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Marius Iosifescu and Serban Grigorescu

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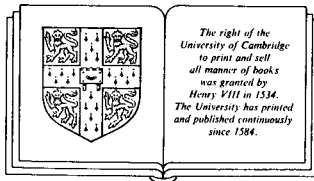
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*Dependence with complete
connections and its
applications*



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Frontmatter

[More information](#)

The original hardback edition of 1990 was dedicated to the memory of Octav Onicescu (1892-1983) and Gheorghe Mihoc (1906-1981), co-founders of the subject. This new paperback edition is published in memory of my co-author and friend Serban Grigorescu (1946-1997) and of our former colleague Adriana Berechet (1939-2006) who took a great interest in our book and suggested several corrections included in the present edition.

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Frontmatter

[More information](#)

Contents

<i>Preface</i>	xi
<i>Abbreviations and notation</i>	xiii
<i>Introduction</i>	1
1 Fundamental notions	5
1.1 The concept of a random system with complete connections	5
1.1.1 The homogeneous case	5
1.1.2 The existence theorem	6
1.1.3 The non-homogeneous case	10
1.1.4 Generalized systems	12
1.2 Examples	15
1.3 Classification problems	31
1.3.1 A communication relation	31
1.3.2 Random systems with complete connections of type (B)	34
Problems	36
2 Ergodicity	38
2.1 Implications of ergodicity	38
2.1.1 Preliminaries	38
2.1.2 A special ergodic theorem	40
2.1.3 A fundamental theorem	43
2.2 The homogeneous case	46
2.2.1 Preliminaries	46
2.2.2 Condition FLS (A_0, ν)	47
2.2.3 Necessary and sufficient conditions for uniform ergodicity	51
2.2.4 A sufficient condition for ergodicity	54
2.3 The non-homogeneous case	55
2.3.1 Weak-ergodicity	55
2.3.2 Strong-ergodicity	58
2.4 An application to the associated Markov chain	60
Problems	64
3 The associated Markov chain	68
3.1 Associated operators	68

Cambridge University Press

978-0-521-10180-6 - Dependence with Complete Connections and its Applications

Marius Iosifescu and Serban Grigorescu

Frontmatter

[More information](#)

viii

Contents

3.1.1 Preliminaries	68
3.1.2 Doeblin's condition and (quasi-) compact operators	72
3.1.3 Doeblin–Fortet operators	76
3.1.4 Markov operators on $C(W)$	86
3.2 Compact Markov chains	93
3.3 Continuous Markov chains	99
3.3.1 Preliminaries	99
3.3.2 Ergodic kernels and transience	101
3.3.3 The case of one ergodic kernel	105
3.3.4 The general case (several ergodic kernels)	110
3.3.5 Subergodic decomposition, ergodicity, regularity	114
3.3.6 The structure of the tail σ -algebra	118
3.4 Applications to ergodicity	122
Problems	126
4 Asymptotic behaviour	132
4.1 Limit theorems for the infinite order chain	132
4.1.1 Preliminaries	132
4.1.2 The law of large numbers	133
4.1.3 The functional central limit theorem	134
4.1.4 The central limit theorem with remainder	141
4.1.5 The law of the iterated logarithm	143
4.1.6 Some non-parametric statistics	144
4.2 Limit theorems for the associated Markov chain	146
4.2.1 Preliminaries	146
4.2.2 The law of large numbers	147
4.2.3 The central limit theorem with remainder	148
4.2.4 An invariance principle for the central limit theorem and the law of the iterated logarithm	155
Problems	156
5 Some special systems	160
5.1 OM chains	160
5.1.1 Preliminaries	160
5.1.2 Ergodicity	162
5.1.3 Alternate linear OM chains	169
5.1.4 The case of an arbitrary state space	173
5.2 The continued fraction expansion	174
5.2.1 Preliminaries	174
5.2.2 Gauss' problem: Lévy's approach	179
5.2.3 Gauss' problem: Kuzmin's approach	183
5.2.4 Limit theorems	186
5.2.5 Generalizations	189
5.3 Piecewise monotonic transformations	190

Cambridge University Press

978-0-521-10180-6 - Dependence with Complete Connections and its Applications

Marius Iosifescu and Serban Grigorescu

Frontmatter

[More information](#)

<i>Contents</i>	ix
5.3.1 Preliminaries	190
5.3.2 The invariant probability: existence and uniqueness	195
5.3.3 The Frobenius–Perron operator	197
5.3.4 The invariant probability: differential properties of its density	205
5.3.5 Limit theorems	209
5.3.6 A few final remarks	214
5.4 f -expansions	215
5.4.1 Preliminaries	215
5.4.2 Gauss’ problem and mixing properties	218
5.4.3 A conjecture	222
5.4.4 The D -ary expansion	222
5.5 Strict-sense infinite-order chains	225
5.5.1 Preliminaries	225
5.5.2 An existence theorem	226
5.5.3 The case of a countable state space	228
5.5.4 The case of a finite state space	232
Problems	234
Appendix 1 Spaces, measures and functions	241
Appendix 2 Notions of functional analysis	256
Appendix 3 Mixing and Markovian dependence	268
<i>Notes and comments</i>	277
<i>Reference</i>	284
<i>Index</i>	303

Cambridge University Press

978-0-521-10180-6 - Dependence with Complete Connections and its Applications

Marius Iosifescu and Serban Grigorescu

Frontmatter

[More information](#)

Preface

Le moment actuel d'un corps vivant ne trouve pas sa raison d'être dans le moment immédiatement antérieur . . . il faut y joindre tout le passé de l'organisme, son hérédité, enfin l'ensemble d'une très longue histoire.

H. Bergson *L'évolution créatrice*, p. 21. Alcan, Paris, 1907.

The theory of dependence with complete connections was initiated by a note published in 1935 in *Comptes Rendus de l'Académie des Sciences de Paris* by the Romanian mathematicians Octav Onicescu and Gheorghe Mihoc. Intended to be a non-trivial extension of Markovian dependence, in the spirit of the Bergsonian quotation above, during its development this theory was condescendingly accompanied by the view that it does not go much beyond its celebrated Markovian predecessor or even reduces to the latter. We hope that the present work will help to rule out this misunderstanding, showing, in particular, that even the associated Markov chain is of a special type and cannot be brought under established theories for Markov chains.

The Romanian version of this work, published in 1982, was the third monograph devoted to dependence with complete connections following *Processes with Complete Connections* by G. Ciucu and R. Theodorescu (1960) and *Random Processes and Learning* by M. Iosifescu and R. Theodorescu (1969). As a consequence, our 1982 book aimed at being a state-of-the-art survey building on the results obtained during the period 1969–1982. Within that period the applicability of the concept was largely extended, a lot of work was done on the associated Markov chain, the classical limit theorems got a functional form, and the rôle played by dependence with complete connections in the metric theory of continued fractions was entirely clarified. All these results appeared in the five chapters of the 1982 book.

The present book is a thoroughly revised, updated and somewhat expanded version of our 1982 work. Scarcely a page has escaped either

Cambridge University Press

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Frontmatter

[More information](#)

xii

Preface

amendment, deletion or addition. Moreover, parts of Chapters 3, 4, and 5 have been completely rewritten, and a new section on piecewise monotonic transformations has been included in Chapter 5. This topic, which has undergone an explosive development in recent years, is the main newcomer to the field of dependence with complete connections.

As with the Romanian version, to make the book self-contained, we have included three appendices containing those notions and results from measure theory, functional analysis, and dependent random variables that are basic to the theory of dependence with complete connections.

The concluding notes and comments aimed at tracing the historical developments of the subject have also been revised and updated to suit the new contents.

The bibliography mainly covers the period 1969–1988. For a more complete bibliographic picture the reader should also consult the reference lists of the 1960 and 1969 volumes mentioned above.

The authors hope that the present work will testify both to the importance and vitality of the concept of dependence with complete connections in its sixth decade of existence and to the fact that interesting work is being done and has yet to be done in the field.

Acknowledgements

Much of our original work included in this book was done in the framework of our association with the Centre of Mathematical Statistics of Bucharest under generous financial support extended over years from the National Committee for Science and Technology of Romania.

Thanks are due to our colleagues and friends Joel E. Cohen (New York), Harry Cohn (Melbourne), Allan Gut (Uppsala), Sofia Kalpazidou (Salonika), Bo Henry Lindqvist (Trondheim), Magda Peligrad (Cincinnati), Helmut Pruscha (Munich), and Petre Tăutu (Heidelberg) for invaluable help with bibliographic materials.

We wish to acknowledge the technical help we have received from Sergiu Celac, Rodica Culcer, H el ene Ely, and Adriana Gr adinaru.

We wish to express our deep gratitude to Joel E. Cohen without whose perseverance and interest this English version might not have existed.

Our colleague and friend Gheorghe Popescu read a first version of the manuscript and with his crystal-clear mind detected some inaccuracies and slips. Expressing our great indebtedness to him, we wish to make it clear that any remaining errors are ours.

Finally, we are especially grateful to our wives  tefania Iosifescu and Monica Grigorescu for their constant support and patience.

Bucharest

Marius Iosifescu
Serban Grigorescu

Abbreviations and notation

a.e.	almost everywhere
a.s.	almost surely
GRSCC	generalized random system with complete connections
iff	if and only if
MC	Markov chain
pw.m.t.	piecewise monotonic transformation
RSCC	random system with complete connections
r.v.	random variable
t.p.f.	transition probability function
w.r.t.	with respect to
\mathbb{N}^*	$\{1, 2, \dots, n, \dots\}$
\mathbb{N}	$\{0, 1, 2, \dots, n, \dots\}$
$-\mathbb{N}$	$\{\dots, -n, \dots, -2, -1, 0\}$
\mathbb{Z}	$(-\mathbb{N}) \cup \mathbb{N}^*$
\mathbb{R}	the collection of all real numbers
\mathbb{R}_+	$\{a \in \mathbb{R}: a \geq 0\}$
\mathbb{R}_-	$\{a: a < 0\}$
\mathbb{C}	the collection of all complex numbers
$[a]$	the integer part of $a \in \mathbb{R}_+$
$\{a\}$	the fractionary part of $a \in \mathbb{R}_+$
Φ	standard normal distribution function
χ_A	the indicator function of the set A

$\mathcal{P}(X)$, 241	$\ \cdot\ _L$, 77, 264
$\mathcal{B}, \mathcal{B}_M (M \subset \mathbb{R})$, 241	$ \cdot $, 259
\mathcal{B}_X , 251	$\ \cdot\ _v$, 265
$\sigma((f_i)_{i \in \Lambda})$, 243	$\ \cdot\ _1$, 265
$\ \mu\ $, μ a measure, 244	BV_λ, L_λ^1 , 198
$E(f)$, 248	$\{(W, \mathcal{W}), (X, \mathcal{X}), u, P\}$, 5
$\mathbf{P} \equiv \mathbf{Q}, \mathbf{P} \ll \mathbf{Q}$, 248	P_j^n , 6
\mathbf{B} , 254	\mathbf{P}_j^∞ , 40, 42, 46
$B(W), B(W, \mathcal{W})$, 259	Q^n , 8, 69
$ba(W, \mathcal{W}), ca(W, \mathcal{W})$, 259	Q_n , 71
$ca(W), rca(W)$, 259	\mathbf{Q}^∞ , 86
$C(W), C_r(W)$, 259	$C = C_r([0, 1])$, 254

Cambridge University Press

978-0-521-10180-6 - Dependence with Complete Connections and its Applications

Marius Iosifescu and Serban Grigorescu

Frontmatter

[More information](#)

xiv

Abbreviations and notation

$D = D([0, 1])$, 255	U, U^n , 8, 70
$s(f)$, 265	U_n , 71
$\text{var } f$, 265	U^∞ , 71
$v(f)$, 265	V, V^n , 70
$E(\sigma)$, 261	V_n , 71
$L(W), L_r(W)$, 77	$\{(W, \mathscr{W}), (X, \mathscr{X}), \Pi, P\}$, 12
R_j, r_j , 79	

Numbering Section 1.2 means Section 2 of Chapter 1; Subsection 1.2.3 means Subsection 3 of Section 1.2; (1.2.3) means equation 3 in Section 1.2; Problem 1 means Problem 1 of the current chapter; Section A1.2 means Section 2 of Appendix 1. Smith (1980, p. 1) means page 1 of the item Smith (1980) of the reference list.

Note Positive means > 0 , non-negative means ≥ 0 , denumerable means either finite or countably infinite, countable means countably infinite, \log denotes the natural logarithm.