PART I

Hydrodynamics of two-phase flows

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The phenomenon of two-phase flows

In Part I we will be concerned exclusively with the fluid dynamics of two-phase flows. A *two-phase flow* is one in which we have dynamic (and sometimes chemical) reactions between two phases or components in a flowing system (e.g., liquid–liquid, liquid–gas, liquid–solid particles, gas–solid particles). Sometimes the two phases consist of the same chemical substance, as in distillation equipment, and sometimes the two phases are of unrelated chemical substances, as for dust particles in air. Flows with two distinct substances are often designated as "two-component flows," to distinguish flows of two phases of a single substance, but this distinction is not of great significance in many flow systems, where the reactions are only of a fluid-dynamical nature.

Theoretical and experimental studies of two-phase flows are becoming increasingly important because of their widespread applications in industry. This relevance is being given a great stimulus by the expanding needs of modern industrial societies for energy supplies from various sources. The application of two-phase-flow research to problems in the petrochemical industries is clear – one only has to consider such systems as boilers, evaporators, distillation towers, and turbines. Transportation of and extraction of the products of oil are other obvious applications. As we move toward such alternative energy sources as coal gasification, nuclear energy, and solar energy, new applications of two-phase-flow technology will become ever more important. Quite apart from industries concerned with the development of energy supplies, two-phase flows occur in such varied industries as food processing, paper manufacturing, and steel manufacture.

Finally, we can find many applications outside industry. Rain (and rain-making), snow, dust storms, and fog and cloud formation all involve interactions of two phases. Bioengineering, from the study of blood flow to the inhalation of air-suspended particulate matter, finds a need for understanding of two-phase-flow phenomena. It is thus abundantly clear that the subject of this book is not without practical importance.

We now define two of the more important quantities that can be used to characterize two-phase flows, or quantities that we may wish to be able to determine or predict, for any such given flow system.

The *void fraction*, ϕ, is defined as the ratio of the volume of gas to the total volume of gas and liquid in a flow (or section of it):

$$\phi = \frac{\text{volume of gas}}{\text{total volume}}$$

Note that the fractional volume of liquid is then $(1 - \phi)$.

The *superficial gas velocity*, v_s, is the ratio of the gas volumetric flow rate, at a given flow cross section, to that cross-sectional area:

$$v_s = Q/A$$

where Q is the gas flow rate and A the local cross-sectional area. In subsequent chapters, methods will be developed to enable these two quantities to be determined, as well as the relationships between them, for various flow conditions, such as gas and liquid densities, viscosities, surface tension and flow rates, and system geometries. The chapters in Part I are concerned primarily with flows in which we have a liquid-containing chamber through which a gas is being forced. However, the results of the various approaches adopted in the study of this kind of flow often have immediate application to the more general (and common) case of the flow of a liquid–gas mixture in a chamber or a duct. Further, the dispersed flow of solid particles in a fluid is considered in Part I, and this will also be found to have wide application.

Traditionally, in the study of two-phase-flow systems, because of the great complexity of such flows compared to single-phase flows, assumptions are made that break the research down into three broad areas of approach. In one approach the two-phase flow is decomposed into what is hoped to be a representative single-phase flow, in which the fictitious fluid's properties are defined in such a way as to maximize the effectiveness of the representation. This is the simplest approach to the problem and, in its simplicity, cannot be expected to furnish results on the detailed behavior of each phase. It is at best a crude approximation to the actual flow.

In "separated" flow models, fluid-dynamic equations are developed separately for each phase. These equations can then be combined to describe the total flow, or boundary conditions can be assumed between the two phases to couple the two sets of equations. The total number of equations will be 12; for each phase there will be three scalar momentum equations, a mass conservation equation, an energy equation, and an equation of state. This corresponds to the 12 unknowns: the gas void fraction, the velocity vectors, temperatures and densities of each phase, and the pressure. Obviously, without the use of many simplifications, this general problem is practically intractable, but with suitable simplifications, this approach has been more productive of results than has the simpler approach described above, and Part I tackles two-phase-flow problems by various techniques with this basic mode of description.

The third, and most phenomenological approach breaks the analysis of two-phase flows down to several flow regimes. Some regimes are almost continuous with others, whereas between other regimes there is a discrete, pronounced structural change.

We adopt a description given elsewhere,[1] which breaks the flow regimes down in terms of the actual structure existing between the two phases. This description is therefore distinctly different from corresponding regime characterizations in single-phase flows. In single-phase flows, theoretical and experimental investigations have led to an ability to characterize flows in terms of nondimensional factors such as the Reynolds number and the Prandtl number. In two-phase flows we cannot, at the present degree of knowledge, so characterize regimes. Instead, we can only define flow regimes in terms of actual resultant flows: there are too many operative factors to be able to predict, with accuracy, which of these regimes will occur in a given flow system. This description of regimes is therefore necessarily of a qualitative, subjective nature.

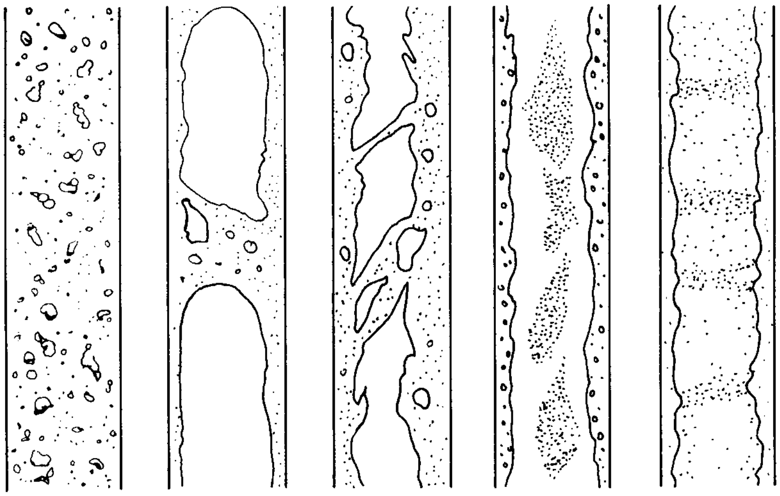
We describe first flow in a vertical direction in a pipe or duct. Perhaps the most obvious example of a two-phase flow is that of gas bubbles in a flowing liquid (see Figure 1.1). In a *bubbly regime,* the bubbles may be small and spherical, or larger, with their shapes nonspherical because of the flow of liquid around them. As the flow rates of the gas and liquid are increased, we develop *slug flow.* Here the gas elements are large enough to almost span the duct diameter, thus creating discrete "slugs" of liquid, connected by liquid flowing along the walls of the duct. Under certain conditions slug flow can make a transition into *churn flow,* where the flow is much more irregular and disturbed, primarily as a result of the breakdown of the gas bubbles. Some sources designate this flow as "slug-annular flow," because of oscillations between the slug-flow condition and annular-like flow, in which the central region of the flow is virtually liquid-free.

In *wispy annular flow,* the central region of the flow is gaseous except for "wisps" of liquid droplets bunched into discrete groupings. Along the walls of the duct we find bubble-impregnated liquid flow. Wisp-annular flow usually occurs at high mass-flow rates, and this explains why it has only recently been distinguished as a distinct flow regime.[2] Finally, in *annular flow,* the central region is gas-dominated, with sparse liquid droplet entrainment from the liquid lying on the walls of the duct. There is no agglomeration of droplets in the central region as in wispy-annular flow.

As earlier described, there is no satisfactory way of predicting which of these regimes will exist in a given system, primarily because so many factors are involved. For example, not only do the physical properties of the phases have an effect, but such things as the mode of gas injection and duct geometry can also have pronounced effects. (It is worth noting that these regimes, in the order described above, have been found to be generated along the direction of flow in a heated duct. The flow is initially wholly liquid, but as heat is absorbed, vapor bubbles appear which expand into slug flow and annular flow, finally becoming a single-phase gas flow.)

However, even with this qualification, attempts have been made to develop flow-pattern maps for specific situations when most parameters are kept constant, resultant regime changes then being a function of fewer parameters. An example is shown in Figure 1.2 for low-pressure air–water and high-pressure steam–water flows in small-diameter (1 to 3 cm, 0.4 to 1.2 in.) vertical tubes.[3] The parameters in which the regimes are mapped are the gas and liquid superficial momentum fluxes, $\rho_g v_s^2$ and $\rho_f v_{s_f}^2$, respectively. Here ρ_g and ρ_f are the gas and liquid densities and v_s and v_{s_f} are the corresponding local superficial velocities. It should be stressed that the boundaries between regimes are very approximate.

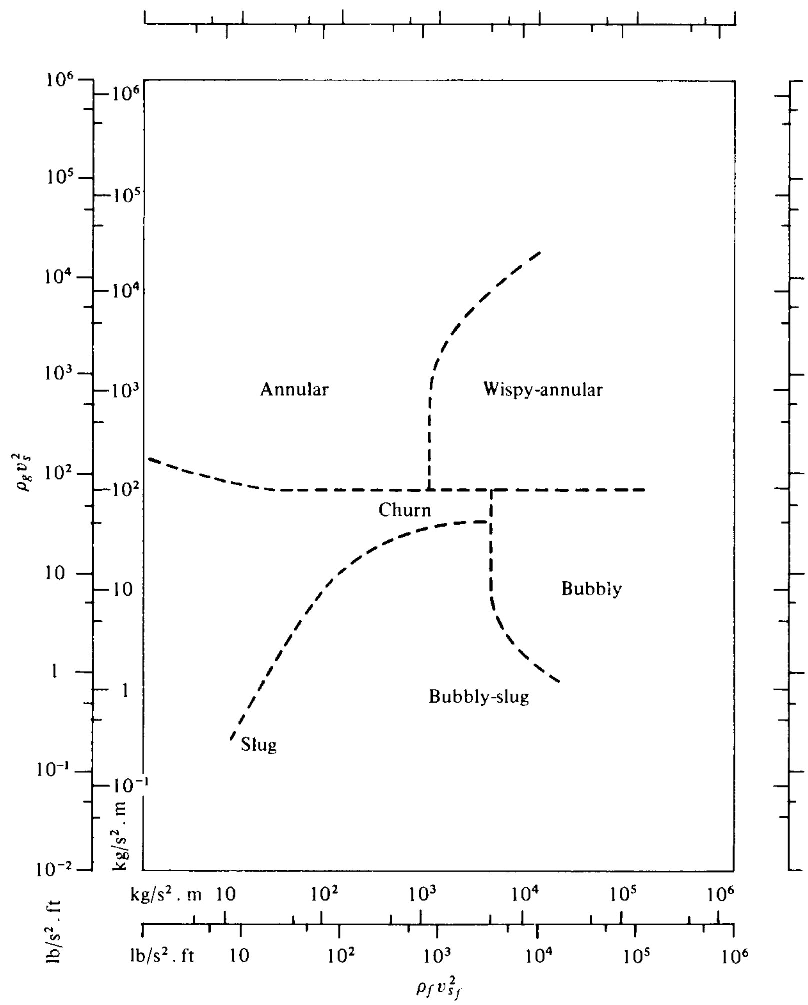
Figure 1.1 Flow patterns in vertical flow. (From J. G. Collier, *Convective Boiling and Condensation,* McGraw-Hill, New York, 1972.)



Although the transition from one regime to another is ill-defined, owing to the subjective visual classification, attempts have been made to determine criteria for predicting the transition points from one regime to another.

Bubbly flows begin to make a transition to slug flow as the likelihood of collision (and thus coalescence) of bubbles increases. Clearly the void fraction ϕ is a significant determinant here, and it has been found[4] that for a void fraction of less than about 0.1, the collision frequency of bubbles is low; hence there is not enough bubble coagulation to generate slug flow. As the

Figure 1.2 Flow pattern map for vertical flow. (From J. G. Collier, *Convective Boiling and Condensation,* McGraw-Hill, New York, 1972.)

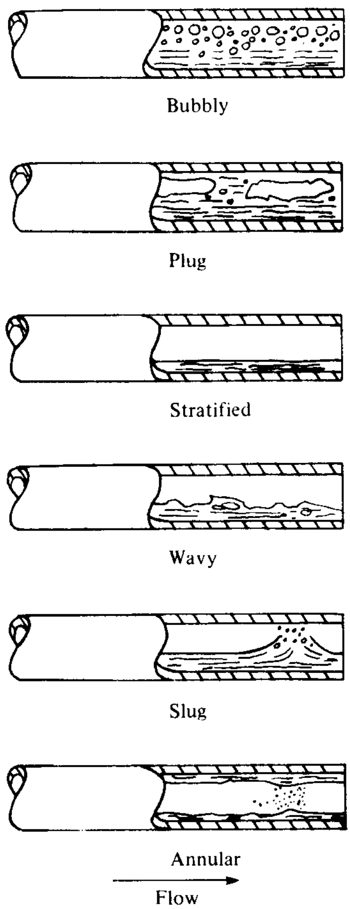


void fraction is increased, we find that for $\phi \simeq 0.3$, the likelihood is great that slug flow will be created.

As the void fraction is further increased, the slug flow begins to break down into *churn flow* at some value of ϕ less than about 0.8. Interactions between the rising gas bubbles and the falling liquid on the walls of the duct or pipe lead to instabilities, generating churn flow as the large bubbles begin to break up. A semiempirical theory has been developed[5] which yields the following approximate criterion for the establishment of churn flow:

$$v_s = 0.105\left[\frac{gD(\rho_f - \rho_g)}{\rho_g}\right]^{1/2}$$

Figure 1.3 Flow patterns in horizontal flow. (From J. G. Collier, *Convective Boiling and Condensation,* McGraw-Hill, New York, 1972.)



where D is the pipe diameter and ρ_f and ρ_g are the liquid and gas densities, respectively.

It is easiest to describe the transition from churn flow to annular flow by considering the effect of the flowing gas on the direction of flow of the liquid on the walls of the pipe.[1] There is a critical gas flow rate for which an increase ensures that the liquid flows upward with the gas and for which a decrease allows the liquid to fall under gravitational force. This *flow-reversal point* has been found to be determined by the following formula:

$$v_s = 0.9\left[\frac{gD(\rho_f - \rho_g)}{\rho_g}\right]^{1/2}$$

This can also be used as an approximate criterion for the churn flow–annular flow transition.[1]

We have concentrated our discussion on upward vertical flows. However, similar results have been found for flows in horizontal pipes or ducts. Given the very approximate nature of the mapping in Figure 1.2, it is known that this mapping also describes horizontal flow transitions. The flow regimes in horizontal flows are slightly different from those in the vertical flow, as can be seen in Figure 1.3.[6] The correspondences between the two flows are quite obvious.

In the following chapters we develop several different (and often very simple) approaches to the types of problems generated in an understanding of the flows we have described. Some of the techniques have been developed by other authors; the remainder constitute new and novel approaches toward obtaining simple, but realistic formulas and conclusions describing two-phase flows.

References

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Single-bubble formation

2.1 Introduction

In the study of the dispersion of gas in a liquid, it is not usually necessary to consider the effect of molecular diffusion on the mixing process. This is because in most practical applications of the two-phase dispersion process, other dynamical effects are present that play a dominant role in mixing. Several such possible dispersion mechanisms are considered in this chapter.

A simple example of a mixing process is one in which gas is forced through an orifice (or orifices) submerged in a liquid (Figure 2.1). Depending on the gas flow rate, the gas will exit from the orifice as individual bubbles (low flow rate) or continuously as a jet (high flow rate). The jet subsequently breaks up into bubbles of various sizes. In this chapter we consider the low-flow-rate bubble-formation phenomenon for the simplified case of a single orifice. Also, we take the gas flow rate through the orifice into the liquid to be constant.

There have been numerous studies on bubble formation from a single orifice submerged in a liquid.[1–29] However, the results obtained in these studies, especially as far as the development of a general theory for bubble size for a two-phase system is concerned, have been inconclusive. Therefore, the need arises for a theoretical investigation of the phenomenon to gain some understanding of the experimental data that have been obtained.

2.2 Buoyancy and surface tension alone

We start with a consideration of the forces acting on each bubble as it forms at the orifice. Each bubble is acted upon by buoyancy, convection currents in the fluid, and by the surface-tension force acting on that section of the bubble that is still in contact with the orifice (Figure 2.2).

For the case of low gas flow, and when the viscosity of the liquid is small and convection is negligible, we can say that the buoyancy and surface-tension forces approximately balance, so that

$$\tfrac{1}{6}\pi d_b^3 g(\rho_f - \rho_g) = \pi D_o \sigma \qquad (2.1)$$

where d_b is the diameter of the bubble at the instant of release; ρ_f and ρ_g the densities of the liquid and gas, respectively; D_o the diameter of the orifice; σ the surface tension; and g the acceleration due to gravity. Note that the factor cos θ (where θ is the angle between the perpendicular and the bubble surface at the orifice), which is usually included on the right-hand side of equation (2.1), has been set equal to unity for simplicity. This gives us

$$d_b = \left[\frac{6\sigma D_o}{(\rho_f - \rho_g)g}\right]^{1/3}$$

The usefulness of this equation is confirmed by experiment for $\mathrm{Re}_o \leq 200$, where $\mathrm{Re}_o = \rho_f v_o D_o/\mu_f$ is the Reynolds number based on the orifice diameter. Here μ_f is the dynamic viscosity of the liquid and v_o the mean velocity of the gas at the orifice. For instance, if we take air (ρ_g = 1.2 kg/m^3, 7.49 × 10^{-2} lb/ft^3) bubbling into water (ρ_f = 10^3 kg/m^3, 62.4 lb/ft^3, σ = 7.36 × 10^{-2} N/m, 4.2 × 10^{-4} lbf/in.) through an orifice 1.3 mm (0.05 in.) in diameter, we find that the diameter of the bubbles will be, according

Figure 2.1 Example of a gas–liquid system.