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Dependence with complete connections and its applications

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The original hardback edition of 1990 was dedicated to the memory of Octav Onicescu (1892-1983) and Gheorghe Mihoc (1906-1981), co-founders of the subject. This new paperback edition is published in memory of my co-author and friend Serban Grigorescu (1946-1997) and of our former colleague Adriana Berechet (1939-2006) who took a great interest in our book and suggested several corrections included in the present edition.

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Preface

Le moment actuel d'un corps vivant ne trouve pas sa raison d'être dans le moment immédiatement anterieur ... il faut y joindre tout le passé de l'organisme, son hérédité, enfin l'ensemble d'une très longue histoire.

H. Bergson *L'évolution créatrice*, p. 21. Alcan, Paris, 1907.

The theory of dependence with complete connections was initiated by a note published in 1935 in *Comptes Rendus de l'Académie des Sciences de Paris* by the Romanian mathematicians Octav Onicescu and Gheorghe Mihoc. Intended to be a non-trivial extension of Markovian dependence, in the spirit of the Bergsonian quotation above, during its development this theory was condescendingly accompanied by the view that it does not go much beyond its celebrated Markovian predecessor or even reduces to the latter. We hope that the present work will help to rule out this misunderstanding, showing, in particular, that even the associated Markov chain is of a special type and cannot be brought under established theories for Markov chains.

The Romanian version of this work, published in 1982, was the third monograph devoted to dependence with complete connections following *Processes with Complete Connections* by G. Ciucu and R. Theodorescu (1960) and *Random Processes and Learning* by M. Iosifescu and R. Theodorescu (1969). As a consequence, our 1982 book aimed at being a state-of-the-art survey building on the results obtained during the period 1969–1982. Within that period the applicability of the concept was largely extended, a lot of work was done on the associated Markov chain, the classical limit theorems got a functional form, and the rôle played by dependence with complete connections in the metric theory of continued fractions was entirely clarified. All these results appeared in the five chapters of the 1982 book.

The present book is a thoroughly revised, updated and somewhat expanded version of our 1982 work. Scarcely a page has escaped either amendment, deletion or addition. Moreover, parts of Chapters 3, 4, and 5 have been completely rewritten, and a new section on piecewise monotonic transformations has been included in Chapter 5. This topic, which has undergone an explosive development in recent years, is the main newcomer to the field of dependence with complete connections.

As with the Romanian version, to make the book self-contained, we have included three appendices containing those notions and results from measure theory, functional analysis, and dependent random variables that are basic to the theory of dependence with complete connections.

The concluding notes and comments aimed at tracing the historical developments of the subject have also been revised and updated to suit the new contents.

The bibliography mainly covers the period 1969–1988. For a more complete bibliographic picture the reader should also consult the reference lists of the 1960 and 1969 volumes mentioned above.

The authors hope that the present work will testify both to the importance and vitality of the concept of dependence with complete connections in its sixth decade of existence and to the fact that interesting work is being done and has yet to be done in the field.

Acknowledgements

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We wish to acknowledge the technical help we have received from Sergiu Celac, Rodica Culcer, Hélène Ely, and Adriana Grădinaru.

We wish to express our deep gratitude to Joel E. Cohen without whose perseverence and interest this English version might not have existed.

Our colleague and friend Gheorghe Popescu read a first version of the manuscript and with his crystal-clear mind detected some inaccuracies and slips. Expressing our great indebtedness to him, we wish to make it clear that any remaining errors are ours.

Finally, we are especially grateful to our wives Ştefania Iosifescu and Monica Grigorescu for their constant support and patience.

Bucharest Marius Iosifescu
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Abbreviations and notation

a.e.	almost everywhere
a.s.	almost surely
GRSCC	generalized random system with complete connections
iff	if and only if
MC	Markov chain
pw.m.t.	piecewise monotonic transformation
RSCC	random system with complete connections
r.v.	random variable
t.p.f.	transition probability function
w.r.t.	with respect to
$\mathbb{N}^*$	$\{1, 2, \ldots, n, \ldots\}$
$\mathbb{N}$	$\{0, 1, 2, \ldots, n, \ldots\}$
$-\mathbb{N}$	$\{\ldots, -n, \ldots, -2, -1, 0\}$
$\mathbb{Z}$	$(-\mathbb{N}) \cup \mathbb{N}^*$
$\mathbb{R}$	the collection of all real numbers
$\mathbb{R}_+$	$\{a \in \mathbb{R}: a \geqslant 0\}$
$\mathbb{R}_-$	$\{a: a < 0\}$
$\mathbb{C}$	the collection of all complex numbers
$[a]$	the integer part of $a \in \mathbb{R}_+$
$\{a\}$	the fractionary part of $a \in \mathbb{R}_+$
Φ	standard normal distribution function
χ_A	the indicator function of the set A

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Numbering Section 1.2 means Section 2 of Chapter 1; Subsection 1.2.3 means Subsection 3 of Section 1.2; (1.2.3) means equation 3 in Section 1.2; Problem 1 means Problem 1 of the current chapter; Section A1.2 means Section 2 of Appendix 1. Smith (1980, p. 1) means page 1 of the item Smith (1980) of the reference list.

Note Positive means >0, non-negative means $\geqslant 0$, denumerable means either finite or countably infinite, countable means countably infinite, log denotes the natural logarithm.