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978-0-521-10152-3 - Continued Fractions: Analytic Theory and Applications

William B. Jones and W. J. Thron

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