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978-0-521-10243-8 - Measurement Theory: With Applications to Decisionmaking, Utility,
and the Social Sciences

Fred S. Roberts

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