before any space probes were launched towards them. The refinements have been valuable and have brought precision and simplicity to what previously may have been approximate and complex, but the great contributions of space research have been elsewhere, to the knowledge of surfaces and surface processes and of magnetic fields. We take it to be almost certain that the magnetic field of the Earth and the changes that the surface of the Earth undergoes are dependent on the internal state of the Earth and are driven by sources of energy within the Earth. The same is very likely true of the Moon and planets, but in different ways and to different degrees; surface features and magnetic fields no doubt contain clues to the nature of the interior, but we do not yet know how to unravel them; we do not understand the connexions in the Earth so how much less can we make use of them in studying the planets. We may nonetheless be fairly confident that it is here that the next major step in understanding the planets will be taken. The relation between the structures of the planets and the natures of the materials, so far as they can be unravelled by dynamical studies and understanding of the physics of condensed matter, is established as far as the major features are concerned. Some ideas of the limits of knowledge are also clear and further studies are, on the whole, likely to lead to refinements of present ideas but not to major changes. If major changes come, it will probably be as a result of understanding the way in which magnetic fields and surface processes are related to internal constitutions.

1.2 The system of the planets

Some of the principal facts about the planets are collected in Table 1.1. It gives the distance of each planet from the Sun, and its radius and mass, all in terms of the Earth's distance, radius and mass, it gives the mean density of each and it gives the period of spin about the polar axis. In Figure 1.1 the masses and densities are plotted against the distance from the Sun. The table and figure demonstrate the well-known division of the planets into two groups: the inner, terrestrial planets, relatively close to the Sun, of low mass and high density, and the outer, major planets, relatively far from the Sun and having high masses and low densities. The size and mass of Pluto, the outermost planet, are poorly known, but it is certainly smaller and denser than the other outer planets.

Table 1.1. *The system of planets*

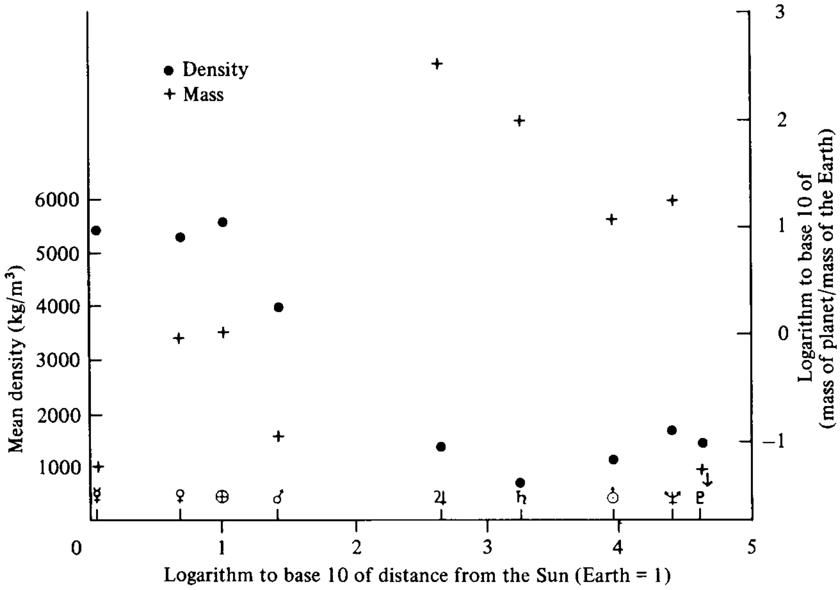
	Distance from Sun (AU)[a]	Radius[b]	Mass[b]	Density (kg/m^3)	Spin period (d)
Moon		0.27	0.0123	3340	28
Mercury	0.387	0.382	0.055	5434	58.6
Venus	0.723	0.949	0.815	5244	243
Earth	1	1	1	5517	1
Mars	1.524	0.532	0.107	3935	1.026
Jupiter	5.203	11.16	317.8	1338	0.411
Saturn	9.539	9.41	95.1	705	0.428
Uranus	19.18	4.02	14.60	1254	0.96?
Neptune	30.06	3.89	16.78	1635	0.92?
Pluto	39.44	0.24	0.002	~1000	6.4

[a] The astronomical unit (AU) is the mean distance of the Earth from the Sun and is equal to 1.496×10^8 km.
[b] The radius and mass are given in terms of the Earth's radius (6378 km) and mass (5.977×10^{24} kg).

Any theory of the origin of the solar system must account for the sharp distinction between the two groups of planets. It is not my purpose to discuss the problem of the origin of the planets in any detail save that at the end I shall draw some conclusions about the origin of the planets from their present structure. The way in which the planets were formed does, however, have consequences for the chemical constitution of the planets, that is for the types of material of which we may suppose them to be made up, and hence for the physical properties of those materials at high pressure and temperature.

It is nowadays commonly supposed that the planets formed at an early stage in the history of the Sun, during the so-called Hiyashi phase, when the Sun was far more extended than it is now. Irregularities of density were brought about by the influence of a second star and led to condensations from which the planets formed. Many people currently favour a hot origin of the inner planets, that is to say, that they formed from the condensation of materials from a hot gas as opposed to accretion from a cloud of cold dust. It is therefore thought that the temperature of a planet immediately after formation would be the temperature of condensation. The distribution of temperature inside the planets is discussed in Chapter 9; here I am concerned to point out that theories of planetary origin entail

Figure 1.1. Masses, densities and distances of the planets. The mass of Pluto is about 0.002 that of the Earth.



certain distributions. They also entail ideas about the compositions of the planets. In particular, if the planets condensed from a hot gas, then the different materials would condense in order of their boiling points, those with the highest points condensing first and so forming the innermost parts of the planets, followed by other materials in succession and leading to zoned structures for the planets. Quite detailed predictions of such sequences have been made, for the thermodynamic properties of materials that may form planets are known in some detail. Thus, if a mechanism for the formation of the planets is postulated, it may be possible to show that it entails a certain internal structure and a certain thermal history. Such an approach has been widely followed in much recent discussion, but it is not adopted here. The fact is that the origin of the solar system remains most obscure and while it may be plausible it is also surely hazardous to base our ideas of the internal structure of the planets on theories which of their nature do not admit of empirical verification.

A different approach is taken in this book. We start with the known dynamical properties of a planet, the size, mass, density and gravitational field, and ask what they imply for the internal distribution of density, and combine that with our knowledge of the properties of likely planetary materials to derive possible models of the internal structures. It may then be possible to make useful comparisons between the models derived in this way and those derived from theories of the origin of the solar system. No more will therefore be said about the origin of the planets until the final chapter when we look at the types of structure to which we shall have been led, and ask if they tell us anything significant about the origin of the planets. In brief, we are going to try to work back from observations of the planets, through models of their interiors, to criticisms of theories of origin, in contrast to going from theories of origin to models of structure which are constrained to fit the observed properties of the planets.

Seismological studies have enabled the internal structure of the Earth to be worked out in great detail, so that we know the density and elastic moduli as functions of pressure from the surface almost to the centre (Chapter 2). We have, in fact, empirical equations of state for the major constituents of the Earth and, by comparing them with equations of state found experimentally in the laboratory, it is possible to identify the chemical constituents of the Earth with some assurance. For no other body is that possible and in consequence other, indirect, evidence must be drawn upon to suggest the nature of the materials of the other planets. The densities of the major planets are so low that they must be composed largely of hydrogen and helium. Table 1.2 shows the densities of the condensed phases of the lighter elements at a pressure of 10^5 Pa,† together with their abundances in the solar system. Evidently the only abundant elements that could form the major planets are hydrogen and helium. The densities of the planets are of course much greater than the densities of hydrogen and helium at one atmosphere, but the pressures in the interiors of those planets are of the order of 10^{12} Pa or more and the compression of hydrogen and helium under such pressures is sufficient (Chapter 7) to account for the mean density of Jupiter and all but account for that of Saturn. Uranus and Neptune, with their greater mean densities but smaller size and so lower pressures, must have heavier elements, such as carbon, nitrogen and oxygen, mixed with the hydrogen and helium.

The increase of density through self-compression is much less in the smaller terrestrial planets, in which the greatest pressure is 3×10^{11} Pa at the centre of the Earth. Accordingly we must suppose that those planets are composed of materials with densities of 3000 kg/m^3 or more at 10^5 Pa. There are three sources of evidence for the nature of such material. First, the surfaces of the Earth and the Moon are composed of silicates of aluminium, magnesium, sodium and iron and similar materials, having densities in the range of about 2500 to 3000 kg/m^3. Secondly, meteorites are composed of similar materials together with free iron and nickel with densities of about 7000 kg/m^3 (Table 1.3). Thirdly, there is the evidence of the internal structure of the Earth. Comparisons between the empirical equations of state of the different zones of the Earth and equations of state determined in the laboratory indicate that the outer zones of the Earth are composed predominantly of silicates of iron,

Table 1.2. *Densities of the condensed phases of the lightest elements at 10^5 Pa*

Element	Solar system abundance (Hydrogen = 1)	Density (kg/m^3)
Hydrogen	1	89
Helium	10^{-1}	120
Lithium	2×10^{-9}	533
Beryllium	2.5×10^{-11}	1846
Boron	2.5×10^{-10}	2030
Carbon	5×10^{-4}	2266

† SI units are used throughout this book. 1 Pa (pascal) = 10^{-5} bar ~ 10^{-5} atmospheres.

magnesium and aluminium and possibly the oxides of iron, magnesium, aluminium and silicon at the greater depths, whilst the inner zone, the core, is composed mainly of iron diluted with some lighter material such as sulphur.

In all the compact bodies of the Universe, the inward attraction of their own gravitation would lead to condensation to an exceedingly high density were it not balanced by some pressure. Stars are hot and the balancing pressure is the radiation pressure of the thermal radiation flowing outwards. White dwarf stars have cooled down so that the radiation pressure is inadequate to balance self-gravitation and in consequence the density of the material has increased until the pressure of the degenerate gas of electrons in the ionized material balances the self-gravitation. Since the pressure depends little on temperature in a degenerate gas, white dwarfs may be considered to be cold. The pressures and temperatures in planets are much less than in stars, whether hot or white dwarfs, and the self-gravitation is balanced by the forces in solids and liquids which prevent their collapse. If the materials are unionized, the forces are the Coulomb forces between electrical charges in crystals; if the materials are ionized and metallic, the forces are those corresponding to the kinetic energy of the conduction electrons and the potential energy of the electrons in the field of the positive ions. In each case, the density is determined by the balance between internal repulsive forces, on the one hand, and internal attractive forces and external pressure, on the other; the repulsive forces arise mainly from the effect of the Pauli exclusion principle on states of electrons and from the kinetic energy of the

Table 1.3. *Composition of the surface of the Earth and chondritic meteorites*

	Crust of Earth	Average chondrite
SiO_2	0.587	0.380
MgO	0.049	0.238
FeO	0.052	0.124
Fe	—	0.188
FeS	—	0.057
Al_2O_3	0.150	0.025
CaO	0.067	0.020
Ni	—	0.013
Na_2O	0.031	0.010
K_2O	0.023	0.002
Fe_2O_3	0.023	—

electrons in metals. None of these forces depends greatly on temperature and so the planets may be considered to be cold bodies. Thermal vibrations lead of course, according to Debye theory, to additional energy and thermal expansion, but the coefficient of thermal expansion decreases rapidly with increase of pressure, so that to a high degree of approximation the densities of materials in the interiors of planets may be calculated as if they were at the absolute zero of temperature.

Let V be the specific volume of a substance, p be the pressure and T the temperature. Then

$$\frac{\partial}{\partial p}\left(\frac{\partial V}{\partial T}\right)=\frac{\partial}{\partial T}\left(\frac{\partial V}{\partial p}\right).$$

But $\partial V/\partial T = \alpha V$, where α is the coefficient of volume thermal expansion, and

$$\frac{\partial V}{\partial p}=-\frac{V}{K}$$

where K is the bulk modulus. Thus

$$\frac{\partial}{\partial p}(\alpha V)=-\frac{\partial}{\partial T}\left(\frac{V}{K}\right)$$

or

$$V\frac{\partial \alpha}{\partial p}-\alpha\frac{V}{K}=-\frac{1}{K}\alpha V+\frac{V}{K^2}\frac{\partial K}{\partial T},$$

that is

$$\frac{\partial \alpha}{\partial p}=\frac{1}{K^2}\frac{\partial K}{\partial T}.$$

The bulk modulus for many materials follows the approximate rule

$$K=K_0+bp,$$

where K_0 is about 3×10^{11} Pa and b about 2; $\partial K/\partial T$ is about -1.5×10^{7} Pa/deg for olivine. Thus, we find $\partial\alpha/\partial p=-1.7\times10^{-16}$/deg Pa at atmospheric pressure and somewhat less at 10^{11} Pa.

Thus the change of α over a range of 10^{11} Pa is -1.7×10^{-5}. But the value of α is about 10^{-5}; it may be inferred that α is negligible at pressures of the order of 5×10^{10} Pa or more.

1.3 Problems of inference

In all studies of the interiors of the Earth and the planets we are faced with having to derive the properties of the interior from observations made at the surface. In mathematical language, we wish to find a distribution of some property, density for example, as a function of radius. What we observe at the surface is some functional of the desired function; for example, the mass, which is equal to

$$\int_{\mathrm{T}} \rho(r)\,\mathrm{d}\tau,$$

where $\mathrm{d}\tau$ denotes the element of volume and T the volume of the planet; or the moment of inertia, which is equal to

$$\tfrac{2}{3}\int_{\mathrm{T}} r^2\rho(r)\,\mathrm{d}\tau.$$

If we are concerned to determine the elastic moduli as functions of radius, then more complex functionals are involved, namely the times of travel of elastic waves from one point on the surface to another, or the periods of various modes of free oscillation of the planet.

The determination of the desired functions from the observed functionals is the key problem of geophysics. It is of the essence of the subject. Quite generally the number of functionals that can be observed is finite and so the detail with which the functions can be estimated is limited. The study of the optimum ways of determining the unknown functions is known as *inverse theory* (Backus, 1970*a*, *b*, *c*; Backus and Gilbert, 1967, 1968, 1970) and has revolutionized our understanding of what can be learnt about the interior of the Earth from observations at the surface.

Seismology, carried out in a systematic way with a worldwide network of instruments and frequent large natural earthquakes, has provided an immense quantity of data from which quite detailed knowledge of the interior of the Earth has been derived. Comparable data are lacking for any of the other planets and are very inadequately represented for the Moon. We are therefore faced with the problem of trying to learn what we may about the planets from the values of two functionals, at the most, namely the mass and moment of inertia, which can be obtained by dynamical analysis without landing space craft on the planet. It may be expected that with such a dearth of information little can be learnt of the interior. The problem has been considered by Parker (1972), who has shown how certain limits may be placed on the models that may be constructed.

Given just the radius, mass and moment of inertia of a planet, the number of parameters by which a model may be characterized is limited. Essentially there are two types of model we may use: one in which the planet is divided into two zones and in which we attempt to determine the radius of the division between them, together with the densities in the two zones; or the other in which there is a continuous distribution of density specified by two parameters. The former is appropriate to the terrestrial planets, for we know the Earth is divided into two major zones, while the density in any zone changes relatively little with pressure. Of course the model is only a first approximation, for, with only the mass and moment of inertia to go on, we cannot determine finer subdivisions nor variations of density within zones. The second type of model may be more suitable for the major planets if, as suggested above, they are composed predominantly of a substance of uniform composition.

The simple models are still indeterminate if only the radius, mass and moment of inertia are given. Consider the model with two zones. Let the radius of the planet be a and the radius of the inner zone be a_1. Let α be the ratio a_1/a.

Let the densities of the inner and outer zones be respectively ρ_2 and ρ_1 and let the mean density of the planet be $\bar{\rho}$. We then have for the mass

$$\tfrac{4}{3}\pi a_1^3\rho_2+\tfrac{4}{3}\pi(a^3-a_1^3)\rho_1=M=\tfrac{4}{3}\pi a^3\bar{\rho}.$$

If we let $\rho_2/\bar{\rho}=\sigma_2$, and $\rho_1/\bar{\rho}=\sigma_1$, this may be written

$$\alpha^3\sigma_2+(1-\alpha^3)\sigma_1=1. \qquad (1.1)$$

We suppose we also know the mean moment of inertia, I. Then

$$\tfrac{8}{15}\pi a_1^5\rho_2+\tfrac{8}{15}\pi(a^5-a_1^5)\rho_1=I.$$

Let us define an *inertial mean density*, $\gamma\bar{\rho}$, by the relation $I=\frac{8}{15}\pi a^5\gamma\bar{\rho}$. Note that

$$\frac{I}{Ma^2}=\tfrac{2}{5}\gamma$$

and that, for a body of uniform density, $\gamma=1$.

We then have the second equation in the form

$$\alpha^5\sigma_2+(1-\alpha^5)\sigma_1=\gamma. \qquad (1.2)$$

It is evident from the form of these equations that given γ from observation the most we can do is obtain a relation between σ_1, σ_2 and α; if a value of a_1, for example, is chosen, ρ_1 and ρ_2 are determined. There are, however, some limits on the range of variables, as is discussed in Appendix 1.

Evidently, if we wish to select a particular model as in some sense the preferred one, we must have some *a priori* principles on which to make