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978-0-521-10180-6 - Dependence with Complete Connections and its Applications

Marius Iosifescu and Serban Grigorescu

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