

Cambridge University Press

978-0-521-10412-8 - The Representation Theory of the Symmetric Group

Gordon James and Adalbert Kerber

Table of Contents

[More information](#)

# Contents

<b>Editor's Statement</b> . . . . .	<b>xi</b>
<b>Section Editor's Foreword</b> . . . . .	<b>xiii</b>
<b>Introduction by G. de B. Robinson</b> . . . . .	<b>xvii</b>
<b>Preface</b> . . . . .	<b>xxi</b>
<b>List of Symbols</b> . . . . .	<b>xxiii</b>
<b>Chapter 1 Symmetric Groups and Their Young Subgroups</b> . . . . .	<b>1</b>
1.1 Symmetric and Alternating Groups . . . . .	1
1.2 The Conjugacy Classes of Symmetric and Alternating Groups . . . . .	8
1.3 Young Subgroups of $S_n$ and Their Double Cosets . . . . .	15
1.4 The Diagram Lattice . . . . .	21
1.5 Young Subgroups as Horizontal and Vertical Groups of Young Tableaux . . . . .	29
Exercises . . . . .	33
<b>Chapter 2 Ordinary Irreducible Representations and Characters of Symmetric and Alternating Groups</b> . . . . .	<b>34</b>
2.1 The Ordinary Irreducible Representations of $S_n$ . . . . .	34
2.2 The Permutation Characters Induced by Young Subgroups . . . . .	38
2.3 The Ordinary Irreducible Characters as $\mathbb{Z}$ -linear Combinations of Permutation Characters . . . . .	45
2.4 A Recursion Formula for the Irreducible Characters . . . . .	58
2.5 Ordinary Irreducible Representations and Characters of $A_n$ . . . . .	65
2.6 $S_n$ is Characterized by its Character Table . . . . .	72
2.7 Cores and Quotients of Partitions . . . . .	75
2.8 Young's Rule and the Littlewood-Richardson Rule . . . . .	87
2.9 Inner Tensor Products . . . . .	95
Exercises . . . . .	100
<b>Chapter 3 Ordinary Irreducible Matrix Representations of Symmetric Groups</b> . . . . .	<b>101</b>
3.1 A Decomposition of the Group Algebra $\mathbb{Q}S_n$ into Minimal Left Ideals . . . . .	101
3.2 The Seminormal Basis of $\mathbb{Q}S_n$ . . . . .	109
3.3 The Representing Matrices . . . . .	115

Cambridge University Press

978-0-521-10412-8 - The Representation Theory of the Symmetric Group

Gordon James and Adalbert Kerber

Table of Contents

[More information](#)

viii

Contents

3.4	The Orthogonal and the Natural Form of $[\alpha]$ . . . . .	126
	Exercises . . . . .	131
<b>Chapter 4</b>	<b>Representations of Wreath Products . . . . .</b>	<b>132</b>
4.1	Wreath Products . . . . .	132
4.2	The Conjugacy Classes of $G \text{ wr } S_n$ . . . . .	138
4.3	Representations of Wreath Products over Algebraically Closed Fields . . . . .	146
4.4	Special Cases and Properties of Representations of Wreath Products . . . . .	155
	Exercises . . . . .	161
<b>Chapter 5</b>	<b>Applications to Combinatorics and Representation Theory . . . . .</b>	<b>162</b>
5.1	The Pólya Theory of Enumeration . . . . .	163
5.2	Symmetrization of Representations . . . . .	184
5.3	Permutrization of Representations . . . . .	202
5.4	Plethysms of Representations . . . . .	218
5.5	Multiply Transitive Groups . . . . .	227
	Exercises . . . . .	237
<b>Chapter 6</b>	<b>Modular Representations . . . . .</b>	<b>240</b>
6.1	The $p$ -block Structure of the Ordinary Irreducibles of $S_n$ and $A_n$ ; Generalized Decomposition Numbers . . . . .	240
6.2	The Dimensions of a $p$ -block; $u$ -numbers; Defect Groups . . . . .	254
6.3	Techniques for Finding Decomposition Matrices . . . . .	265
	Exercises . . . . .	292
<b>Chapter 7</b>	<b>Representation Theory of <math>S_n</math> over an Arbitrary Field . . . . .</b>	<b>294</b>
7.1	Specht Modules . . . . .	294
7.2	The Standard Basis of the Specht Module . . . . .	301
7.3	On the Role of Hook Lengths . . . . .	306
	Exercises . . . . .	318
<b>Chapter 8</b>	<b>Representations of General Linear Groups . . . . .</b>	<b>319</b>
8.1	Weyl Modules . . . . .	320
8.2	The Hyperalgebra . . . . .	327
8.3	Irreducible $GL(m, F)$ -modules over $F$ . . . . .	334
8.4	Further Connections between Specht and Weyl Modules . . . . .	341
	Exercises . . . . .	346

Cambridge University Press

978-0-521-10412-8 - The Representation Theory of the Symmetric Group

Gordon James and Adalbert Kerber

Table of Contents

[More information](#)

Contents	ix
Appendix I: Tables	348
I.A Character Tables	348
I.B Class Multiplication Coefficients	356
I.C Representing Matrices	368
I.D Decompositions of Symmetrizations and Permutrizations	380
I.E Decomposition Numbers	413
I.F Irreducible Brauer Characters	430
I.G Littlewood-Richardson Coefficients	436
I.H Character Tables of Wreath Products of Symmetric Groups	442
I.I Decompositions of Inner Tensor Products	451
Appendix II: Notes and References	459
II.A Books and Lecture Notes	459
II.B Comments on the Chapters	460
II.C Suggestions for Further Reading	468
II.D References	468
Index	507